proof Suppose  $W_1 \cup W_2$  is a subspace of V, we assume that  $W_1W_2$  and  $W_2W_1$ . Let  $x \in W_1 \backslash W_2$ ,  $y \in W_2 \backslash W_1$ , then  $x + y \in W_1$  or  $W_2$ , say  $W_1$ . But  $y = (x + y) - x \in W_1$ , which is a contradiction. So we have  $W_1 \subseteq W_2$  or  $W_2 \subseteq W_1$ . Conversely is trivial.