中山大学软件学院 **2010** 级软件工程专业(2011学年秋季学期)

《数值分析》期末试题(B卷)

(考试形式：闭 卷 考试时间：2小时)

警示

《中山大学授予学士学位工作细则》第六条

考试作弊不授予学士学位

方向：_____ 姓名：_____ 学号：_____

1. Fill in the blanket with proper answers (5 marks each, total 20 marks)

1) Suppose $x = 29.56789...$, then the approximate value $x^* = 30.00789...$ has _____ significant digits.

2) The error term of Lagrange polynomial approximation for the function **f(x)** at the nodes $a \le x_0 < x_1 < ... < x_n \le b$ is

_____.

3) The recursive rule of **Secant methods** for solving nonlinear equation f(x)=0 is

_____.

and its speed of convergence is ： _____ .

4) The fast algorithm for evaluate the Newton polynomial $y = \sum_{k=0}^{n} a_k \prod_{i=0}^{k-1} (x - x_i)^k$ is somewhat like Horner's algorithm, and the recursive rule is：

_____.

5) Using the Seidel iteration method to solve the following nonlinear system,

$$\begin{cases} 2x_1 + x_2 = -12 \\ -x_1 + 2x_2 = 20 \end{cases}$$

Suppose the iteration format be the following matrix form：

$$X^{(k+1)} = BX^{(k)} + f$$

then $B =$ _____。

2. (20 marks)Given the function constrain table

x	−1	2	3	5
f(x)	−7	−19	−31	5

first construct the divided difference table，and then find the Newton interpolation polynomial.

3. (20 marks)Consider the nonlinear system

$$\begin{cases} 2x^2 - y^2 + 4x - 5 = 0 \\ x - 2y + 1 = 0 \end{cases}$$

1) Find analytically the zeros of the system;
2) Write out the Newton iteration for the system.

4. (20 marks) In order to solve the nonlinear equation $f(x)=e^{x}+10x-2=0$, we design the following fixed point iteration:

$$\begin{cases} x_0 = 0 \\ x_k = \dfrac{2-e^{x_{k-1}}}{10} \quad k > 0 \end{cases}$$

1、 Show that the equation has **unique** root;

2、 Show that for any initial value in [-1,1], the fixed point iteration converges to the unique root.（Hint：Verify that on [-1,1]， $\varphi(x)$ is a contraction mapping）

5. (20 marks)Given the 4×4 matrix

$$A=\begin{bmatrix}1 & 2 & 1 & 4\\ 2 & 0 & 4 & 3\\ 4 & 2 & 2 & 1\\ -3 & 1 & 3 & 2\end{bmatrix}$$

1、Find out the PA=LU factorization for A. The factorization should be correspondent to Gauss elimination.（10 marks）

2、If you have had the PA=LU factorization, analyze the computational complexity of finding the inverse of A.（10 marks）