Assignment 3 of CISC 1006

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1.1

When $0 \leq y \leq 2$

$$f(y) = P(Y = y) = \frac{C_3^y C_3^{2-y}}{C_6^2}$$

elsewhere,

$$f(y) = P(Y = y) = 0$$

y	$f(y) = P(Y = y)$
0	0.2000
1	0.6000
2	0.2000
elsewhere	0

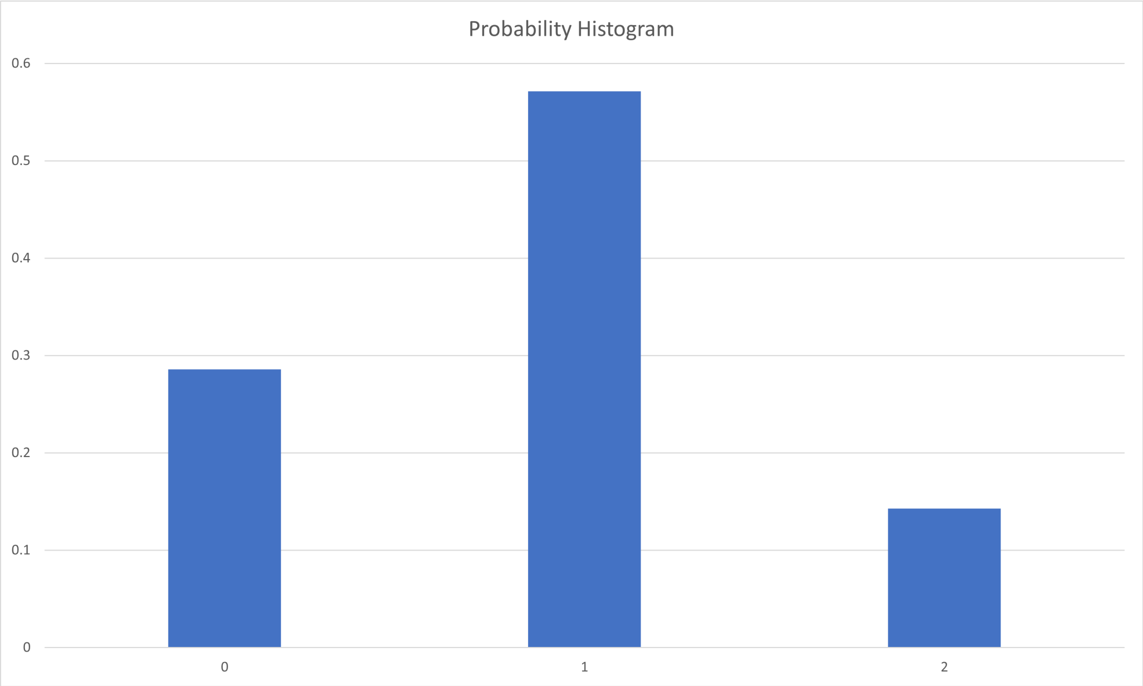
1.2

- $\forall y \in \mathbb{Z}$
$$f(y) \geq 0$$
- $$\sum_y P(Y = y) = 1$$
- $\forall y \in \mathbb{Z}$
$$f(y) = P(Y = y)$$

