Quantum Computation HW2

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Acknowledgement:
All the code for this homework can be seen on the corresponding github repo
https://github.com/Bowen-777/Quantum-Computation-X-Homework

Problem 1

1. By turning the method of `AerSimulator` to "unitary", and use the `array_to_latex` command, we can get the result of U,

$$U = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}.$$

   Note that I have already use the `reverse_bits()` command to avoid mismatch between the reverse and standard basis.

2. The trails are done by setting the method to "automatic". We can also directly get the statevector by setting method to "statevector". The histogram is shown in Figure 1

Problem 2

The quantum circuit is shown in Figure 2 One the few results are

```
The result is: {'101010': 1}
Given (a, b) are: ( 31 , 11 )
The answer of a+b is (in decimal): 42
```

For more results, please refer to the source code $\boxed{\text{problem2.ipynb}}$ and run it directly.

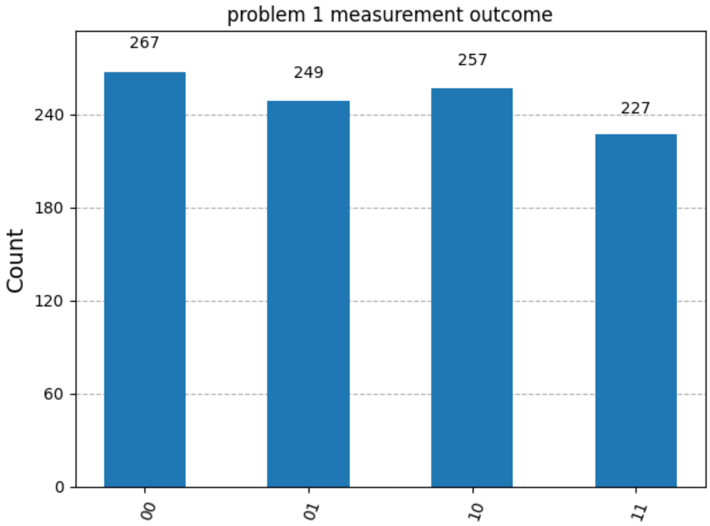

Figure 1: Distribution of 1000 trials

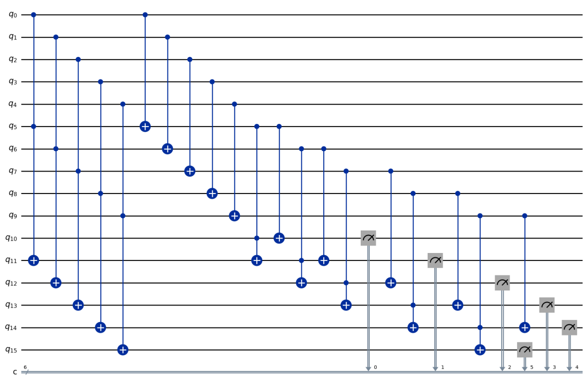

Figure 2: Circuit for n-bits full adder

Problem 3

(a) As shown in $\boxed{\text{problem3.ipynb}}$ The logical states are
$|0_L\rangle$:

```
|0000000>: (0.35355339059327384+0j)
|0001111>: (0.3535533905932738+0j)
|0110011>: (0.3535533905932738+0j)
|0111100>: (0.35355339059327373+0j)
|1010101>: (0.3535533905932738+0j)
|1011010>: (0.35355339059327373+0j)
|1100110>: (0.35355339059327373+0j)
|1101001>: (0.3535533905932737+0j)
```

$|1_L\rangle$:

```
|0010110>: (0.3535533905932737+0j)
|0011001>: (0.35355339059327373+0j)
|0100101>: (0.35355339059327373+0j)
|0101010>: (0.3535533905932738+0j)
|1000011>: (0.35355339059327373+0j)
|1001100>: (0.3535533905932738+0j)
|1110000>: (0.3535533905932738+0j)
|1111111>: (0.35355339059327384+0j)
```

