中 山 大 学 软 件 学 院 **2011** 级 软 件 工 程 专 业 (2012 学年秋季学期)

《 数 值 分 析 》 期 中 试 题 **(B 卷)**

(考 试 形 式： 闭 卷   考 试 时 间：**2** 小 时)

警 示   《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：_____   姓名：_____   学号：_____

1. Fill in the blanket with proper answers (5 marks each, total 30 marks)

   1) Suppose $x = 0.544987104184$, then the approximate value 0.544986720817 has __6__ significant digits.

   2) Let $A = \begin{pmatrix} 0 & 1 & 1 & 4 \\ 5 & 2 & -1 & 0 \\ 1 & 5 & 2 & 2 \\ 2 & -3 & 0 & 6 \end{pmatrix}$, then $\|A\|_1 = $ __12__ , $\|A\|_\infty = $ __11__ .

   3) The recursive rule of secant method for solving nonlinear equation f(x)=0 is

   $$p_{k+1} = p_k - \frac{f(p_k)}{f(p_k) - f(p_{k-1})}(p_k - p_{k-1})$$

   and its speed of convergence is $\dfrac{1+\sqrt{5}}{2}$.

   4) The function $g(x) = -4 + 4x - \frac{1}{2}x^2$ has _____2_____ fixed points, and the fixed point $\underline{x=4}$ is attractive.

   5) The process of Gaussian elimination with back substitution for solving the linear systems $Ax = b$, where A is an $n \times n$ matrix, requires $\dfrac{n^3 - n}{3}$ multiplications and $\dfrac{n(n-1)}{2}$ divisions.

   6) For the linear system,

   $$\begin{cases} 2x - y = -4 \\ x + 3y = -2 \end{cases}$$

   starting with $(x_0, y_0) = (1, 2)$, the Gauss-Seidel iteration process yields

   $$(x_1, y_1) = \left(-1, -\frac{1}{3}\right).$$

the Jacobi iteration process yields $\left(x_1, y_1 \right) = \underline{(-1, -1)}$.

2. (20 marks)Consider the division of two numbers x, y with the approximate value x^*, y^*, analyzing the propagation of relative error.

解：记 $\varepsilon_x = x^* - x$，$\varepsilon_y = y^* - y$ 则

$$\frac{x^*}{y^*} - \frac{x}{y} = \frac{x + \varepsilon_x}{y + \varepsilon_y} - \frac{x}{y} = \frac{y\varepsilon_x - x\varepsilon_y}{y(y + \varepsilon_y)}$$

$$= \frac{\varepsilon_x}{(y + \varepsilon_q)} - \frac{x\varepsilon_y}{y(y + \varepsilon_y)}$$

$$\approx \frac{1}{y}\varepsilon_x - \frac{x}{y^2}\varepsilon_y$$

$$= \frac{1}{y}\left(\varepsilon_x - \frac{x}{y}\varepsilon_y \right)$$

所以

$$\frac{\frac{x^*}{y^*} - \frac{x}{y}}{\frac{x}{y}} \approx \frac{\frac{1}{y}\left(\varepsilon_x - \frac{x}{y}\varepsilon_y \right)}{\frac{x}{y}} = \frac{1}{x}\left(\varepsilon_x - \frac{x}{y}\varepsilon_y \right) = \frac{\varepsilon_x}{x} - \frac{\varepsilon_y}{y}$$

即除法的相对误差等于分子的相对误差与分母的相对误差之差！

3. (20 marks)Write down the cubic Lagrange interpolation to the function

$$f(x) = x^4 - 2x^3$$

where interpolation is to be exact at the four nodes $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 3$.

解：

x	-1	0	1	3
f(x)	3	0	-1	27

于是

$$L(x) = 3 \cdot \frac{x(x-1)(x-3)}{(-1) \cdot (-2) \cdot (-4)} - \frac{x(x+1)(x-3)}{1 \cdot 2 \cdot (-2)} + 27 \cdot \frac{x(x+1)(x-1)}{4 \cdot 3 \cdot 2}$$

4. (10 marks)How to select the initial value x_0, so that the fixed point iteration $\{x_n\}$ defined

by $x_n = \varphi(x_{n-1})$, where $\varphi(x) = \dfrac{2-e^x}{5}$, always converges？

解：可以根据压缩映射定理来选择初值。容易证明 $\varphi(x)$ 是闭区间 $[-1,1]$ 上的压缩映射。

事实上，$\varphi(x)$ 单调下降，于是当 $x \in [-1,1]$ 时

$$-1 \le \varphi(1) \le \varphi(x) \le \varphi(-1) \le 1$$

$$\left|\varphi^{'}(x)\right| = \left|-\frac{e^x}{5}\right| \le \frac{e}{5} < 1$$

于是初值 x_0 可以选 $[-1,1]$ 中的任意一点。

5. (20 marks) Find the triangular factorization $PA = LU$ for the following matrix

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 6 \\ 2 & 2 & 4 \end{bmatrix}$$

解：

$$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 6 \\ 2 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 6 \\ 2 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$p = (1,2,3) \qquad p = (3,2,1) \qquad\qquad p = (3,2,1)$$

$$\rightarrow \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ 2 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ 2 & 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 2 & 2 & 4 \end{bmatrix}$$

$$p = (3,2,1) \qquad p = (3,1,2) \qquad p = (3,1,2)$$

所以

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & 2 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{pmatrix}$$