1. Vector Space

§1-3 Subspace.

Ex This is Question

Solution.
This is Solution. ■

10 Prove that $W_1 = \{ (a_1, a_2, \cdots, a_n) \in \mathrm{F}^n \mid a_1x_1 + \cdots + a_nx_n = 0 \}$ is a subspace of F^n, but $W_2 = \{ (a_1, a_2, \cdots, a_n) \in \mathrm{F}^n \mid a_1 + \cdots + a_n = 1 \}$ is not.

13 Let S be a nonempty set and F a field. Prove that for any $s_0 \in S$,$\{ f \in F(S, F) \mid f(s_0) = 0 \}$, is a subspace of $F(S, F)$.

14 Let S be a nonempty set and F a field. Let $C(S, F)$ denote the set of all functions $f \in F(S, F)$ such that $f(s) = 0$ for all but a finite number of elements of S. Prove that $C(S, F)$ is a subspace of $F(S, F)$

20 Max

23 Let W_1 and W_2 be subspaces of a vector space V.

(a) Prove that $W_1 + W_2$ is a subspace of V that contains both W_1 and W_2.

(b) Prove that any subspace of V that contains both W_1 and W_2 must also contain $W_1 + W_2$.

30 Let W_1 and W_2 be subspaces of a vector space V Prove that V is the direct sum of W_1 and W_2 if and only if each vector in V can be uniquely written as $x_1 + x_2$, where $x_1 \in W_1$ and $x_2 \in W_2$.

§1-4 Linear Combination.

(a) Show that if S_1 and S_2 are subsets of a vector space V such that $S_1 \subseteq S_2$,then $\mathrm{span}(S_1) \subseteq \mathrm{span}(S_2)$. In particular, if $S_1 \subseteq S_2$ and $\mathrm{span}(S_1) = \mathrm{V}$, deduce that $\mathrm{span}(S_2) = \mathrm{V}$

(b) Show that if S_1 and S_2 are arbitrary subsets of a vector space V, then $\mathrm{span}(S_1 \cup S_2$ $)= \mathrm{span}(S_1) + \mathrm{span}(S_2)$.

§1-5 Linear Independent.

(a) Let V be a vector space over a field of characteristic not equal to two.

i. Let u and v be distinct vectors in V. Prove that $\{ u, v \}$ is linearly independent if and only if $\{ u + v, u - v \}$ is linearly independent.

ii. Let u, v and w be distinct vectors in V. Prove that $\{ u, v, w \}$ is linearly independent if and only if $\{ u + v, u + w, v_w \}$ is linearly independent.

(b) Prove that a set S of vectors is linearly independent if and only if each finite subset of S is linearly independent.

(c) Let $f, g \in F(R, R)$ be the functions defined by $f(t) = e^{et}$ and $g(t) = e^{st}$, where $r \neq s$. Prove that f and g are linearly independent in $F(R, R)$.