Absolute or Relative World
Uses and Abuses of Percentages

A. Uses of Percentages

1a. Use percentage to describe a relative amount in total

$$Percentage\ of\ an\ Amount\ in\ Total = \frac{Amount}{Total\ Amount}\ 100\% \quad p\% = \frac{A}{T}\ 100\%$$

$$A = Total \cdot p\%$$

1b. Use percentage to compare two values in relative term

$$Percentage\ of\ a\ Compared\ value = \frac{Compared\ valure}{Reference\ value}\ 100\% \quad p\% = \frac{V_n}{V_0}\ 100\%$$

$$Compared\ value = reference\ value \cdot p\%$$

$$V_n = V_0 \cdot p\%$$

	Greater than 1 Greater than 100%	Less than 1 Less than 100%
Rate	4 to 1	1 to 4
Multiple/Fraction	4 times of	¼ of
Decimals	4.5 times of	0.25 of
Percentage	400% of	25% of

Example 1: (Uses of Percentages)

Country	Population (millions*) 2017	GDP per capita (current US$) 2017
China	1420 (1416.48)	8830 (8827)
USA	327 (327.36)	59500 (59532)
World Total	7650 (7654.12)	

*correct to three significant number

(a) China's population is _____ % of world population

$$p\% = \frac{A}{T} \cdot 100\% = \frac{1420}{7650} \cdot 100\% = 18.56\% \quad or \quad 19\% \quad or \quad \frac{1}{5}$$

(b) USA's population is _____ % of China's population

$$p\% = \frac{V_n}{V_0} \cdot 100\% = \frac{327}{1420} \cdot 100\% = 23.03\% \quad or \quad 23\% \quad less\ than\ \frac{1}{4}$$

(c) China's per capita GDP is _____ % that of USA

$$p\% = \frac{V_n}{V_0} \cdot 100\% = \frac{8830}{59500} \cdot 100\% = 14.84\% \quad or \quad 15\% \quad or \quad < \frac{1}{7}$$

(d) US's per capita GDP is _____ % that of China

$$p\% = \frac{V_n}{V_0} \cdot 100\% = \frac{59500}{8830} \cdot 100\% = 673.84\% \quad or \quad 674\%, \quad > 6.7\ times$$

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2. Percentage Change

Percentage change is to describe a change of a value/difference of values in relative terms

$$\text{Percentage Change} = \frac{change}{Original\ value} \cdot 100\%$$

Note: a change can be an increase/growth or a decrease/decay/

2a. Relative Change in a Value/ Growth Rate

$p.c.\% = \dfrac{New\ value - Original\ value}{Original\ value} \cdot 100\%$	$p.c.\% = \dfrac{V_n - V_o}{V_o} \times 100\%$
New value = Original value $(1 + p.c.\%)$	$V_n = V_o(1 + p.c.\%)$

Example 2: (Describe Changes in World Population in Absolute Term and Relative Term)
World population was 2.6 billion in 1950 and 7.65 billion in 2018. Describe the absolute change and relative change in world population from 1950 to 2018.

- absolute change: $= V_n - V_o = 7.65 - 2.6 = 5.05$ (billion)
- relative change: $p.c.\% = \frac{V_n - V_o}{V_o} \cdot 100\% = \frac{7.65 - 2.6}{2.6} \cdot 100\% = 194\%$

2b. Relative Comparison of Two Values

Use percentage change to describe the difference of two values in relative terms

$p.c.\% = \dfrac{compared\ value - reference\ value}{reference\ value} \cdot 100\%$	$p.c.\% = \dfrac{V_n - V_o}{V_o} \times 100\%$
Compared value = Reference value $(1 + p.c.\%)$	
$V_n = V_o(1 + p.c.\%)$	V_o : Reference Value

Example 3: (Uses of Percentage change)

(a) China's population is _____ % more than US's population

$$p.c.\% = \frac{V_n - V_o}{V_o} \cdot 100\% = \frac{1420 - 327}{327} \cdot 100\% \approx 334\%$$

(b) US's population is _____ % less than China's population

$$p.c.\% = \frac{V_n - V_o}{V_o} \cdot 100\% = \frac{327 - 1420}{1420} \cdot 100\% = -77\%$$

