Assignment 6 of CISC 2002

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1

$$
\begin{aligned}
\lim_{x\to 0} f(x) =& \lim_{0}^{1} \frac{\sin x}{x} \\
=& \lim_{0}^{1} \frac{\cos x}{1} \\
=& 1 \\
=& f(0)
\end{aligned}
$$

Thus, $f(x)$ is continuous on $[0, 1]$

1.1

$$
\begin{aligned}
I - h \approx& hf(\frac{h}{2}) \\
=& 1f(\frac{1}{2}) \\
=& 2\sin\frac{1}{2} \\
\approx& 0.9589
\end{aligned}
$$

1.2

$$
\begin{aligned}
I_h \approx& \frac{h}{2}(f(0) + f(h)) \\
=& \frac{1}{2}(1 + sin 1) \\
\approx& 0.9207
\end{aligned}
$$

1.3

$$
\begin{aligned}
I_h \approx & \frac{h}{6}(f(0)+4f(\frac{h}{2})+f(h)) \\
= & \frac{1}{6}(1+4\times 2\sin\frac{1}{2}+\sin 1) \\
\approx & 0.9461
\end{aligned}
$$

1.4

$$
\begin{aligned}
h = & \frac{1-0}{6} \\
= & \frac{1}{6}
\end{aligned}
$$

$$
\begin{aligned}
I_h \approx & \frac{h}{2}(f(x_0)+2(f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5))+f(x_6)) \\
= & \frac{1}{12}(1+2(6\sin\frac{1}{6}+3\sin\frac{2}{6}+2\sin\frac{3}{6}+\frac{3}{2}\sin\frac{4}{6}+\frac{6}{5}\sin\frac{5}{6})+\sin 1) \\
\approx & 0.9454
\end{aligned}
$$

1.5

$$
\begin{aligned}
I_h \approx & \frac{3}{h}(f(x_0)+4(f(x_1)+f(x_3)+f(x_5))+2(f(x_2)+f(x_4))) \\
\approx & 0.9461
\end{aligned}
$$

2

2.1

$$
I_h \approx \frac{1}{3\times 2^n}[f(x_0)+4\sum_{i=1}^{2^{n-1}} f(x_{2i-1})+2\sum_{i=1}^{2^{n-1}-1} f(x_{2i})+f(x_{2^n})]
$$

```
clear
clc
y=1:1:20;
x=1:1:20;
for i=1:1:20
    i
    e1=abs(sin(10)-simpson(i));
    e2=abs(sin(10)-simpson(i+1));
    e=e1/e2
    y(i)=e1;
end
```

```
semilogy(x,y)


function answer=simpson(n)
    answer=f(0);
    d=10.0/2^n;
    for i=1:1:2^(n-1)
        answer=answer+f(10*(2*i-1)/2^n)*4;
    end
    for i=1:1:2^(n-1)-1
        answer=answer+f(10*(2*i)/2^n)*2;
    end
    answer=answer+f(10);
    answer=d/3*answer;
end

function y=f(x)
    y=cos(x);
end
```

Listing 1: Code

```
i =

     1


e =

    7.4241


i =

     2


e =

   40.3726


i =

     3


e =

   18.6527


i =

     4
```

```


e =

    16.5811


i =

       5


e =

    16.1409


i =

       6


e =

    16.0350
```

