中 山 大 学 软 件 学 院 **2010** 级 软 件 工 程 专 业 (2011 学年秋季学期)

《数值分析》期末试题(A 卷)

(考 试 形 式 ： 闭 卷 考 试 时 间 ： 2 小 时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：____________________ 姓名：__________ 学号：____________

1. Fill in the blanket with proper answers (5 marks each, total 20 marks)

1) Suppose $\pi \approx 3.1415926$, then the approximate value 3.141601 has _____ significant digits.

3.141601

3.1415926

0.000009 5位有效数字

2) The error term of Lagrange polynomial approximation for the function **f(x)** at the nodes

$a \le x_0 < x_1 < ... < x_n \le b$ is

$R_n(x) = \dfrac{f^{(n+1)}(\xi)}{(n+1)!}\omega_{n+1}(x)$ ，其中$\xi \in (a,b)$。

3) The recursive rule of Newton's methods for solving nonlinear equation f(x)=0 is:

$x_{k+1} = x - \dfrac{f(x_k)}{f'(x_k)}$， and its speed of convergence is order 1 near a multiple root.

4) The fast algorithm for evaluate the polynomial $y = \sum_{i=0}^{n} a_i x^i$ is called horner's algorithm, and the recursive rule is：

$$\begin{cases} b_n = a_n \\ b_k = a_k + x b_{k+1} \quad k = n-1, n-2, ..., 0 \end{cases}.$$

$$b_0 = P(x)$$

5) For the following nonlinear system, write out the Seidel iteration formula

$$\begin{cases} 3x + y - z = 3 \\ 2x - 5y - z = -4 \\ x + 3y - 6z = -2 \end{cases}$$

$$\begin{cases} 3x^{(k+1)} + y^{(k)} - z^{(k)} = 3 \\ 2x^{(k+1)} - 5y^{(k+1)} - z^{(k)} = -4 \\ x^{(k+1)} + 3y^{(k+1)} - 6z^{(k+1)} = -2 \end{cases}$$

2. (20 marks)Given the function constrain table

x	0	2	3	4
f(x)	5	5	23	69

first construct the divided difference table，and then find the Newton interpolation polynomial.（20 marks）

可以使用 Lagrange 插值公式，也可以使用 Newton 插值公式。以下我们使用 Newton 插值公式。首先计算均差。

x_k	$f(x_k)$			
0	5			
2	5	0		
3	23	18	6	
4	69	46	14	2

$$\begin{aligned} N_3(x) &= 5 + 6x(x-2) + 2x(x-2)(x-3) \\ &= 2x^3 - 4x^2 + 5 \end{aligned}$$

3. (20 marks) In order to solve the nonlinear equation $f(x)=e^x+10x-2=0$, we design the following fixed point iteration

:

$$\begin{cases} x_0 = 0 \\ x_k = \dfrac{2-e^{x_{k-1}}}{10} \quad k>0 \end{cases}$$

1、 Show that the equation has **unique** root;

2、 Show that for any initial value in [-1,1], the fixed point iteration converges to the unique root.（Hint：Verify that on [-1,1]，$\varphi(x)$ is a contraction mapping）

证明 对 f 求导数，$f'(x)=e^x+10>0$。于是 f(x)是一个单调上升的函数。由于 $f(-1)=\frac{1}{e}-10-2<0$，$f(1)=e+10-2>0$，故 f 仅有唯一一个根，此根在[-1,1]。

迭代对应于 $\varphi(x) = \dfrac{2-e^x}{10}$，以下检查在[-1,1]满足压缩映射定理。

1、 显然 $\varphi(x)$ 是一个单调下降的函数，于是 $\forall x \in [-1,1]$，

$$1 \ge \varphi(-1) \ge \varphi(x) \ge \varphi(1) \ge \frac{2-e^1}{10} > -1$$

所以 $\forall \varphi(x) \in [-1,1]$。

2、 $L = \max_{x\in[-1,1]} |\varphi'(x)| = \max_{x\in[-1,1]} \left|\frac{e^x}{10}\right| = \frac{e}{10} < 1$

