Problem 2.

$x(\lambda)$ intersects a quadric Q. $\quad X = \lambda X_1 + (1-\lambda) X_2$.

$\therefore X^T Q X = 0$

$= (\lambda X_1 + (1-\lambda) X_2)^T Q (\lambda X_1 + (1-\lambda) X_2)$

$= \lambda^2 X_1^T Q X_1 + (\lambda X_1^T Q (1-\lambda) X_2) + (1-\lambda) X_2^T Q \lambda X_1 + (1-\lambda) X_2^T Q (1-\lambda) X_2$.

$= (X_1^T Q X_1 + X_2^T Q X_2 - X_1^T Q X_2 - X_2^T Q X_1)\lambda^2 + (X_1^T Q X_2 + X_2^T Q X_1 - 2 X_2^T Q X_2)\lambda$
$\quad + X_2^T Q X_2$.

X_1 : 4*1 matrix $\quad Q$: 4*4 (symmetric matrix) $\quad X_2$: 4*1.

Let $X_1^T Q X_2 = a$ where a is a scale. $\xrightarrow{\text{transpose.}} X_2^T Q^T X_1 = a$.

$X_2^T Q X_1 = b$

$\because Q$ is symm $\therefore Q = Q^T$

$\therefore X_2^T Q X_1 = a = b$

$\therefore X_1^T Q X_2 = X_2^T Q X_1$

$\therefore X^T Q X = (X_1^T Q X_1 - 2X_1^T Q X_2 + X_2^T Q X_2)\lambda^2 + 2(X_1^T Q X_2 - X_2^T Q X_2)\lambda$
$\quad + X_2^T Q X_2 = 0$

$\therefore C_2 = X_1^T Q X_1 - 2X_1^T Q X_2 + X_2^T Q X_2$

$C_1 = 2(X_1^T Q X_2 - X_2^T Q X_2)$

$C_0 = X_2^T Q X_2$.