中 山 大 学 软 件 学 院 **2011** 级 软 件 工 程 专 业 (2012 学年秋季学期)

《数 值 分 析》 期 中 试 题 (A 卷)

(考 试 形 式： 闭 卷 考 试 时 间：2 小 时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：________________ 姓名：________ 学号：__________

1. Fill in the blanket with proper answers (5 marks each, total 30 marks)

1) Near an attractive fixed point P, the fixed point iteration $\{p_n\}$, defined by $p_n = g(p_{n-1})$, satisfies the error estimation:

$$\left|P-p_n\right| \le \frac{K^n}{1-K}\left|p_1-p_0\right| \text{或} \left|P-p_n\right| \le \frac{K}{1-K}\left|p_n-p_{n-1}\right|$$

其中 K 是压缩映射定理中的常数，可以取 $K \approx g'(P)$。

2) The error term of Lagrange polynomial approximation for the function $f(x)$ at the nodes $a \le x_0 < x_1 < ... < x_n \le b$ is

$$f(x) - P_N(x) = \frac{f^{(N+1)}(\xi)}{(N+1)!}\omega_{N+1}(x)$$

3) The recursive rule of Newton's methods for solving nonlinear equation f(x)=0 is

$$p_k = g(p_{k-1}) = p_{k-1} - \frac{f(p_{k-1})}{f'(p_{k-1})} \qquad k = 1, 2, ...$$

and its speed of convergence is ____1____ near a multiple root.

4) The function $g(x) = -4 + 4x - \frac{1}{2}x^2$ has ____2____ fixed points, and the fixed point $x = 4$ is attractive.

5) The fast algorithm for evaluate the polynomial $y = \sum_{i=0}^{n} a_i x^i$ is called 秦九韶或 Horner 算法, and the recursive rule is

$$\begin{cases} b_n = a_n \\ b_k = a_k + x_0 b_{k+1} \quad k = n-1, n-2, ..., 0 \end{cases}$$

$$b_0 = P(x_0)$$

2. For the linear system,

$$\begin{cases} 3x+y-z=3 \\ 2x-5y-z=-4 \\ x+3y-6z=-2 \end{cases}$$

Write out the Gauss-Seidel iteration formula in matrix form $x^{(k+1)}=Bx^{(k)}+f$.

$$\begin{cases} 3x=-y+z+3 \\ 2x-5y=z-4 \\ x+3y-6z=-2 \end{cases}$$

$$\begin{pmatrix} 3 & 0 & 0 \\ 2 & -5 & 0 \\ 1 & 3 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}+\begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 3 & 0 & 0 \\ 2 & -5 & 0 \\ 1 & 3 & -6 \end{pmatrix}^{-1}\begin{pmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}+\begin{pmatrix} 3 & 0 & 0 \\ 2 & -5 & 0 \\ 1 & 3 & -6 \end{pmatrix}^{-1}\begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ \frac{2}{15} & -\frac{1}{5} & 0 \\ \frac{11}{90} & -\frac{1}{10} & -\frac{1}{6} \end{pmatrix}\begin{pmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}+\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ \frac{2}{15} & -\frac{1}{5} & 0 \\ \frac{11}{90} & -\frac{1}{10} & -\frac{1}{6} \end{pmatrix}\begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{2}{15} & -\frac{1}{15} \\ 0 & -\frac{11}{90} & \frac{1}{45} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}+\begin{pmatrix} 1 \\ \frac{6}{5} \\ \frac{11}{10} \end{pmatrix}$$

所以

$$B=\begin{pmatrix} 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{2}{15} & -\frac{1}{15} \\ 0 & -\frac{11}{90} & \frac{1}{45} \end{pmatrix}, f=\begin{pmatrix} 1 \\ \frac{6}{5} \\ \frac{11}{10} \end{pmatrix}$$

3. (20 marks)Write down the cubic Newton interpolation to the function

$$f(x) = 2x^4 - 5x^3$$

where interpolation is to be exact at the four nodes $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 3$.

解：

x	f(x)	一阶均差	二阶均差	三阶均差
−1	7			
0	0	−7		
1	−3	−3	2	
3	27	15	6	1

于是

$$N(x) = 7 - 7(x+1) + 2x(x+1) + x(x+1)(x-3)$$

4. (20 marks) Find the triangular factorization $A = LU$ for the following matrix

$$A=\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 6 \\ -5 & 2 & -1 \end{bmatrix}$$

解：

$$A=\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 6 \\ -5 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & -3 \\ -5 & 2 & 14 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & -3 \\ -5 & 2 & 20 \end{bmatrix}$$

所以

$$\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 6 \\ -5 & 2 & -1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -5 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 20 \end{pmatrix}$$

5. (20 marks)Consider the nonlinear system

$$\begin{cases} x^2 - 2x - y + 0.5 = 0 \\ x^2 + 4y^2 - 4 = 0 \end{cases}$$

Write out the Newton iteration for the system.

解：

$$J = \begin{pmatrix} 2x-2 & -1 \\ 2x & 8y \end{pmatrix}$$

$$J^{-1} = \frac{1}{2x-16y+16xy}\begin{pmatrix} 8y & 1 \\ -2x & 2x-2 \end{pmatrix}$$

Newton iteration is

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} - \frac{1}{2x_k-16y_k+16x_ky_k}\begin{pmatrix} 8y_k & 1 \\ -2x_k & 2x_k-2 \end{pmatrix}\begin{pmatrix} x_k^2-2x_k-y_k+0.5 \\ x_k^2+4y_k^2-4 \end{pmatrix}$$