根据压缩映射定理，$\forall x_0 \in [-1,1]$，迭代收敛。

4. (20 marks)Given the 3×3 matrix

$$A=\begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

1、Find out the PA=LU factorization for A. The factorization should be correspondent to Gauss elimination.（10 marks）

2、Using the factorization to solve the linear system.（10 marks）

$$\begin{cases} -4x_2 - 2x_3 = -16 \\ x_1 + x_2 + x_3 = 4 \\ 2x_1 - 2x_2 + x_3 = -6 \end{cases}$$

$$\underset{p=(1,2,3)}{\begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}} \Rightarrow \underset{p=(3,2,1)}{\begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}} \Rightarrow \underset{p=(3,2,1)}{\begin{bmatrix} 0 & -4 & -2 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ 2 & -2 & 1 \end{bmatrix}}$$

$$\Rightarrow \underset{p=(3,1,2)}{\begin{bmatrix} 0 & -4 & -2 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ 2 & -2 & 1 \end{bmatrix}} \Rightarrow \underset{p=(3,1,2)}{\begin{bmatrix} 0 & -4 & -2 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 2 & -2 & 1 \end{bmatrix}}$$

$$P=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, L=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}, U=\begin{bmatrix} 2 & -2 & 1 \\ 0 & -4 & -2 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$
\begin{aligned}
AX &= b \\
PAX &= Pb \\
LUX &= Pb
\end{aligned}
$$

$$
\begin{cases} LY = Pb \\ UX = Y \end{cases}
$$

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -16 \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -16 \\ -1 \end{pmatrix}
$$

$$
\begin{bmatrix} 2 & -2 & 1 \\ 0 & -4 & -2 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -16 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}
$$

5. (20 marks)Consider the nonlinear system

$$
\begin{cases} 2x^2 - y^2 + 4x - 5 = 0 \\ x - 2y + 1 = 0 \end{cases}
$$

1) Find analytically the zeros of the system;
2) Write out the Newton iteration for the system.

$$
y = \frac{x+1}{2}
$$

$$
2x^2 - \left(\frac{x+1}{2}\right)^2 + 4x - 5 = 0
$$

$$
8x^2 - \left(x^2 + 2x + 1\right) + 16x - 20 = 0
$$

$$
7x^2 + 14x - 21 = 0
$$

$$
x^2 + 2x - 3 = 0
$$

$$
\left(x+3\right)\left(\mathrm{x}-1\right) = 0
$$

$$
\begin{cases} x_1 = -3 \\ y_1 = -1 \end{cases}, \begin{cases} x_2 = 1 \\ y_1 = 1 \end{cases}
$$

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \end{pmatrix} = \begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix} - \begin{pmatrix} 4x^{(k)}+4 & -2y^{(k)} \\ 1 & -2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2\left(x^{(k)}\right)^2 - \left(y^{(k)}\right)^2 + 4x^{(k)} - 5 \\ x^{(k)} - 2y^{(k)} + 1 \end{pmatrix}$$

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \end{pmatrix} = \begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix} - \frac{1}{-8x^{(k)}+2y^{(k)}-8} \begin{pmatrix} -2 & 2y^{(k)} \\ -1 & 4x^{(k)}+4 \end{pmatrix} \cdot \begin{pmatrix} 2\left(x^{(k)}\right)^2 - \left(y^{(k)}\right)^2 + 4x^{(k)} - 5 \\ x^{(k)} - 2y^{(k)} + 1 \end{pmatrix}$$

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \end{pmatrix} = \begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix} - \frac{1}{-8x^{(k)}+2y^{(k)}-8} \begin{pmatrix} -2 & 2y^{(k)} \\ -1 & 4x^{(k)}+4 \end{pmatrix} \cdot \begin{pmatrix} 2\left(x^{(k)}\right)^2 - \left(y^{(k)}\right)^2 + 4x^{(k)} - 5 \\ x^{(k)} - 2y^{(k)} + 1 \end{pmatrix}$$