中 山 大 学 软 件 学 院 **2010** 级 软 件 工 程 专 业 (2011 学年秋季学期)

《SE-205+数值计算方法》 期末试题答案(B)

1. Fill in the blankets with proper answers (5 marks each, total 40 marks)

1) Let $X=(1,2,3,4)$ and $A=\begin{pmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 4 & 5 & -6\end{pmatrix}$, then $\|X\|_2=\underline{\sqrt{30}}$, $\|A\|_1=\underline{8}$.

2) The Simpson formula for calculating definite integral is

$$\int_a^b f(x)dx=\underline{\frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)},$$

and it has__3__degree of precision.

3) Let $f\in C^2[-1,1]$ and $f(-0.1)=5.05, f(0)=5, f(0.2)=5.20$, by the relationship between derivative and divided deference, $f''(0)\approx\underline{\ \ 10\ \ }$.

4) The Chebyshev polynomial of degree 3 is

$$\underline{T_3(x)=4x^3-3x},$$

and its points of alternation are $\underline{-1,-\frac{1}{2},\frac{1}{2},1}$.

5) The recursive rule of secant methods for solving nonlinear equation f(x)=0 is: $\underline{p_{n+1}=p_n-\frac{f(p_n)}{f(p_n)-f(p_{n-1})}(p_n-p_{n-1})}$, and its speed of convergence is $\underline{\frac{\sqrt{5}+1}{2}}$.

6) The fast algorithm for evaluate the polynomial $y=\sum_{i=0}^{n}a_nx^n$ is called 秦九韶算法或 **Horner 算法**, and the recursive rule is：

$$\begin{cases}S_n=a_n\\ S_{k-1}=S_k\cdot x+a_{n-1} & k=n,n-1,\dots,1.\end{cases}$$
$$y=a_0$$

7) For the following linear system, write out the Seidel iteration formula

$$\begin{cases}3x+y-z=3\\2x-4y+z=-4\\x-3y-5z=-2\end{cases}$$

$$\begin{cases}x_{k+1}=-\frac{1}{3}y_k+\frac{1}{3}z_k+1\\y_{k+1}=\frac{1}{2}x_{k+1}+\frac{1}{4}z_k+1\\z_{k+1}=\frac{1}{5}x_{k+1}-\frac{3}{5}y_{k+1}+\frac{2}{5}\end{cases}$$

8) Newton iteration for the following nonlinear system

$$\begin{cases}y=2x-1\\x^2+y^2=1\end{cases}$$

is

$$\begin{bmatrix}x_{k+1}\\y_{k+1}\end{bmatrix}=\begin{bmatrix}x_k\\y_k\end{bmatrix}-J^{-1}\begin{bmatrix}f_1\\f_2\end{bmatrix}$$

where $J=\begin{pmatrix}-2 & 1\\2x_k & 2y_k\end{pmatrix}$ and. $f_1=y_k-2x_k+1$.

2. **（13 marks）Find the three moment equation（三弯矩方程）for the cubic spline S(x) that satisfies clamped cubic spline that passes through the end points （-3,2),（-2,0),（1,3）and（4,1）with the first derivative boundary conditions** $S'(-3)=-1, S'(4)=1$.

解：1、首先计算二阶均差。在首尾端点的前后分别补充一个节点，以便将端点当成内部节点来处理。

xk	f(xk)	一阶均差	二阶均差
-3	2		
-3	2	$S'(-3)=-1$	
-2	0	$f[x_0,x_1]=-2$	$\frac{g_0}{6}=-1$
1	3	$f[x_1,x_2]=1$	$\frac{g_1}{6}=f[x_0,x_1,x_2]=\frac{3}{4}$
4	1	$f[x_2,x_3]=-\frac{2}{3}$	$\frac{g_2}{6}=f[x_1,x_2,x_3]=-\frac{5}{18}$
4	1	$S'(4)=1$	$\frac{g_3}{6}=\frac{5}{9}$

2、计算各个λ_i、μ_i，并列方程组。

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1/4 & 2 & 3/4 & 0 \\ 0 & 1/2 & 2 & 1/2 \\ 0 & 0 & 1 & 2 \end{pmatrix}\begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 9/2 \\ -5/3 \\ 10/3 \end{bmatrix}$$

3. **（15 marks）Given the** 3×3 **matrix**

$$A=\begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

1、Find out the PA=LU factorization for A. The factorization should be correspondent to Gauss elimination.

2、If you have had the PA=LU factorization, analyze the computational complexity of finding the inverse of A.

解 1、三阶矩阵做高斯消去法需要两次消去的过程，具体如下：

$$\underset{p=(1,2,3)}{\begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}} \Rightarrow \underset{p=(3,2,1)}{\begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}} \Rightarrow \underset{p=(3,2,1)}{\begin{bmatrix} 0 & -4 & -2 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ 2 & -2 & 1 \end{bmatrix}}$$

$$\Rightarrow \underset{p=(3,1,2)}{\begin{bmatrix} 0 & -4 & -2 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ 2 & -2 & 1 \end{bmatrix}} \Rightarrow \underset{p=(3,1,2)}{\begin{bmatrix} 0 & -4 & -2 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 2 & -2 & 1 \end{bmatrix}}$$

$$P=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, L=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}, U=\begin{bmatrix} 2 & -2 & 1 \\ 0 & -4 & -2 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

2、设 $A^{-1}=\left[X_1, X_2, \dots, X_n\right], I=\left[e_1, e_2, \dots, e_n\right]$，则

$$AX_k = e_k \qquad k=1,2,\dots,n$$
$$PAX_k = Pe_k$$
$$LUX_k = Pe_k$$
$$\begin{cases} LY_k = Pe_k \\ UX_k = Y_k \end{cases}$$

$$求解LY_k = Pe_k的计算量为\frac{n^2-n}{2}$$

$$求解UX_k = Y的计算量为\frac{n^2+n}{2}$$

$$总的计算量=n\left(\frac{n^2-n}{2}+\frac{n^2+n}{2}\right)=n^3=27$$

4. （10 marks）Use the Romberg algorithm to find the numerical integration for $\int_0^1 \frac{\sin x}{x}dx$.

