

Problem 2 Write your answers using MS Word or Latex and save it as a single PDF file:

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Independent Events and Bayes Theorem [20 pts]

1.1. [5 Points] For events A, B prove:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

($\neg A$ denote the event that A does not occur.)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Using the law of probability where,

$$P(A) = \sum P(A \cap B_n) = \sum P(A|B_n)P(B_n)$$

Our new function becomes,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

1.2. Let X, Y, and Z be random variables taking values in {0, 1}. The following table lists the probability of each possible assignment of 0 and 1 to the variables X, Y, and Z:

| | Z = 0 | | Z = 1 | | Total |
|-------------|-------|-------|-------------|-------|---------|
| | X = 0 | X = 1 | X = 0 | X = 1 | Y=0,Y=1 |
| Y = 0 | 0.1 | 0.05 | 0.1 | 0.1 | 0.35 |
| Y = 1 | 0.2 | 0.1 | 0.175 | 0.175 | 0.65 |
| | 0.3 | 0.15 | 0.275 | 0.275 | |
| X=0, X=1 | 0.575 | 0.325 | Z=0, Z=1 | 0.45 | 0.55 |

(a) [5 Points] Is X independent of Y? Why or why not?

$$\begin{aligned} X \text{ is independent if } P(X \cap Y) &= P(X) * P(Y), \\ P(X = 0) &= 0.1 + 0.05 + 0.1 + 0.175 = 0.575 \\ P(Y = 0) &= 0.1 + 0.05 + 0.1 + 0.1 = 0.35 \\ P(X) * P(Y) &= 0.575 * 0.35 = .201 \\ P(Y = 0 \cap X = 0) &= 0.1 + 0.1 = 0.2 \\ X \text{ is not independent of } Y \end{aligned}$$

(b) [5 Points] Is X conditionally independent of Y given Z? Why or why not?

$$\begin{aligned} X \text{ is conditionally independent of } Y \text{ given } Z \text{ if } P(X|Y \cap Z) &= P(X|Z) \\ P(X|Z) &= \frac{P(X \cap Z)}{P(Z)} = \frac{0.45 * 0.575}{0.45} = 0.575 \\ P(X|Y \cap Z) &= \frac{P(X \cap (Y \cap Z))}{P(Y \cap Z)} = \frac{0.575 * (0.15)}{0.15} = 0.575 \end{aligned}$$

X is conditionally independent of Y given Z

(c) [5 Points] Calculate $P(X \neq Y | Z = 0)$.

$$P(X \neq Y|Z = 0) = 0.05 + 0.2 = 0.25$$

