中山大学软件学院 **2011** 级软件工程专业(2012 学年秋季学期)

《数值分析》期中试题(B卷)

(考试形式：闭 卷　　考试时间：2 小时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：________________　　姓名：________　　学号：__________

1. Fill in the blanket with proper answers (5 marks each, total 30 marks)

1) Suppose $x = 0.544987104184$, then the approximate value 0.544986720817 has __6__ significant digits.

2) Let $A = \begin{pmatrix} 0 & 1 & 1 & 4 \\ 5 & 2 & -1 & 0 \\ 1 & 5 & 2 & 2 \\ 2 & -3 & 0 & 6 \end{pmatrix}$, then $\|A\|_1 =$ __12__ , $\|A\|_\infty =$ __11__ .

3) The recursive rule of secant method for solving nonlinear equation f(x)=0 is

$$p_{k+1} = p_k - \frac{f(p_k)}{f(p_k) - f(p_{k-1})}(p_k - p_{k-1})$$

and its speed of convergence is $\frac{1+\sqrt{5}}{2}$.

4) The function $g(x) = -4 + 4x - \frac{1}{2}x^2$ has __2__ fixed points, and the fixed point __x=4__ is attractive.

5) The process of Gaussian elimination with back substitution for solving the linear systems $Ax = b$, where A is an $n \times n$ matrix, requires $\underline{\frac{n^3}{3} + \frac{n^2}{2} - \frac{5n}{6}}$ multiplications and $\underline{\frac{n(n+1)}{2}}$ divisions.

说明：消去的过程乘法的次数为$\underline{\frac{n^3 - n}{3}}$， 除法的次数为$\underline{\frac{n(n-1)}{2}}$。回带是解上三角方程组，乘法的次数为$\underline{0+1+2+\ldots+(n-1) = \frac{n(n-1)}{2}}$， 除法的次数为$\underline{n}$。

6) For the linear system,

$$\begin{cases} 2x - y = -4 \\ x + 3y = -2 \end{cases}$$

starting with $(\mathrm{x}_0, \mathrm{y}_0) = (1, 2)$,the Gauss-Seidel iteration process yields

$$(\mathrm{x}_1, \mathrm{y}_1) = \underline{\left(-1, -\frac{1}{3}\right)}.$$

the Jacobi iteration process yields $(\mathrm{x}_1, \mathrm{y}_1) = \underline{(-1, -1)}$.

2. (20 marks)Consider the division of two numbers x, y with the approximate value x^*, y^*, analyzing the propagation of relative error.

解：记 $\varepsilon_x = x^* - x$， $\varepsilon_y = y^* - y$ 则

$$\frac{x^*}{y^*} - \frac{x}{y} = \frac{x+\varepsilon_x}{y+\varepsilon_y} - \frac{x}{y} = \frac{y\varepsilon_x - x\varepsilon_y}{y(y+\varepsilon_y)}$$

$$= \frac{\varepsilon_x}{(y+\varepsilon_q)} - \frac{x\varepsilon_y}{y(y+\varepsilon_y)}$$

$$\approx \frac{1}{y}\varepsilon_x - \frac{x}{y^2}\varepsilon_y$$

$$= \frac{1}{y}\left(\varepsilon_x - \frac{x}{y}\varepsilon_y\right)$$

所以

$$\frac{\frac{x^*}{y^*} - \frac{x}{y}}{\frac{x}{y}} \approx \frac{\frac{1}{y}\left(\varepsilon_x - \frac{x}{y}\varepsilon_y\right)}{\frac{x}{y}} = \frac{1}{x}\left(\varepsilon_x - \frac{x}{y}\varepsilon_y\right) = \frac{\varepsilon_x}{x} - \frac{\varepsilon_y}{y}$$

即除法的相对误差等于分子的相对误差与分母的相对误差之差！

3. (20 marks)Write down the cubic Lagrange interpolation to the function

$$f(x) = x^4 - 2x^3$$

where interpolation is to be exact at the four nodes $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 3$.

解：

x	-1	0	1	3
f(x)	3	0	-1	27

于是

$$L(x)=3\cdot\frac{x(x-1)(x-3)}{(-1)\cdot(-2)\cdot(-4)}-\frac{x(x+1)(x-3)}{1\cdot 2\cdot(-2)}+27\cdot\frac{x(x+1)(x-1)}{4\cdot 3\cdot 2}$$
$$=x^3+x^2-3x$$

4. (10 marks)How to select the initial value x_0, so that the fixed point iteration $\{x_n\}$ defined by $x_n=\varphi(x_{n-1})$, where $\varphi(x)=\frac{2-e^x}{5}$, always converges?

解：可以根据压缩映射定理来选择初值。容易证明 $\varphi(x)$ 是闭区间 $[-1,1]$ 上的压缩映射。

事实上，$\varphi(x)$ 单调下降，于是当 $x\in[-1,1]$ 时

$$-1\le\varphi(1)\le\varphi(x)\le\varphi(-1)\le 1$$

$$|\varphi'(x)|=\left|-\frac{e^x}{5}\right|\le\frac{e}{5}<1$$

于是初值 x_0 可以选 $[-1,1]$ 中的任意一点。

5. (20 marks) Find the triangular factorization $PA=LU$ for the following matrix

$$A=\begin{bmatrix}0&2&3\\1&2&6\\2&2&4\end{bmatrix}$$

解：

$$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 6 \\ 2 & 2 & 4 \end{bmatrix} \to \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 6 \\ 2 & 2 & 4 \end{bmatrix} \to \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$p=(1,2,3) \qquad p=(3,2,1) \qquad p=(3,2,1)$$

$$\to \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ 2 & 2 & 4 \end{bmatrix} \to \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ 2 & 2 & 4 \end{bmatrix} \to \begin{bmatrix} 0 & 2 & 3 \\ \frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 2 & 2 & 4 \end{bmatrix}$$

$$p=(3,2,1) \qquad p=(3,1,2) \qquad p=(3,1,2)$$

所以

$$P=\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, L=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}, U=\begin{pmatrix} 2 & 2 & 4 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{pmatrix}$$