Listing 2: Output

2.2

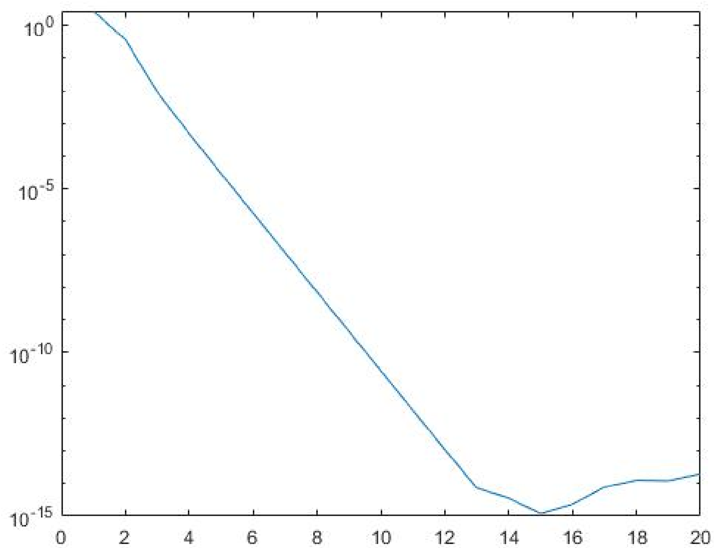

Figure 1: Figure

3

$$
\begin{aligned}
f(x) =& f(\frac{3h}{2}) + f'(\frac{3h}{2})(x - \frac{3h}{2}) + \frac{f''(\frac{3h}{2})}{2!}(x - \frac{3h}{2})^2 \\
&+ \frac{f^{(3)}(\frac{3h}{2})}{3!}(x - \frac{3h}{2})^3 + + \frac{f^{(4)}(\frac{3h}{2})}{4!}(x - \frac{3h}{2})^4 \\
&+ \frac{f^{(5)}(\frac{3h}{2})}{5!}(x - \frac{3h}{2})^5 + r_5(x - \frac{3h}{2})^5 + ...
\end{aligned}
$$

$$
\begin{aligned}
\int_0^{3h} f(x)dx =& 3hf(\frac{3h}{2}) + \frac{9}{4}h^3\frac{f''(\frac{3h}{2})}{2!} + \frac{243}{80}h^5\frac{f^{(4)}(\frac{3h}{2})}{4!} + ... \\
e =& \int_0^{3h} f(x)dx - \frac{3h}{8}(f(0) + 3f(h) + 3f(2h) + f(3h))
\end{aligned}
$$

$$
\begin{aligned}
f(0) =& f(\frac{3h}{2}) - \frac{3h}{2}f'(\frac{3h}{2}) + \frac{3h}{2}^2\frac{f''(\frac{3h}{2})}{2!} - \frac{3h}{2}^3\frac{f^{(3)}(\frac{3h}{2})}{3!} + ...\\
f(h) =& f(\frac{3h}{2}) - \frac{h}{2}f'(\frac{3h}{2}) + \frac{h}{2}^2\frac{f''(\frac{h}{2})}{2!} - \frac{h}{2}^3\frac{f^{(3)}(\frac{3h}{2})}{3!} + ...\\
f(2h) =& f(\frac{3h}{2}) + \frac{h}{2}f'(\frac{3h}{2}) + \frac{h}{2}^2\frac{f''(\frac{h}{2})}{2!} + \frac{h}{2}^3\frac{f^{(3)}(\frac{3h}{2})}{3!} + ...\\
f(3h) =& f(\frac{3h}{2}) + \frac{3h}{2}f'(\frac{3h}{2}) + \frac{3h}{2}^2\frac{f''(\frac{3h}{2})}{2!} + \frac{3h}{2}^3\frac{f^{(3)}(\frac{3h}{2})}{3!} + ...
\end{aligned}
$$

$$
\begin{aligned}
e =& -\frac{9}{10}h^5\frac{f^{(4)}(\frac{3h}{2})}{4!}\\
=& O(h^5)
\end{aligned}
$$