This means that the encoded logic qubits are

$$\begin{aligned}|0_L\rangle =&\frac{1}{2\sqrt{2}}(|0000000\rangle + |0001111\rangle + |0110011\rangle + |0111100\rangle \\ &+ |1010101\rangle + |1011010\rangle + |1100110\rangle + |1101001\rangle)\end{aligned}$$

$$\begin{aligned}|1_L\rangle =&\frac{1}{2\sqrt{2}}(|0010110\rangle + |0011001\rangle + |0100101\rangle + |0101010\rangle \\ &+ |1000011\rangle + |1001100\rangle + |1110000\rangle + |1111111\rangle)\end{aligned}$$

(b) The circuit is shown in Figure 3. For the eigenvalues, we can infer it from the ancilla qubits' result.

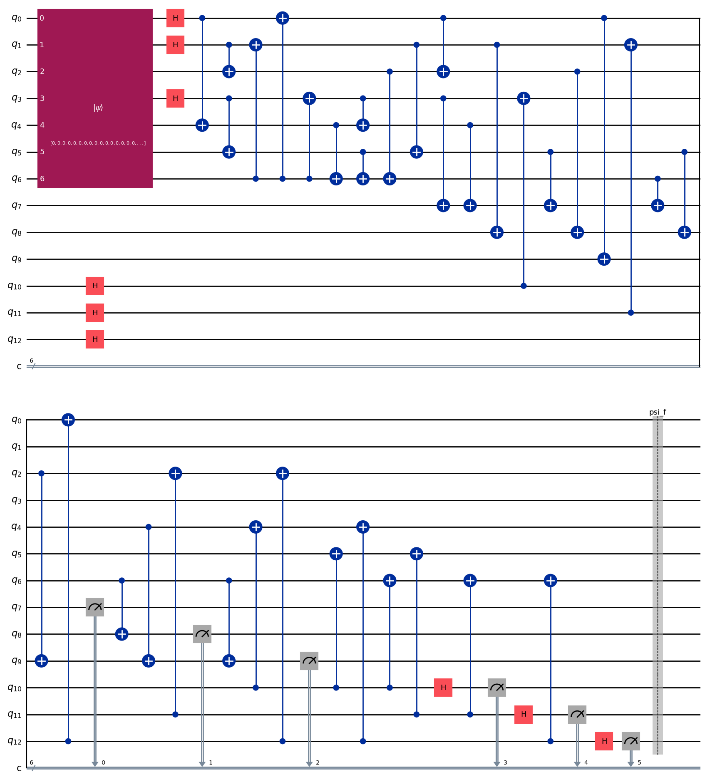

Figure 3: Stablizers

|0010110000000>: (0.3535533905932738-6.4946704217662e-17j)
|0011001000000>: (0.3535533905932738-6.494670421766199e-17j)
|0100101000000>: (0.3535533905932738-6.494670421766199e-17j)
|0101010000000>: (0.3535533905932738-6.494670421766198e-17j)
|1000011000000>: (0.3535533905932738-6.494670421766199e-17j)
|1001100000000>: (0.3535533905932738-6.494670421766199e-17j)
|1110000000000>: (0.3535533905932738-6.494670421766199e-17j)
|1111111000000>: (0.3535533905932738-6.494670421766198e-17j)

Since all the ancilla qubits still return $|0\rangle$ state, we know that the eigenvalues of logical state $|1_L\rangle$ are all 1. The result is similar for logical state $|0_L\rangle$.

(c) Please refer to the code or the circuit in the next sub-question.

