ST745, Spring 2016

Homework 1 Due: Thursday, 1/28/2016

1. Prove the formula (7) in lecture note 1 on page 7, i.e.

$$mrl(t_0) = E[T - t_0 | T \geq t_0] = \frac{\int_{t_0}^{\infty} S(t)dt}{S(t_0)}.$$

2. The time in days to development of a tumor for rats exposed to a carcinogen follows a Weibull distribution with $\alpha = 2$ and $\lambda = 0.5$.

   (a) Find the probabilities that a (random) rat will be tumor free at 30 days.

   (b) What is the average time to tumor development? (Hint: $\Gamma(0.5) = \sqrt{\pi}$, where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} \mathrm{e}^{-t} dt$)

   (c) Find the hazard rate of time to tumor development at 30 days.

   (d) Find the median time to tumor development.

3. Suppose we have a small data set with different kinds of censoring: 2^+, 3, 4, 5^-, 6, 7^+, $[5, 7]$, where $^+$ ($^-$) means right (left) censored observations and $[a, b]$ means an interval censored observation. Suppose the distribution of the underlying survival time is an exponential distribution with a constant hazard λ. Write down the likelihood function of λ for this given data set.

4. For the following small data set of survival time: 3+, 4, 5, 6, 6+, 8, 11+, 14+, 15, 16+, where "+" means a right censored survival time, do the following (here we assume that they are from a Weibull distribution):

   (a) Write down the log-likelihood function of the data.

   (b) Find the score and information matrix from this model and then evaluate them under the hypothesis that the data are from an exponential distribution.

   (c) Perform the score test to test whether or not the survival times are from an exponential distribution.

5. Using the lung cancer data (`http://www.biostat.mcw.edu/homepgs/klein/4.7.4.html`) in problem 4.3 of the textbook, do the following by using statistical software (such as `SAS` or R):

   (a) Fit a Weibull model to the censored survival data

   (b) Perform Wald test to test whether or not the survival times are from an exponential distribution.

   (c) Perform likelihood ratio test to test whether or not the survival times are from an exponential distribution.

   (d) Suggest ways to check the Weibull model assumption and conduct the diagnostics.