中 山 大 学 软 件 学 院 **2010** 级 软 件 工 程 专 业 (2011 学年秋季学期)

《数 值 分 析》 期 末 试 题 (B 卷)

(考 试 形 式： 闭 卷 考 试 时 间：2 小 时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：____________________ 姓名：__________ 学号：____________

1. Fill in the blanket with proper answers (5 marks each, total 20 marks)

   1) Suppose $x = 29.56789...$, then the approximate value $x^* = 30.00789...$ has __2__ significant digits.

$$\begin{aligned} x^* &= 30.00789... \\ x &= 29.56789... \\ |x^* - x| &= 0.44000 \end{aligned}$$

   2) The error term of Lagrange polynomial approximation for the function **f(x)** at the nodes $a \le x_0 < x_1 < ... < x_n \le b$ is

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\omega_{n+1}(x) \quad ，其中\xi \in (a,b)。$$

   3) The recursive rule of Secant methods for solving nonlinear equation f(x)=0 is

$$x_{k+1} = x_k - \frac{(x_k - x_{k-1}) f(x_k)}{f(x_k) - f(x_{k-1})} \qquad k = 2,3,4,...$$

and its speed of convergence is ： $\frac{1+\sqrt{5}}{2}$

   4) The fast algorithm for evaluate the Newton polynomial $y = \sum_{k=0}^{n} a_k \prod_{i=0}^{k-1} (x - x_i)^k$ is somewhat like Horner's algorithm, and the recursive rule is：

$$\begin{cases} b_n = a_n \\ b_k = a_k + (x - x_k) b_{k+1} \quad k = n-1, n-2, ..., 0 \end{cases}.$$

$$b_0 = P(x)$$

   5) Using the Seidel iteration method to solve the following nonlinear system,

$$\begin{cases} 2x_1 + x_2 = -12 \\ -x_1 + 2x_2 = 20 \end{cases}$$

Suppose the iteration format be the following form:

$$X^{(k+1)} = BX^{(k)} + f$$

then $B = \underline{\begin{pmatrix} 0 & -\frac{1}{2} \\ 0 & -\frac{1}{4} \end{pmatrix}}$。

2. (20 marks)Given the function constrain table

x	-1	2	3	5
f(x)	-7	-19	-31	5

first construct the divided difference table, and then find the Newton interpolation polynomial.

可以使用 Lagrange 插值公式，也可以使用 Newton 插值公式。以下我们使用 Newton 插值公式。首先计算均差。

x_k	$f(x_k)$			
-1	-7			
2	-19	-4		
3	-31	-12	-2	
5	5	18	10	2

$$N_3(x) = -7 - 4(x+1) - 2(x+1)(x-2) + 2(x+1)(x-2)(x-3)$$
$$= 2x^3 - 10x^2 + 5$$

3. (20 marks) In order to solve the nonlinear equation $f(x)=e^x+10x-2=0$, we design the following fixed point iteration
:

$$\begin{cases} x_0 = 0 \\ x_k = \dfrac{2-e^{x_{k-1}}}{10} \quad k>0 \end{cases}$$

1、 Show that the equation has **unique** root;

2、 Show that for any initial value in [-1,1], the fixed point iteration converges to the unique root.（Hint：Verify that on [-1,1]，$\varphi(x)$ is a contraction mapping）

证明 对 f 求导数，$f'(x)=e^x+10>0$。于是 f(x)是一个单调上升的函数。由于 $f(-1)=\frac{1}{e}-10-2<0$，$f(1)=e+10-2>0$，故 f 仅有唯一一个根，此根在[-1,1]。

迭代对应于 $\varphi(x)=\dfrac{2-e^x}{10}$，以下检查在[-1,1]满足压缩映射定理。

1、显然 $\varphi(x)$ 是一个单调下降的函数，于是 $\forall x\in[-1,1]$，

$$1\ge\varphi(-1)\ge\varphi(x)\ge\varphi(1)\ge\frac{2-e^1}{10}>-1$$

所以 $\forall\varphi(x)\in[-1,1]$。

2、$L=\max\limits_{x\in[-1,1]}|\varphi'(x)|=\max\limits_{x\in[-1,1]}\left|\dfrac{e^x}{10}\right|=\dfrac{e}{10}<1$