1) **The following tableau illustrates Romberg integration process, but the values on last line are omitted intentionally. Please calculate them out according Romberg algorithm.**
2) **Estimate the significant digits for each of these unknown values.**

R(0, 0)=0.9207355			
R(1, 0)=0.9397933	**R(1, 1)=0.9461459**		
R(2, 0)=0.9445135	**R(2, 1)=0.9460869**	**R(2, 2) =0.9460830**	
R(3, 0)=?	**R(3, 1)=?**	**R(3, 2)=?**	**R(3, 3)=?**

解：

$$h=\frac{1}{8},\ f(x)=\frac{\sin x}{x}$$

$$R(3,0)=\frac{R(2,0)}{2}+h\left[f\left(\frac{1}{8}\right)+f\left(\frac{3}{8}\right)+f\left(\frac{5}{8}\right)+f\left(\frac{7}{8}\right)\right]=0.9456909$$

$$R(3,1)=\frac{4}{3}R(3,0)-\frac{1}{3}R(2,0)=0.9460833$$

$$R(3,2)=\frac{16}{15}R(3,1)-\frac{1}{15}R(2,1)=0.9460831$$

$$R(3,3)=\frac{64}{63}R(3,2)-\frac{1}{63}R(2,2)=0.9460831$$

由于 R(3,3)与 R(3,2)完全相同，我们可以把 R(3,3)的值看做准确值，并以其为标准来计算各个值的绝对误差，据此得到各个值的有效数字。

近似值	绝对误差	有效数字
$R(3,0)=0.9456909$	0.0003922	3
$R(3,1)=0.9460833$	0.0000002	6
$R(3,2)=0.9460831$	0	7
$R(3,3)=0.9460831$	0	7

5.　（10 marks）Given the differential equation,

$$\begin{cases} \dfrac{dy}{dt} = f(t) \\ y(a) = 0 \end{cases}$$

and the step size is chosen to be $h = \dfrac{b-a}{n}$**,where n is some positive number.**

1、　Find the Euler recursive rule for numerically solving the equation;

2、　Find out the approximate solution at $t = b$**.**

解：**1、微分方程离散化**

$$\frac{y_{k+1} - y_k}{h} = f(t_k) = f(a+kh)$$

即可得到 Euler 递推法则

$$y_{k+1} = y_k + hf(a+kh) \qquad k = 0,1,2,...,n-1$$

2、 $\therefore$

$$\sum_{k=0}^{n-1} y_{k+1} = \sum_{k=0}^{n-1} y_k + h\sum_{k=0}^{n-1} f(a+kh)$$

$$y_1 + y_2 + ... + y_n = y_0 + y_1 + ... + y_{n-1} + h\sum_{k=0}^{n-1} f(a+kh)$$

$$y_n = y_0 + h\sum_{k=0}^{n-1} f(a+kh)$$

$$y(b) \approx y_n = y(a) + h\sum_{k=0}^{n-1} f(a+kh) = h\sum_{k=0}^{n-1} f(a+kh)$$

6.　（12 marks）Given four points on the plane: $(-1, y_1), (0, y_2), (1, y_3), (2, y_4)$**,**

and two base functions

$$f_1(x) = 1,\ f_2(x) = x + c$$

where c is some constant.

1、 Derive the normal equation for the linear least square problem for the linear combination.

$$f(x) = c_1 f_1(x) + c_2 f_2(x)$$

2、 Find out the proper value for c, so that the normal equation is easy to solve.

解：1、N=4

$$F = \begin{bmatrix} f_1(x_1) & f_2(x_1) \\ f_1(x_2) & f_2(x_2) \\ f_1(x_3) & f_2(x_3) \\ ... & ... \\ f_1(x_N) & f_2(x_N) \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \\ y_N \end{bmatrix}, C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$F'FC = F'Y$$

$$\begin{bmatrix} f_1(x_1) & f_1(x_2) & f_1(x_3) & \cdots & f_1(x_N) \\ f_2(x_1) & f_2(x_2) & f_2(x_3) & \cdots & f_2(x_N) \end{bmatrix} \begin{bmatrix} f_1(x_1) & f_2(x_1) \\ f_1(x_2) & f_2(x_2) \\ f_1(x_3) & f_2(x_3) \\ \cdots & \cdots \\ f_1(x_N) & f_2(x_N) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = F' \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdots \\ y_N \end{bmatrix}$$

$$\begin{bmatrix} \sum_{k=1}^{N} f_1^2(x_k) & \sum_{k=1}^{N} f_1(x_k) f_2(x_k) \\ \sum_{k=1}^{N} f_1(x_k) f_2(x_k) & \sum_{k=1}^{N} f_2^2(x_k) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N} f_1(x_k) y_k \\ \sum_{k=1}^{N} f_2(x_k) y_k \end{bmatrix}$$

$$\begin{bmatrix} N & cN + \sum_{k=1}^{N} x_k \\ cN + \sum_{k=1}^{N} x_k & \sum_{k=1}^{N} (x_k + c)^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N} y_k \\ \sum_{k=1}^{N} x_k y_k + c \sum_{k=1}^{N} y_k \end{bmatrix}$$

2、If $cN + \sum_{k=1}^{N} x_k = 0$, ie $c = -\frac{1}{N} \sum_{k=1}^{N} x_k$, then the normal equation is diagonal matrix and is easy to solve.