COSC1125/1127 Artificial Intelligence

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Semester 1, 2018
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Tutorial Sheet 10
Probability Reasoning

We use $\Pr(\cdot)$ (sometimes also written $P(\cdot)$) to refer to a probability function or probability distribution. When using an upper letter (e.g., X or $Cavity$), we refer to the random variable; when using lowercases we refer to a specific value of the corresponding random variable. So, $\Pr(Cavity)$ is a probability distributio over variable $Cavity$; whereas $\Pr(cavity)$ is a shorthand for $\Pr(Cavity = true)$ and $\Pr(\neg cavity)$ is a shorthand for $\Pr(Cavity = false)$.

1. Prove, formally, that $\Pr(A \mid B \wedge A) = 1$.

2. Consider the domain of dealing 5-card poker hands from a standard deck of 52 cards, under the assumption that the dealer is fair.

   (a) How many atomic events are there in the joint probability distribution (i.e., how many 5-card hands are there)?

   (b) What is the probability of each atomic event?

   (c) What is the probability of being dealt a royal straight flush (the ace, king, queen, jack and ten of the same suit)?

   (d) What is the probability of four of a kind?

3. Given the full joint distribution shown in the table below, calculate the following:

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	.108	.012	.072	.008
$\neg cavity$	.016	.064	.144	.576

   (a) $\Pr(toothache)$

   (b) $\Pr(Cavity)$

   (c) $\Pr(Toothache \mid cavity)$

   (d) $\Pr(Cavity \mid toothache \vee catch)$.

4. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

5. Prove, formally, that $\Pr(A \wedge B \wedge C) = \Pr(A \mid B \wedge C) \times \Pr(B \mid C) \times \Pr(C)$.

6. Prove Bayes' Theorem: $\Pr(A \mid B) = \dfrac{\Pr(B \mid A) \times \Pr(A)}{\Pr(B)}$.