(d) The combined circuit is shown in Figure 4. The single bit error here is $[I = i(X_4 + Y_4 + Z_4)]/2$. The result after correction and decoding is

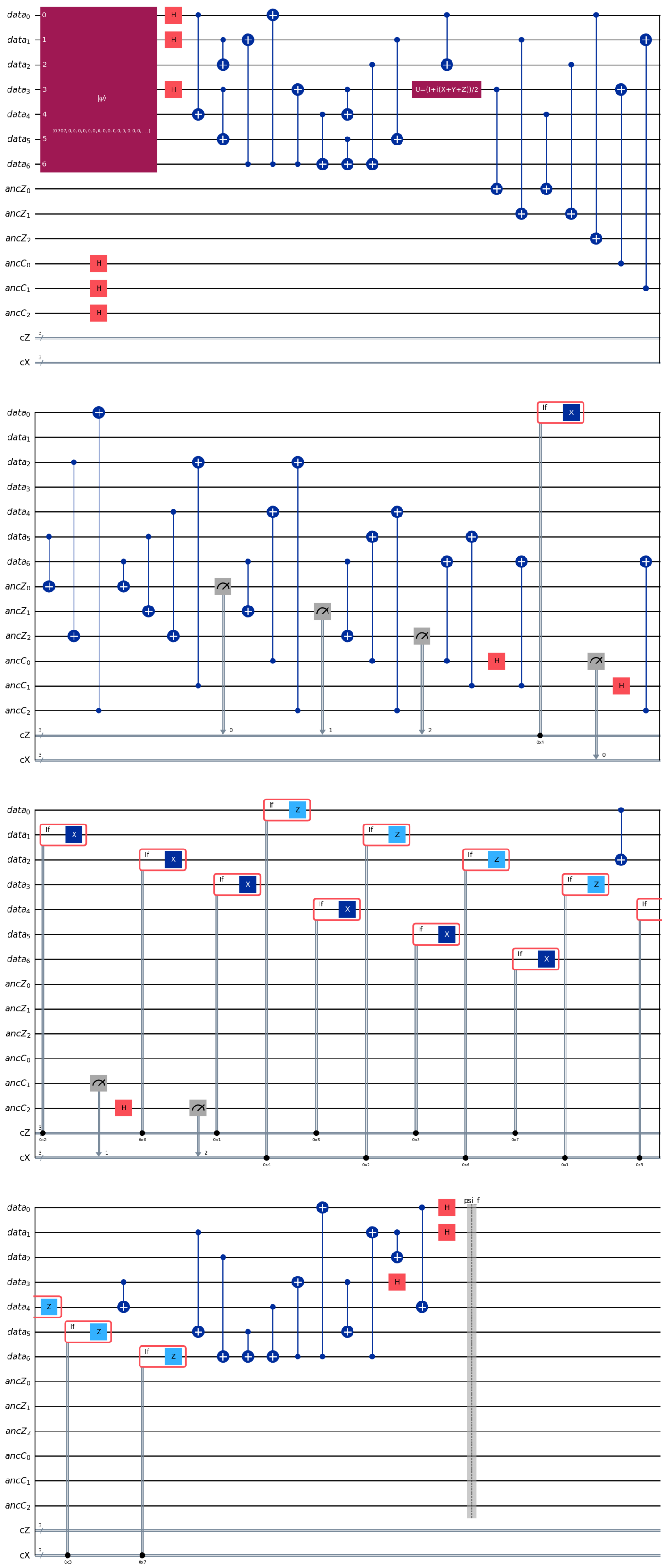

Figure 4: Combined circuit

```
|0000000000000>: (0.7071067811865475-2.0176939725871287e-16j)
|0000001000000>: (3.1461979272660923e-16+0.7071067811865475j)
```

This means that the initial state has been corrected. For more details, please refer to the code.

