ST745, Spring 2006

Midterm, 1:30-2:45, Thursday, 3/2/2006

This is an in-class, open-note and open-book midterm exam.

1. (20 pts) Suppose the hazard function of a random survival time (in years) T takes the form $\lambda(t) = (\lambda/2)t^{-1/2}$, where $\lambda > 0$. Do the following:

   (a) Find the cumulative hazard function, survival function and density function of T. (8 pts)

   (b) Given a random sample $(\tilde{t}_1, \delta_1), (\tilde{t}_2, \delta_2), ..., (\tilde{t}_n, \delta_n)$, where $\tilde{t}_i$ is the observed failure time or right censoring time; $\delta_i = 1$ if $\tilde{t}_i$ is a failure time and $\delta_i = 0$ if $\tilde{t}_i$ is a censoring time. Find the MLE of λ and its asymptotic variance. (8 pts)

   (c) Suppose we have a sample $1, 4+, 4, 9$, where $+$ indicates a right censored observation. Find the MLE of λ and its variance. (4 pts)

2. (20 pts) The following table gives grouped survival data in years for patients with certain disease:

time interval i	n_i	d_i	w_i
$i = 1$: [0, 2)	20	2	2
$i = 2$: [2, 4)	16	2	4
$i = 3$: [4, 6)	10	2	2

   where n_i is the number of patients at the beginning of time interval i, d_i and w_i are the number of deaths and censorings observed in that interval. **Assume** all censorings in an interval occurred at the **middle** of that interval. Do the following:

   (a) Find the life-table estimate of $S(6) = P(T \geq 6)$. (10 pts)

   (b) Find the variance estimate for the estimate you got in (a). (10 pts)

3. (40 pts) The survival times of 10 patients with certain heart disease are given as follows:

   $$2,\ 3,\ 4+,\ 5,\ 5+,\ 6+,\ 8,\ 10,\ 12,\ 15+,$$

   where "+" means a right censored survival time, do the following:

   (a) Find the Nelson-Aalen estimator of the cumulative hazard $\Lambda(8)$ and the Kaplan-Meier estimator of $S(8) = P(T > 8)$. (15 pts)

(b) Find the variance estimates for the Nelson-Aalen and Kaplan-Meier estimators you got in (a). (15 pts)

(c) Construct a 95% confidence interval for $S(8)$ based on the Nelson-Aalen and Kaplan-Meier estimators you got in (a). (10 pts)

4. (20 pts) In a clinical trial, we want to compare a new treatment to the standard one on time (in months) to tumor recurrence for patients with certain type of cancer. Assume that enough patients will be available at the **beginning** of the trial and half of them will be randomly assigned to each treatment. It is known that the average tumor recurrence time is 2 month for the standard treatment; while for the new treatment, it is expected that it will extend the average tumor recurrence time to 4 month. The trial will last 10 months. Suppose that the tumor recurrence time for each treatment group follows an exponential distribution and the censoring time is uniformly distributed on the interval [0, 10]. Do the following:

   (a) Find the probability for each treatment that a patient will experience the tumor recurrence before the end of the trial. (10 pts)

   (b) Suppose we will use the log-rank test to test the difference in time to tumor recurrence for these two treatments with significance level $\alpha = 0.05$ and 99% power to detect the expected difference. How many patients should be included in the study? (10 pts)