Linear Algebra Sabrina Edition

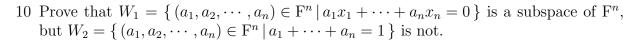
## 1. Vector Space

## §1-3 Subspace.

Ex This is Question

Solution.

This is Solution.



- 13 Let S be a nonempty set and F a field. Prove that for any  $s_0 \in S$ ,  $\{ f \in F(S, F) \mid f(s_0) = 0 \}$ , is a subspace of F(S, F).
- 14 Let S be a nonempty set and F a field. Let C(S, F) denote the set of all functions  $f \in F(S, F)$  such that f(s) = 0 for all but a finite number of elements of S. Prove that C(S, F) is a subspace of F(S, F)
- 20 Max
- 23 Let  $W_1$  and  $W_2$  be subspaces of a vector space V.
  - (a) Prove that  $W_1 + W_2$  is a subspace of V that contains both  $W_1$  and  $W_2$ .
  - (b) Prove that any subspace of V that contains both  $W_1$  and  $W_2$  must also contain  $W_1 + W_2$ .
- 30 Let  $W_1$  and  $W_2$  be subspaces of a vector space V Prove that V is the direct sum of  $W_1$  and  $W_2$  if and only if each vector in V can be uniquely written as  $x_1 + x_2$ , where  $x_1 \in W_1$  and  $x_2 \in W_2$ .

## §1-4 Linear Combination.

- (a) Show that if  $S_1$  and  $S_2$  are subsets of a vector space V such that  $S_1 \subseteq S_2$ , then  $\operatorname{span}(S_1) \subseteq \operatorname{span}(S_2)$ . In particular, if  $S_1 \subseteq S_2$  and  $\operatorname{span}(S_1) = V$ , deduce that  $\operatorname{span}(S_2) = V$
- (b) Show that if  $S_1$  and  $S_2$  are arbitrary subsets of a vector space V, then  $\operatorname{span}(S_1 \cup S_2) = \operatorname{span}(S_1) + \operatorname{span}(S_2)$ .

## §1-5 Linear Independent.

- (a) Let V be a vector space over a field of characteristic not equal to two.
  - i. Let u and v be distinct vectors in V. Prove that  $\{u, v\}$  is linearly independent if and only if  $\{u + v, u v\}$  is linearly independent.

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- ii. Let u, v and w be distinct vectors in V. Prove that  $\{u, v, w\}$  is linearly independent if and only if  $\{u + v, u + w, v_w\}$  is linearly independent.
- (b) Prove that a set S of vectors is linearly independent if and only if each finite subset of S is linearly independent.
- (c) Let  $f, g \in F(R, R)$  be the functions defined by  $f(t) = e^{et}$  and  $g(t) = e^{st}$ , where  $r \neq s$ . Prove that f and g are linearly independent in F(R, R).