

## 《数值分析》期中试题(A 卷)

(考试形式：闭 卷 考试时间：2 小时)



《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：\_\_\_\_\_ 姓名：\_\_\_\_\_ 学号：\_\_\_\_\_

1. Fill in the blanket with proper answers (5 marks each, total 20 marks)

1) Suppose  $\pi \approx 3.1415926$ , then the approximate value 3.141601 has \_\_\_\_\_ significant digits.

2) The error term of Lagrange polynomial approximation for the function  $f(x)$  at the nodes

$$a \leq x_0 < x_1 < \dots < x_n \leq b \text{ is}$$

\_\_\_\_\_.

3) The recursive rule of Newton's methods for solving nonlinear equation  $f(x)=0$  is:

\_\_\_\_\_.

and its speed of convergence is \_\_\_\_\_ near a multiple root.

4) The fast algorithm for evaluate the polynomial  $y = \sum_{i=0}^n a_i x^i$  is called

\_\_\_\_\_, and the recursive rule is:

\_\_\_\_\_.

5) For the following nonlinear system, write out the Seidel iteration formula

$$\begin{cases} 3x + y - z = 3 \\ 2x - 5y - z = -4 \\ x + 3y - 6z = -2 \end{cases}$$

\_\_\_\_\_

2. (20 marks) Given the function constrain table

x	0	2	3	4
f(x)	5	5	23	69

first construct the divided difference table, and then find the Newton interpolation polynomial.

3. (20 marks) In order to solve the nonlinear equation  $f(x)=e^x+10x-2=0$ , we design the following fixed point iteration:

$$\begin{cases} x_0 = 0 \\ x_k = \frac{2 - e^{x_{k-1}}}{10} \quad k > 0 \end{cases}$$

- 1、 Show that the equation has unique root;
- 2、 Show that for any initial value in  $[-1,1]$ , the fixed point iteration converges to the unique root. (Hint: Verify that on  $[-1,1]$ ,  $\varphi(x)$  is a contraction mapping)

4. (20 marks) Consider the nonlinear system

$$\begin{cases} 2x^2 - y^2 + 4x - 5 = 0 \\ x - 2y + 1 = 0 \end{cases}$$

- 1) Find analytically the zeros of the system;
- 2) Write out the Newton iteration for the system.

5. (20 marks) Given the  $3 \times 3$  matrix

$$A = \begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

1、Find out the  $PA=LU$  factorization for  $A$ . The factorization should be correspondent to Gauss elimination. (10 marks)

2、Using the factorization to solve the linear system. (10 marks)

$$\begin{cases} -4x_2 - 2x_3 = -16 \\ x_1 + x_2 + x_3 = 4 \\ 2x_1 - 2x_2 + x_3 = -6 \end{cases}$$