# Assignment 4 of MATH 2005

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## 1

$$
\begin{aligned}
E[X] &= \sum_{n=1}^{k} n f(n) \\
&= \frac{1}{k} \sum_{n=1}^{k} n \\
&= \frac{1}{k} \frac{k(k+1)}{2} \\
&= \frac{k+1}{2} \\
var(X) &= \sum_{n=1}^{k} (n - E[X])^2 f(n) \\
&= \frac{1}{k} \sum_{n=1}^{k} (n - \frac{k+1}{2})^2 \\
&= \frac{1}{k} \sum_{n=1}^{k} (n^2 - n(k+1) - \frac{(k+1)^2}{4}) \\
&= \frac{1}{k} (\frac{k(k+1)(2k+1)}{6} - \frac{k(k+1)}{2}(k+1) + \frac{k(k+1)^2}{4}) \\
&= \frac{(k+1)(2k+1)}{6} - \frac{(k+1)^2}{2} + \frac{(k+1)^2}{4} \\
&= \frac{(k+1)(2k+1)}{6} - \frac{(k+1)^2}{4} \\
&= (k+1) \frac{k-1}{12} \\
&= \frac{k^2-1}{12}
\end{aligned}
$$

## 2

$$\begin{aligned}
b(x;n,\theta) =& C_n^x \theta^x (1-\theta)^{n-x} \\
=& C_n^{n-x} (1-\theta)^{n-x} \theta^x \\
=& b(n-x;n,1-\theta)
\end{aligned}$$

### 2.1

$$\begin{aligned}
B(n-x;n,1-\theta) - B(n-x-1;n,1-\theta) =& \sum_{y=0}^{n-x} b(y;n,1-\theta) \\
& - \sum_{y=0}^{n-x-1} b(y;n,1-\theta) \\
=& b(n-x;n,1-\theta) \\
=& b(x;n,\theta)
\end{aligned}$$

### 2.2

$$B(n;n,1-\theta) = \sum_{y=0}^{n} b(y;n,\theta) = 1$$

$$\begin{aligned}
B(x;n,\theta) =& \sum_{y=0}^{x} b(y;n,\theta) \\
=& \sum_{y=0}^{x} [B(n-y;n,1-\theta) - B(n-y-1;n,1-\theta)] \\
=& B(n;n,1-\theta) - B(n-x-1;n,1-\theta) \\
=& 1 - B(n-x-1;n,1-\theta)
\end{aligned}$$