Your Name: **Your student ID:**

MATHEMATICAL PHYSICS, Phys 227, Marcel den Nijs

Midterm 1: [100 points total]

Thursday January 27 0:30 -11:20 am.

This an online open book exam. Write your solutions on several sheets of paper.
They needs to be handwritten in your own handwriting. Upload your work into Canvas.
The upload needs to happen before 11:25 am. The upload window is set to close at 11:30 am

Email your work in case the upload fails, or when you loose internet during the test,
or encounter other emergencies.

$$\log(1-x) = \sum_{n=1}^{\infty} \frac{1}{n} x^n \quad ; \quad e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

PROBLEM 1 [15 points]

Find the first 4 terms in the Taylor series expansions of.

$$w(z) = \exp(e^z - 1)$$

evaluated at $z = 0$, using one of the above listed series. Show your derivation

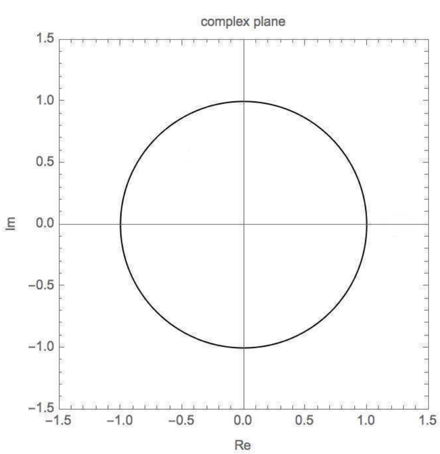
PROBLEM 2 [15 points]

Derive using complex numbers that:

$$4\cos^4(z) + 4\sin^4(z) = 3 + \cos(4z)$$

PROBLEM 3 [15 points]

Derive all the roots of the polynomial $z^{10} + z^6 = 0$ and mark the locations of those roots in the complex plane below. Show a derivation and comment on your choices. How many roots in total are there?



PROBLEM 4 [30 points]

Simplify each of the following expressions into the basic complex number Cartesian form $z = x + iy$ or polar form $z = re^{i\theta}$

$$\frac{6i-9}{2+3i} =$$

$$\frac{5i}{1+\frac{i}{1+i}} - 1 =$$

$$\sqrt{\frac{e^{i\pi/2}}{i^3}} =$$

$$\log(e^{i\pi/3} + e^{i5\pi/3}) =$$

PROBLEM 5 [15 points]

a. [5] Write down the Taylor series for $w(z) = \log[\frac{1}{2}(1-2z)(2-z)]$ expanded about $z = 0$ in the complex plane.

b. [4] Describe in a few sentences the Alternating Series Convergence Test.

c. [3] At what values of z in the complex plane does the Taylor series in part (a) obey the Alternating Series Convergence test? Show your derivation.

d. [3] What is the radius of convergence of the Taylor series of part (a)?

PROBLEM 6 [10 points]

What is the value of the complex number z represented by the continued fraction

$$z = i + \cfrac{1}{2i + \cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{i}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cfrac{1}{2i+\cdots}}}}}}}}}}}}}}}$$