根据压缩映射定理，$\forall x_0\in[-1,1]$，迭代收敛。

4. (20 marks)Given the 4×4 matrix

$$A=\begin{bmatrix}1&2&1&4\\2&0&4&3\\4&2&2&1\\-3&1&3&2\end{bmatrix}$$

1、Find out the PA=LU factorization for A. The factorization should be correspondent to Gauss elimination.（10 marks）

2、If you have had the PA=LU factorization, analyze the computational complexity of finding the inverse of A.（10 marks）

解：

$$\begin{bmatrix}1&2&1&4\\2&0&4&3\\4&2&2&1\\-3&1&3&2\end{bmatrix}\to\begin{bmatrix}1&2&1&4\\2&-4&2&-5\\4&-6&-2&-15\\-3&7&6&14\end{bmatrix}\to\begin{bmatrix}1&2&1&4\\2&-4&2&-5\\4&1.5&-5&-7.5\\-3&-1.75&9.5&5.25\end{bmatrix}\to\begin{bmatrix}1&2&1&4\\2&-4&2&-5\\4&1.5&-5&-7.5\\-3&-1.75&-1.9&-9\end{bmatrix}$$

$$L=\begin{bmatrix}1&0&0&0\\2&1&0&0\\4&1.5&1&0\\-3&-1.75&-1.9&1\end{bmatrix},U=\begin{bmatrix}1&2&1&4\\0&-4&2&-5\\0&0&-5&-7.5\\0&0&0&-9\end{bmatrix}$$

设 $A^{-1}=\left[X_1,X_2,...,X_n\right],I=\left[e_1,e_2,...,e_n\right]$，则

$$AX_k=e_k \qquad k=1,2,...,n$$
$$PAX_k=Pe_k$$
$$LUX_k=Pe_k$$
$$\begin{cases}LY_k=Pe_k\\UX_k=Y_k\end{cases}$$

求解$LY_k=Pe_k$的计算量为$\dfrac{\mathrm{n}^2-n}{2}$

求解$UX_k=Y$的计算量为$\dfrac{\mathrm{n}^2+n}{2}$

总的计算量=$\mathrm{n}\left(\dfrac{\mathrm{n}^2-n}{2}+\dfrac{\mathrm{n}^2+n}{2}\right)=n^3=64$

5. (20 marks)Consider the nonlinear system

$$\begin{cases}2x^2-y^2+4x-5=0\\x-2y+1=0\end{cases}$$

1) Find analytically the zeros of the system;
2) Write out the Newton iteration for the system.

$$y=\frac{x+1}{2}$$
$$2x^2-\left(\frac{x+1}{2}\right)^2+4x-5=0$$
$$8x^2-\left(x^2+2x+1\right)+16x-20=0$$
$$7x^2+14x-21=0$$
$$x^2+2x-3=0$$
$$\left(x+3\right)\left(\mathrm{x}-1\right)=0$$
$$\begin{cases}x_1=-3\\y_1=-1\end{cases},\begin{cases}x_2=1\\y_1=1\end{cases}$$

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \end{pmatrix} = \begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix} - \begin{pmatrix} 4x^{(k)}+4 & -2y^{(k)} \\ 1 & -2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2\left(x^{(k)}\right)^2 - \left(y^{(k)}\right)^2 + 4x^{(k)} - 5 \\ x^{(k)} - 2y^{(k)} + 1 \end{pmatrix}$$

$$\begin{pmatrix} x^{(k+1)} \\ y^{(k+1)} \end{pmatrix} = \begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix} - \frac{1}{-8x^{(k)}+2y^{(k)}-8} \begin{pmatrix} -2 & 2y^{(k)} \\ -1 & 4x^{(k)}+4 \end{pmatrix} \cdot \begin{pmatrix} 2\left(x^{(k)}\right)^2 - \left(y^{(k)}\right)^2 + 4x^{(k)} - 5 \\ x^{(k)} - 2y^{(k)} + 1 \end{pmatrix}$$

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