(c) USA's per capita GDP is _____ % more than that of China

$$p.c.\% = \frac{V_n - V_o}{V_o} \cdot 100\% = \frac{59500 - 8830}{8830} \cdot 100\% = 574\%$$

2c Percentage and Percentage Change

$$V_n = V_o(1 + p.c.\%) = V_o(p\%)$$

$$1 + p.c.\% = p\%$$

Example 4: (Compare "of" with "more than/less than")
A population that triples in size from 200 to 600.
- Using *more than*: the new population is 200% more than the original population
- Using *of*: the new population is 300% of the original population.

$$\text{: "more than": } p.c.\% + 1 = p\% \text{ : } p.c.\% = p\% - 1 = 300\% - 1 = 200\%$$

$$\text{: "of": } p\% = \frac{V_n}{V_0} \cdot 100\% = \frac{600}{200} \cdot 100\% = 300\%$$

(Salary Difference) Carol earns 50% more than Peter. How many times is her income of his?

$$p.c.\% = 50\% \text{ : } p\% = 1 + p.c.\% = 1 + 50\% = 150\%$$

3. *Percentage change* of Percentages
Percentage change in percentages is to describe the change of a percentage value or difference of two percentage values in relative term.

Percentage change of the percentage	$p.c.\% = \dfrac{p_n\% - p_0\%}{p_0\%} \times 100\%$

$$p_n\% = p_0\%(1 + p.c.\%)$$

Example 5: Percentage point (in absolute term) versus **Percentage change% (in relative term)** Suppose your bank increases the interest rate on your savings account from 4% to 6%, describe this change of interest rate (1) in absolute term and (2) in relative term.

$$abs. \ change = p_n\% - p_0\% = 6\% - 4\% = 2\% \quad 2 \text{ percentage points}$$

$$relative \ change: \ p.c.\% = \frac{p_n - p_0}{p_0} \cdot 100\% = \frac{6\% - 4\%}{4\%} \cdot 100\% = 50\%$$

Example 6: China's GDP grow at a rate of 8%; while energy consumption grow at a rate of 15%. Compare these rates (1) in absolute term and (2) in relative term.

$$absolute \quad Difference = 15\% - 8\% = 7 \text{ percentage points}$$

energy consumption growth rate is 7 percentage points more than GDP growth rate.

$$rel. \ Diff: \ p.c.\% = \frac{15\% - 8\%}{8\%} \cdot 100\% =$$

Example 7: *Compare* percentage points *with* percentage
Assume 40% votes are Republicans in city A.
a. The percentage of voters registered as republicans is 25% higher in city B than in city A. *(p.c.%)* What percentage of the registered voters in City B are Republicans?

$$G: \ p_0\% = 40\% \ ; \ p.c.\% = 25\% \ .$$

$$F: \ p_n\% = ? \quad p_n = p_0(1 + p.c.\%) = 40\%(1 + 25\%) = 50\%$$

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b. The percentage of voters registered as republicans is 25 percentage points higher in city B than in city A. What percentage of the registered voters in City B are Republicans?

$$P_o = 40\%, \qquad P_n = ?$$

$$P_n - P_o = 25 \text{ percentage points}$$

$$P_n = P_o + 25\% = 40\% + 25\% = 65\%$$

B. Abuses of Percentages

Example 8: Shifting reference values

Your employer promise to raise your salary by 10% two months later, after cutting your salary by 10% now, will you accept the offer?

$$\boxed{V_n = V_o \left(1 + p.c\%\right)} \qquad p.c\% = -10\%, +10\% ?$$

$$V_n = \left[V_o \left(1 - 10\%\right)\right]\left(1 + 10\%\right)$$

$$= V_o \left(0.9\right)\left(1.1\right)$$

$$= V_o \left(0.99\right) \qquad \boxed{V_n = V_o \left(1 - 1\%\right)}$$

$$= V_o \left(1 - 0.01\right)$$

Example 9: Adding and Subtracting % decreased by 1%.