Problem 4

The total circuit is shown in Figure 5

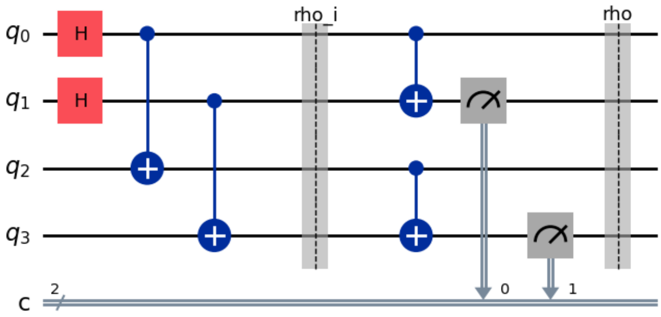

Figure 5: Purification circuit

(a) The distribution is shown in Figure 6 The statevector and fidelity result are as

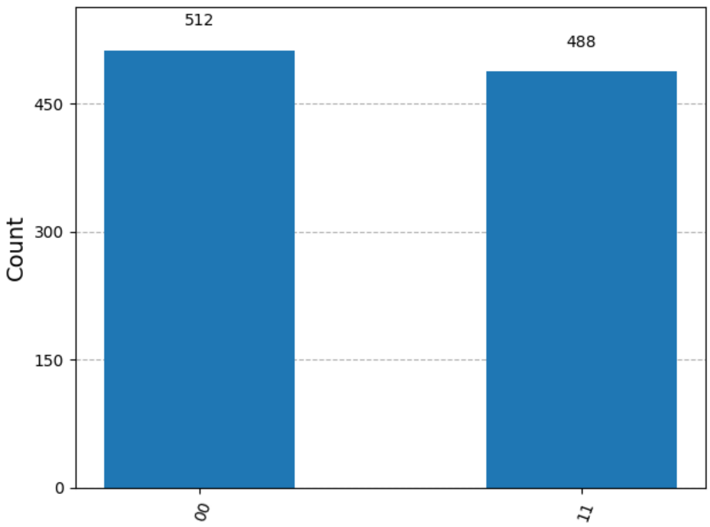

Figure 6: Perfect EPR state measurement

follows:

```
When conditioned on measurement 00:
|0000>: (0.7071067811865477+0j)
|1010>: (0.7071067811865475+0j)
When conditioned on measurement 11:
|0101>: (0.7071067811865476+0j)
|1111>: (0.7071067811865476+0j)
Fidelity for measurement outcome 00: 1.0
Fidelity for measurement outcome 11: 1.0
```

(b) The distribution is shown in Figure 7 The fidelity results are

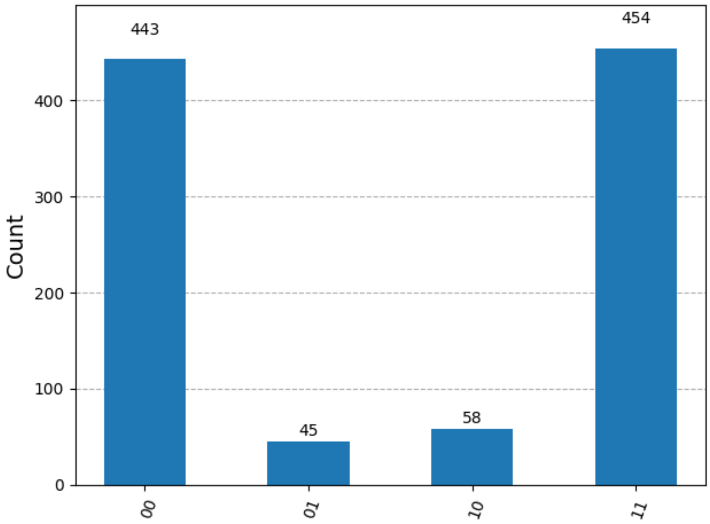

Figure 7: Perfect EPR state measurement

```
Initial fidelity for qubit 0 and 2: 0.9219999999999998
Initial fidelity for qubit 1 and 3: 0.9309999999999999
Final fidelity conditioned on 00: 0.9525959367945823
Final fidelity conditioned on 11: 0.962555066079295
{'11': 454, '01': 45, '10': 58, '00': 443}
```