中山大学软件学院 **2009** 级软件工程专业 (2010 学年秋季学期)

《SE-205 数值计算方法》期末试题 (A卷)

(考试形式：闭 卷 考试时间：2 小时)

警示 《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：_____ 姓名：_____ 学号：_____

(允许使用简单的计算器，使用智能手机、电脑等设备当作作弊处理)

1. Fill in the blanket with proper answers (5 marks each, total 40 marks)

   1）Suppose $\pi \approx 3.1415926$，then the approximate value 3.141601 has _____ significant digits.

   2）The numerical integration formula

   $$\int_{-1}^{1} f(x)dx \approx \frac{1}{3}\left[f\left(-\frac{1}{2}\right)+4f(0)+f\left(\frac{1}{2}\right)\right]$$ has _____ degrees of precision.

   3）For the numerical integration formula $\int_{a}^{b} f(x)dx \approx \sum_{i=1}^{n} A_i f(x_i)$, if _____, then we think it is **not** stable.

   4）The error term of Lagrange polynomial approximation for the function **f** at the nodes

   $$a \le x_0 < x_1 < ... < x_n \le b$$

   is

   _____
   _____.

   5）The Chebyshev polynomial of degree 4 is_____, and it has_____ points of alternation.

   6）The recursive rule of Newton's methods for solving nonlinear equation f(x)=0 is:

   _____,
   and its speed of convergence is_____.

   7）The fast algorithm for evaluate the polynomial $y = \sum_{i=0}^{n} a_n x^n$ is called _____, and the recursive rule is：

_____

_____.

8) For the following nonlinear system, write out the Seidel iteration formula

$$\begin{cases} 3x + y - z = 3 \\ 2x - 5y - z = -4 \\ x + 3y - 6z = -2 \end{cases}$$

_____

_____

2. Write out the three-moment equation(三弯矩方程) of the cubic spline that pass through $(-1, y_1)$, $(0, y_2)$ and $(1, y_3)$ with the clamped condition $S'(x_0) = y_0'$ and $S'(x_2) = y_2'$. (11 marks)

3. Given the function constrain table

x	0	2	3	4
f(x)	5	5	23	69

first construct the divided difference table，and then find the Newton interpolation polynomial.（12 marks）

4. Given the 3×3 matrix

$$A = \begin{bmatrix} 0 & -4 & -2 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

1、Find out the PA=LU factorization for A. The factorization should be correspondent to Gauss elimination.（10 marks）

2、Using the factorization to solve the linear system.（5 marks）

$$\begin{cases} -4x_2 - 2x_3 = -16 \\ x_1 + x_2 + x_3 = 4 \\ 2x_1 - 2x_2 + x_3 = -6 \end{cases}$$

5. Using the Romberg algorithm to find the numerical integration for $\int_0^1 x^2 dx$. It's only to calculate up to that [0,1] is divided into 4 subintervals.（12 marks）

6. Given the differential equation,

$$\begin{cases} \dfrac{dy}{dx} = 0.5y \\ y(0) = 2 \end{cases}$$

and the step size is chosen to be $h = 0.2$,

    1、 Write the Euler recursive formula.（6 marks）

    2、 Find out the approximate solution at x=1.（4 marks）

(EXTRA WORK) Given three points on the plane: $(-1, y_1), (0, y_2), (1, y_3)$, and three base functions

$$f_1(x) = 1$$
$$f_2(x) = x$$
$$f_3(x) = a + bx + x^2$$

where a and b are some fixed constants.

1. Derive the normal equation for the linear least square problem for the linear combination: (6 marks)
$$f(x) = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x)$$

2. Do you think what are the best choice for a and b, so that the normal equation is easy to solve?   （4 marks）