4

4.1

```
clear
clc
t=0:0.1:3*pi;
x=sin(t);
y=cos(2.*t);
plot3(x,y,t)
```

Listing 3: Code

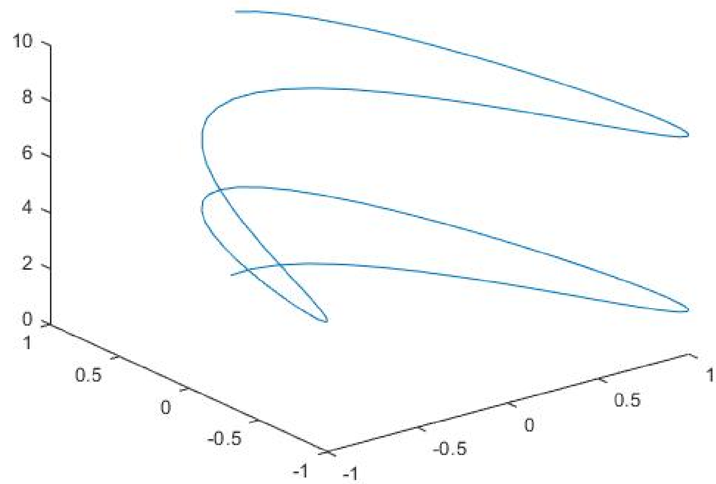

Figure 2: Figure

4.2

$$
\begin{aligned}
s &= \int_0^{3\pi} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2} dt \\
&= \int_0^{3\pi} \sqrt{\cos^2 t + 4\sin^2 2t + 1} dt
\end{aligned}
$$

$$h = \frac{3\pi}{14}$$

$$s = \frac{h}{3}(f_0 + 4f_1 + 2f_2 ... 4f_{14} + f_{15})$$

```
clear
clc
h=(3*pi)/14;
answer=f(0)+f(3*pi);
for i=1:1:7
    a=2*i-1;
    x=a*h;
    answer=answer+4*f(x);
    a=2*i;
    x=a*h;
```

```
    answer=answer+2*f(x);
end
answer=answer*h/3

function y=f(x)
    y=sqrt((cos(x))^2+4*(sin(2*x))^2+1);
end
```

Listing 4: Code

```
answer =

   17.8194
```

Listing 5: Output

4.3

```
clear
clc
f=quad(@(x) sqrt(cos(x).^2+4*sin(2*x).^2+1),0,3*pi)
```

Listing 6: Code

```
f =

   17.1914
```

Listing 7: Output

5

5.1

```
clear
clc
g=@(x) besselj(0,x);
x=0:0.01:20;
y=g(x);
plot(x,y)
h=20/30;
answer=g(0)+g(20);
for i=0:1:14
    a=2*i+1;
    x=a*h;
    answer=answer+4*g(x);
    a=2*i;
    x=a*h;
    answer=answer+2*g(x);
end
answer=answer-2*g(0);
answer=answer*h/3
```

Listing 8: Code

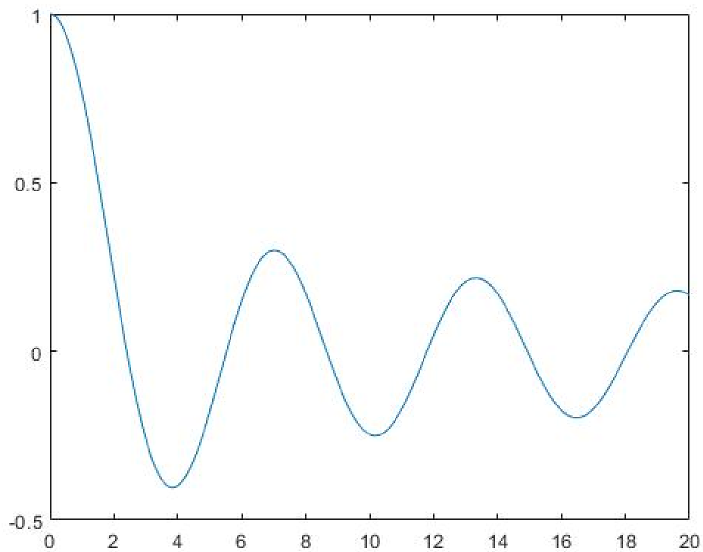

Figure 3: Figure

5.2

```
answer =

    1.0585
```

Listing 9: Output

5.3

```
clear
clc
f=quad(@(x) besselj(0,x),0,20)
```

Listing 10: Code

```
output
f =

    1.0584
```

Listing 11: Output