Applied Multivariate Statistical Analysis: Homework 2

Homework format: all homework must be written in latex. You must turn in both your tex and pdf files. Attach your code and computer output if there is any programming.

1. (a) Let $X_1, \ldots, X_n$ be i.i.d. $N(0,1)$, for any $a \in \Re^n$ with $\|a\| > 0$, find the conditional distribuition of $\sum_{i=1}^n X_i^2$ given that $\sum_{i=1}^n a_i X_i = 0$.

   (b) show that the range $R_n = X_{(n)} - X_{(1)}$ is independent of the sample mean $\bar{X}_n$, where $X_{(i)}$ is the ith order statistic.

2. Given a $p \times p$ positive definite matrix B and a scalar $b > 0$, show that

$$\frac{1}{|\Sigma|^b} \exp\{-\mathrm{tr}(\Sigma^{-1}B)/2\} \leq \frac{1}{|B|^b}(2b)^{pb}e^{-pb}$$

   for all $p \times p$ positive definite matrices Σ, with equality holding only for $\Sigma = B/(2b)$.

3. Use matrix differentiation to show that $\bar{x}_n = n^{-1}\sum_{i=1}^n x_i$ and $S_n = n^{-1}\sum_{i=1}^n x_i x_i'$ are the stationary points of the likelihood function $L(\mu, \Sigma | x_1, \ldots, x_n)$, i.e., $\partial L/\partial \mu = 0$ and $\partial L/\partial \Sigma = 0$,, where $\{x_1, \ldots, x_n\}$ is a random sample from $N_p(\mu, \Sigma)$.

4. If X is a normal data matrix from $N_p(\mu, \Sigma)$, let the $m \times q$ data matrix $Y = AXB$, then Y is also a normal data matrix if and only if (i) $A1_n = \alpha 1_n$ for some scalar α, or $B'\mu = 0$; (ii)$AA' = \beta I_n$ for some scalar β, or $B'\Sigma B = 0$. If both of these conditions are satisfied, then Y is a normal data matrix from $N_q(\alpha B'\mu, \beta B'\Sigma B)$.

5. If X is a normal data matrix from $N_p(\mu, \Sigma)$, and if $C_1, \ldots, C_k$ are symmetric matrices, then $X'C_1X, \ldots, X'C_kX$ are jointly independent if $C_rC_s = 0$ for all $1 \leq r \neq s \leq k$.

Johnson-Wichern 6th Edition, 4.2, 4.4, 4.16, 4.21, 4.26, 4.39, 4.40.