清華大學

Grover algo revision! 7'52"
1: 12'20"
2.3. 22'53"
4: 11'04"
5: 47'52"
6.7.8: 22'12"

Quantum Computing HW1 房俐文
2024010794

【1】

(a) $\vec{a}\cdot\vec{\sigma} = \sum_i a_i\sigma_i$

$\therefore$ LHS $= \sum_i a_i \sigma_i \cdot \sum_j b_j \sigma_j$

$= \sum_{i,j} a_i b_j \sigma_i \sigma_j$

$= \sum_{i=j} a_i b_i I + \sum_{i\neq j} a_i b_j i\varepsilon_{ijk}\sigma_k$

$= (\vec{a}\cdot\vec{b})I + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

(b) $\exp(i\theta(\vec{v}\cdot\vec{\sigma}))$

$= I + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$

where $t = i\theta(\vec{v}\cdot\vec{\sigma})$

$t^2 = -\theta^2(\vec{v}\cdot\vec{\sigma})(\vec{v}\cdot\vec{\sigma})$

$= -\theta^2[(\vec{v}\cdot\vec{v})I + i(\vec{v}\times\vec{v})\cdot\vec{\sigma}]$

$= -\theta^2 I$

$\therefore$ the $\exp(i\theta(\vec{v}\cdot\vec{\sigma}))$

$= I + i(\vec{v}\cdot\vec{\sigma})\theta - I\frac{\theta^2}{2} - i(\vec{v}\cdot\vec{\sigma})\frac{\theta^3}{3!} + \cdots$

$= I\left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} - \cdots\right) + i(\vec{v}\cdot\vec{\sigma})\cdot\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)$

$= I\cos\theta + i\vec{v}\cdot\vec{\sigma}\sin\theta$

【2】

(a) $|01\rangle \to \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|1\rangle \to \frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$

$\to \frac{1}{\sqrt{2}}\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} - |1\rangle\cdot\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)$

$\to \frac{1}{2}(|00\rangle - |11\rangle - |10\rangle + |01\rangle)$

$|00\rangle \to \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)|0\rangle \to \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$

$\to \frac{1}{\sqrt{2}}\left(|0\rangle\frac{|0\rangle+|1\rangle}{\sqrt{2}} - |1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$

$\to \frac{1}{2}(|00\rangle+|11\rangle - |10\rangle + |01\rangle)$

$|10\rangle \to \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)|0\rangle \to \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

$\to \frac{1}{\sqrt{2}}\left(|0\rangle\frac{|0\rangle+|1\rangle}{\sqrt{2}} + |1\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$

$\to \frac{1}{2}(|00\rangle + |11\rangle + |10\rangle - |01\rangle)$

$|11\rangle \to \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)|1\rangle \to \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

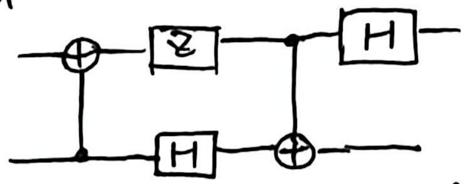
$\to \frac{1}{\sqrt{2}}\left(|0\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} + |1\rangle\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)$

$\to \frac{1}{2}(|00\rangle - |11\rangle + |10\rangle + |01\rangle)$

$$U = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

(b)

U^{-1}:



(c) uniformly distributed on four computational ~~base~~ basis state since

$|00\rangle \to \frac{1}{2}(|00\rangle+|01\rangle-|10\rangle+|11\rangle)$

【3】

$|00\rangle\times|0\rangle \to |000\rangle \to |000\rangle$

$|01\rangle\times|0\rangle \to |010\rangle \to |010\rangle$

$|10\rangle\times|0\rangle \to |100\rangle \to |110\rangle$

$|11\rangle\times|0\rangle \to |111\rangle \to |101\rangle$

This satisfies the truth table for a half adder.

Since the "sum" actually carries $|a\oplus b\rangle$. ~~we~~ If we want to get $|b\rangle$, we just need to apply the same CNOT gate to $CNOT|a, b'\rangle \to |a, b\rangle$ to get $|b\rangle$.

【4】

for each qubit, we ~~only need to~~ express it as

$|\psi_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$

we only need to store α_i & β_i (two complex number) using $O(n)$ space

~~Thus, w~~

Also, we can still use a constant number of ~~a~~ ~~opera~~ operation C_0 for every gate in $\{H, S, T, CNOT\}$, since there is not entanglement.

∴ The time complexity is still poly(n)

【5】

(a) $$U_\omega = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

(b) $$U_s = 2|\psi_0\rangle\langle\psi_0| - I = \begin{pmatrix} -\frac12 & \frac12 & \frac12 & \frac12 \\ \frac12 & -\frac12 & \frac12 & \frac12 \\ \frac12 & \frac12 & -\frac12 & \frac12 \\ \frac12 & \frac12 & \frac12 & -\frac12 \end{pmatrix}$$

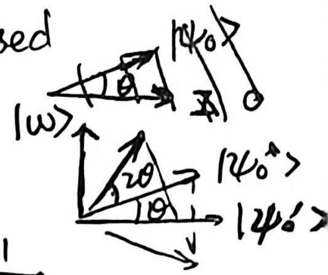
$$U_s U_\omega = \begin{pmatrix} -\frac12 & -\frac12 & \frac12 & \frac12 \\ \frac12 & \frac12 & \frac12 & \frac12 \\ \frac12 & -\frac12 & -\frac12 & \frac12 \\ \frac12 & -\frac12 & \frac12 & -\frac12 \end{pmatrix}$$

$$U_s = \frac32|\psi'\rangle\langle\psi'| + \frac{\sqrt3}{2}(|\psi'\rangle\langle\omega| - |\omega\rangle\langle\psi'|) + \frac12|\omega\rangle\langle\omega| - I$$