(Shifting Investment Value) A stockbroker offers the following defense to angry investors: "I admit that the value of your investment fell 60% during my first year on the job. This year, however, their value has increased by 75%, so you are now 15% ahead!". Evaluate stockbrokers defense.

$$V_n = \left[V_o \left(1 - 60\%\right)\right]\left(1 + 75\%\right) = V_o \left(0.6\right)\left(1.75\right)$$

$$= V_o \left(0.7\right)$$

$$= V_o \left(1 - 30\%\right)$$

decreased by 30%

Example 10: Average of two percentages (different base)

Suppose you got 70% of the questions correct on a midterm exam (10 questions) and 90% correct on the final exam (20 questions). Can you conclude that you answered 80% of all the questions correct?

$$p\% = \frac{A_1 + A_2}{T} = \frac{70\%(10) + 90\%(20)}{10 + 20} = \frac{7 + 18}{30} \cdot 100\%$$

$$= 83.33\%.$$

Impossible Percentage: Less than nothing

A store advertises that it will take "150% off" the price of all merchandise. What should happen when you go to the counter to buy a $500 item?

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<div align="center">Classwork .   Average Annual Growth Rate</div>

Example 1: Compare two values in absolute term and in relative term.

(a) GDP per capita of US is _____ $50705 \ USD$ _____ more than that of China in 2017 in absolute term.

$$Change = V_n - V_0 = 59532 - 8827 = 50705 \ (USD)$$

(b) GDP per capita of US is _____ 574% _____ more than that of China in 2017 in relative term (in percentage change).

$$P.C\% = \frac{V_n - V_0}{V_0} \cdot 100\% = \frac{59532 - 8827}{8827} \cdot 100\% = 574\%$$

Example 2: Compare two percentages in absolute term and in relative term

(a) GDP growth rate of China is _____ $4.63 \ percentage \ points$ _____ more than that of US in 2017 in absolute term.

$$P_n - P_0 = 6.9\% - 2.27\% = 4.63 \ (percentage \ points)$$

(b) GDP growth rate of China is _____ 204% _____ more than that of US in 2017 in relative term (in percentage change).

$$\frac{P_n - P_0}{P_0} \cdot 100\% = \frac{6.9\% - 2.27\%}{2.27\%} \cdot 100\% = 204\%$$

Example 3: Total Growth Rate over a period of time

(a) "If the GDP in China grow by 7.3% in 2014, 6.9 % in 2015 and 6.7 % in 2016, then a total growth of 20.9 % over the three-year period." True or False? Why?

$$False$$

(b) Find the true total growth rate of GDP in China over the three years in 2013 to 2016.

$$V_n = V_0 (1 + 7.3\%)(1 + 6.9\%)(1 + 6.7\%) = V_0 (1 + 0.2239)$$

Increase by 22.39% over the 3 years from 2013 to 2016.

Example 4: Annual Growth Rate over a period of time

(a) What is the (average) annual growth rate of GHG emissions (kt of oil equivalent) in China between 2000 and 2012. (correct to 1 decimal places) $t = 2012 - 2000 = 12$

$$V_n = V_0 (1+r)^t;$$

$$12454710.6 = 5082325.6(1+r)^{12}$$

$$\boxed{r = \left(\frac{V_n}{V_0}\right)^{\frac{1}{t}} - 1} \qquad r = 7.8\%$$

(b) What is the (average) annual growth rate of GHG emissions (kt of oil equivalent) per capita in US between 2000 and 2012. (correct to 1 decimal places)

$$V_n = V_0 (1+r)^t;$$

$$6343840.5 = 6969123.8(1+r)^{12}$$

$$r = \left(\frac{V_n}{V_0}\right)^{\frac{1}{t}} - 1 \qquad r = -0.8\%$$

Exercise 1:

(a) CO2 emissions in metric tons per capita of China is __8.95 metric tons per capita__ less than that of US in 2014 in absolute term.

(b) CO2 emissions in metric tons per capita of China is __54.25%__ less than that of US in 2014 in relative term (in percentage change).

(c) GDP per capita growth rate of China is __4.76 percentage points__ more than that of US in 2017 in absolute term.

(d) GDP per capita growth rate of China is __307.50%__ more than that of US in 2017 in relative term (in percentage change).

Exercise 2:

(a) What is the annual growth rate of CO2 emissions (metric tons per capita) in China between 2000 and 2014. (correct to 2 decimal places)

$$r = \left(\frac{V_n}{V_0}\right)^{\frac{1}{14}} - 1 = 7.6\%$$

(b) What is the annual growth rate of energy use per capita in China between 2000 and 2014. (correct to 2 decimal places)

$$r = \left(\frac{V_n}{V_0}\right)^{\frac{1}{14}} - 1 = -1.4\%$$