Assignment 2 of CISC 1006

ZHANG HUAKANG
DB92760

Computer Science,
Faculty of Science and Technology

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1

Let A be the event thatan automobile being filled with gasoline will also need an oil change, and B be the event that it needs a new oil filter.

$$
\begin{aligned}
P(A) =& 0.25 \\
P(\overline{A}) =& 0.75 \\
P(B) =& 0.4 \\
P(\overline{B}) =& 0.6 \\
P(A \cap B) =& 0.14
\end{aligned}
$$

1.1

$$
\begin{aligned}
P(B|A) =& \frac{P(A \cap B)}{P(A)} \\
=& \frac{0.14}{0.25} \\
=& 0.56
\end{aligned}
$$

1.2

$$
\begin{aligned}
P(A|B) =& \frac{P(A \cap B)}{P(B)} \\
=& \frac{0.14}{0.4} \\
=& 0.35
\end{aligned}
$$

2

Let A_n be the event that the probability that a specific engine is available when needed, where $n = 1, 2$

$$\begin{aligned} P(A_n) =& 0.96 \\ P(\overline{A_n}) =& 0.04 \end{aligned}$$

2.1

$$P_a = P(\overline{A_1} \cap \overline{A_2})$$

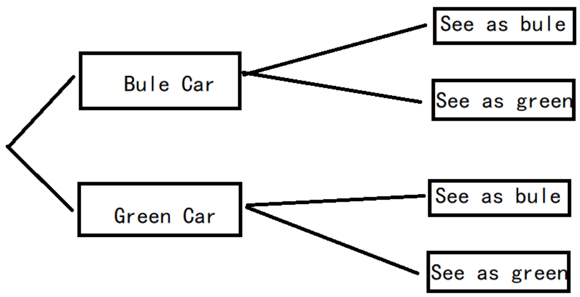
Because fire enginer operates independently.

$$\begin{aligned} P(\overline{A_1} \cap \overline{A_2}) =& P(\overline{A_1}) \times P(\overline{A_2}) \\ =& 0.04^2 \\ =& 1.6 \times 10^{-3} \end{aligned}$$

2.2

$$\begin{aligned} P_b =& 1 - P_a \\ =& 0.9984 \end{aligned}$$

3



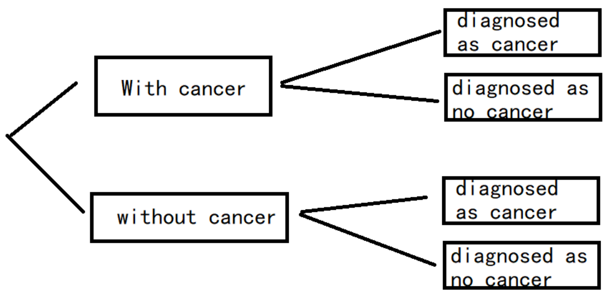
Let A be the event that the witness sees a car as blue, and B be the event that the car is blue. we can know that

$$
\begin{aligned}
P(B) =& \frac{1}{1+99} \\
=& 0.01 \\
P(\overline{B}) =& 0.99 \\
P(A|B) =& 0.99 \\
P(A|\overline{B}) =& 0.02 \\
P(\overline{A}|B) =& 0.01 \\
P(\overline{A}|\overline{B}) =& 0.98
\end{aligned}
$$

$$
\begin{aligned}
P(B|A) =& \frac{P(B \cap A)}{P(A)} \\
=& \frac{P(B)P(A|B)}{P(A)} \\
=& \frac{P(B)P(A|B)}{P(A \cap B) + P(A \cap \overline{B})} \\
=& \frac{P(B)P(A|B)}{P(B)P(A|B) + P(\overline{B})P(A|\overline{B})} \\
=& \frac{1}{3}
\end{aligned}
$$

Thus the probability that when a car is blue the witness see it as bule is only $\frac{1}{3}$ which is a very low probability. So the probability that the car driver is innocence is $\frac{2}{3}$

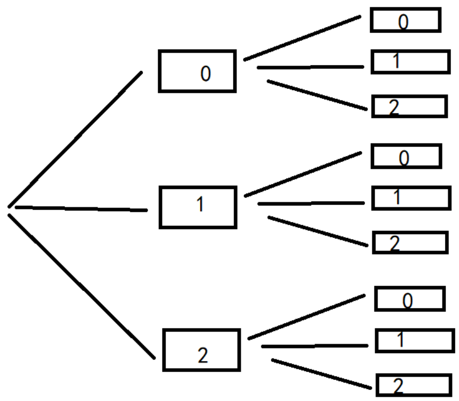
4



Let C be the event that a person has cancer, D is that a person is diagnosed as cancer.

$$
\begin{aligned}
P(C) &= 0.05 \\
P(\overline{C}) &= 0.95 \\
P(D|C) &= 0.78 \\
P(\overline{D}|C) &= 0.22 \\
P(D|\overline{C}) &= 0.06 \\
P(\overline{D}|\overline{C}) &= 0.94 \\
P(D) &= P(D \cap C) + P(D \cap \overline{C}) \\
&= P(C)P(D|C) + P(\overline{C})P(D|\overline{C}) \\
&= 0.096 \\
P(C|D) &= \frac{P(C \cap D)}{P(D)} \\
&= \frac{P(C)P(D|C)}{P(D)} \\
&= 0.40625
\end{aligned}
$$

5



Let D_n be the event that lots contain n defective components and d_n be the event that n defective exist in the lot.

$$
\begin{aligned}
P(D_0) =& 0.6 \\
P(D_1) =& 0.3 \\
P(D_2) =& 0.1 \\
P(d_0|D_0) =& 1 \\
P(d_0|D_1) =& \frac{C_{19}^2 C_1^0}{C_{20}^2} \\
=& 0.9 \\
P(d_0|D_2) =& \frac{C_{18}^2 C_2^0}{C_{20}^2} \\
=& \frac{153}{190} \\
P(d_0) =& P(D_2)P(d_0|D_2) + P(D_1)P(d_0|D_1) + P(D_0)P(d_0|D_0) \\
=& \frac{903}{950}
\end{aligned}
$$

5.1

$$
\begin{aligned}
P(D_0|d_0) =& \frac{P(D_0 \cap d_0)}{P(d_0)} \\
=& \frac{P(D_0)P(d_0|D_0)}{P(d_0)} \\
=& \frac{190}{301}
\end{aligned}
$$

5.2

$$
\begin{aligned}
P(D_1|d_0) =& \frac{P(D_1 \cap d_0)}{P(d_0)} \\
=& \frac{P(D_1)P(d_0|D_1)}{P(d_0)} \\
=& \frac{171}{602}
\end{aligned}
$$

5.3

$$
\begin{aligned}
P(D_2|d_0) =& \frac{P(D_2 \cap d_0)}{P(d_0)} \\
=& \frac{P(D_2)P(d_0|D_2)}{P(d_0)} \\
=& \frac{51}{602}
\end{aligned}
$$

6

6.1

$$
\begin{aligned}
P(W) =& P(A \cap D \cap B \cap C) + P(A \cap D \cap \overline{B} \cap C) + P(A \cap D \cap B \cap \overline{C}) \\
=& P(A) \times P(D) \times (P(B)P(C) + P(\overline{B})P(C) + P(B)P(\overline{C})) \\
=& \frac{2538}{3125}
\end{aligned}
$$

6.2

$$
P(W|\overline{A}) = 0
$$

6.3

$$
P(\overline{A}|W) = 0
$$