(c) ① $U_\omega = I - 2|\omega\rangle\langle\omega|$

$U_s = 2|\psi_0\rangle\langle\psi_0| - I$

every time the angle is ~~incre~~ increased by 2θ where



$\sin\theta = \langle\omega|\psi_0\rangle = \frac{1}{\sqrt N}$

$= \frac12$

$\Rightarrow \theta = \frac{\pi}{6}$

$\sin\theta$

$(2k+1)\frac{\pi}{6} = \frac{\pi}{2}$ ∴ $k=3$

$(2k+1)\theta = \frac{\pi}{2}$

$2k+1 = \frac{\pi}{2\theta} \approx \frac{\pi}{2}\sqrt N$

② when $N=4$

$\sin\theta = \langle\omega|\psi_0\rangle = \frac12$

$\theta = \frac{\pi}{6}$

$\Rightarrow (2k+1)\frac{\pi}{6} = \frac{\pi}{2}$ $k=1$

~~1° first iter.~~

$|\psi_0^{(1)}\rangle = U_s\frac12(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$

$= U_s \frac{\sqrt3}{2}\frac{1}{\sqrt3}(|00\rangle + |10\rangle + |11\rangle) - \frac12|01\rangle$

let $|\psi_0'\rangle = \frac{1}{\sqrt3}|00\rangle + |10\rangle + |11\rangle$

~~$|\psi_0^{(1)}\rangle = U_s\frac{\sqrt3}{2}|\psi_0'\rangle - \frac12|\omega\rangle$~~

~~$U_s = 2\frac{\sqrt{}}{2}$~~

$|\psi_0\rangle = \frac{\sqrt3}{2}|\psi'\rangle - \frac12|\omega\rangle$

$U_s = -\frac12|\psi'\rangle\langle\psi'| + \frac{\sqrt3}{2}(|\psi'\rangle\langle\omega| + |\omega\rangle\langle\psi'|) - \frac12|\omega\rangle\langle\omega|$

After ~~four~~ ~~three~~ one iteration.

$|\psi_0^{(1)}\rangle$ would be come $|\omega\rangle$

∴ the probability is 100%

is one more is applied $\theta^{(2)} = \frac{\pi}{2} + \frac{\pi}{3}$

$|\psi_0^{(2)}\rangle = \sin(\theta^{(2)})|\omega\rangle + \cos\theta^{(2)}|\psi_0'\rangle$

$= \frac12|\omega\rangle + \frac{\sqrt3}{2}|\omega_0'\rangle$

$P = (\sin\theta^{(2)})^2 = \frac14$

【6】

Using computer to calculate the ~~ssi~~ order of 3 (mod 85) is 16

$3^{16} \equiv 1 \pmod{85}$ $r=16$

also

$3^{8} \equiv 16 \pmod{85}$ $\frac{r}{2}=8$

$\therefore 3^{\frac{r}{2}}-1=15$ $3^{\frac{r}{2}}+1=17 \neq 1$

$\gcd(15,85)=5$

$\gcd(17,85)=17$

$\Rightarrow 85=5\times 17$

【7】

(a)

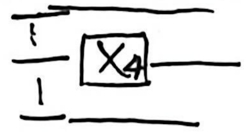
	$X_4X_5X_6X_7$	$X_2X_3X_6X_7$	$X_1X_3X_5X_7$	$Z_4Z_5Z_6Z_7$	$Z_2Z_3Z_6Z_7$	$Z_1Z_3Z_5Z_7$
X_1						-1
X_2					-1	
X_3					-1	-1
X_4				-1		
X_5				-1		-1
X_6				-1	-1	
X_7				-1	-1	-1
Z_1			-1			
Z_2		-1				
Z_3		-1	-1			
Z_4	-1					
Z_5	-1		-1			
Z_6	-1	-1				
Z_7	-1	-1	-1			

(b)

X_1X_5	1	1	1	-1	1	1

(b) the most likely error is X_4

if we want to correct this error

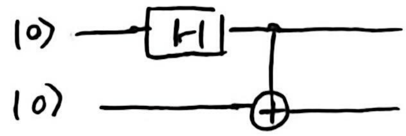


we would pass through a X_4 operator.

however, the error becomes

$X_1X_4X_5$, which is not corrected

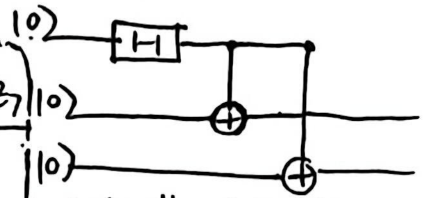
【8】



Creates

$|00\rangle \rightarrow \frac{|0\rangle+|1\rangle}{\sqrt{2}}|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$

$\therefore$ For three qubits



actually creates

$\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$

which is a GHZ state

Since in GHZ state,

all three qubit are ~~in~~ always in the same position,

~~$\Rightarrow$ Using~~ with no error

$\boxed{P_1}$. $\boxed{P_2}$ $\boxed{P_3}$ are all applied with 50% probability (corresponding)

which is the same as using one ancilla qubit (still 50% ~~per~~ probability and all P_i corresponds to $|1\rangle_{ancilla}$ state.