2

2.1

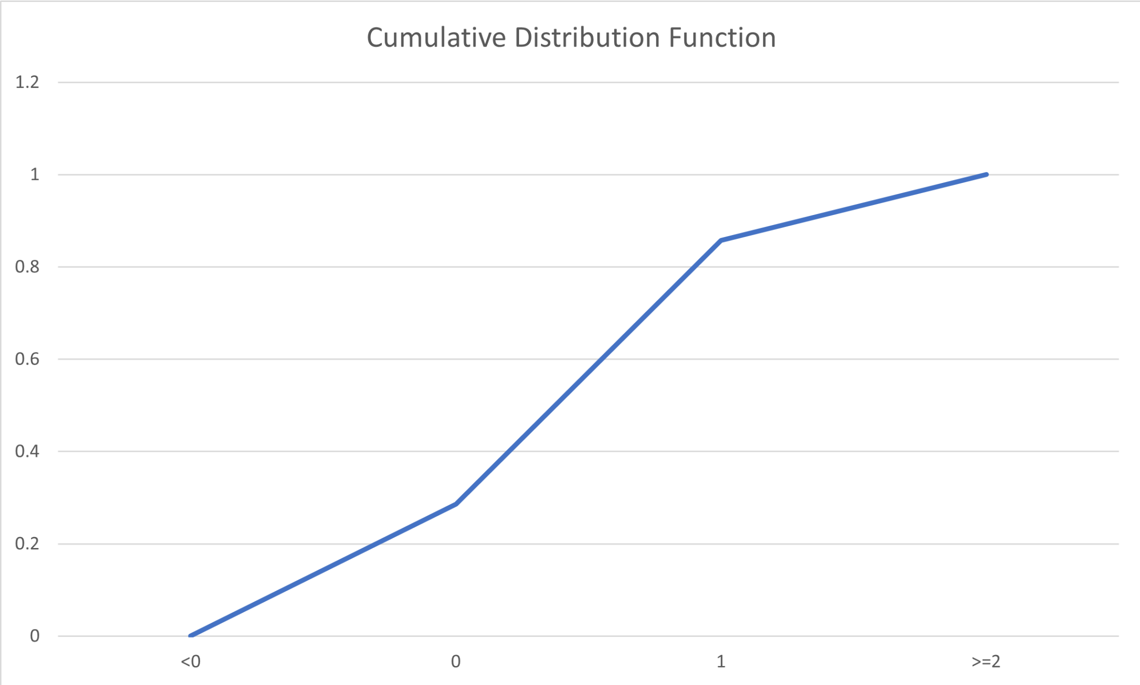
$$P(X = x) = \frac{C_2^x C_5^{3-x}}{C_7^3}, (0 \leq x \leq 2)$$



2.2

$$
\begin{aligned}
F(X=x) =& P(X \leq x) \\
=& \sum_{t=0}^{x} \frac{C_2^t C_5^{3-t}}{C_7^3}, (0 \leq x \leq 2) \\
& \text{or} \\
=& 0, (x < 0) \\
& \text{or} \\
=& 1, (x > 2) \\
P(X=1) =& F(X=1) - F(X=0) \\
\approx& 0.5714 \\
P(0 < X \leq 2) =& F(X=2) - F(X=0) \\
\approx& 0.7142
\end{aligned}
$$

2.3



3

3.1

$$
\begin{aligned}
\int_{-\infty}^{\infty} f(x)dx =& \int_{-1}^{1} k(2-x^2)dx + 0 \\
=& k(3x - \frac{x^3}{3})|_{-1}^{1} \\
=& 2k(3 - \frac{1}{3}) \\
=& \frac{16}{3}k \\
\frac{16}{3}k =& 1 \\
k =& \frac{3}{16}
\end{aligned}
$$

3.2

$$
\begin{aligned}
P(X \leq \frac{1}{2}) =& \int_{-\infty}^{\frac{1}{2}} f(x)dx \\
=& 0 + \int_{-1}^{\frac{1}{2}} \frac{3}{16}(3 - x^2)dx \\
=& \frac{99}{128} \\
\approx& 0.7734
\end{aligned}
$$

3.3

$$
\begin{aligned}
P(|X| > 0.8) =& P(X < -0.8 + P(X > 0.8) \\
=& \int_{-\infty}^{-0.8} f(x)dx + \int_{0.8}^{\infty} f(x)dx \\
=& \int_{-1}^{-0.8} f(x)dx + \int_{0.8}^{1} f(x)dx \\
=& \frac{41}{500} + \frac{41}{500} \\
=& \frac{41}{250} \\
=& 0.1640
\end{aligned}
$$

4

4.1

$$
\begin{aligned}
& \text{When } x \geq 0 \\
F(X) =& \int_{-\infty}^{x} f(t)dt \\
=& \int_{0}^{x} \frac{e^{-\frac{t}{2000}}}{2000} dt \\
=& \int_{0}^{x} -e^{-\frac{t}{2000}} \\
=& 1 - e^{-\frac{x}{2000}} \\
& \text{When } x < 0 \\
F(x) =& 0
\end{aligned}
$$

4.2

$$
\begin{aligned}
P(X \geq 1000) =& \int_{1000}^{\infty} f(x)dx \\
=& \lim_{t \to \infty} \int_{1000}^{t} \frac{e^{-\frac{t}{2000}}}{2000} dt \\
=& e^{-\frac{1}{2}} \\
\approx& 0.6065
\end{aligned}
$$

4.3

$$
\begin{aligned}
P(X \leq 2000) =& \int_{-\infty}^{2000} f(x)dx \\
=& \int_{0}^{2000} f(x)dx \\
=& -e^{-\frac{x}{2000}} \Big|_{0}^{2000} \\
=& 1 - \frac{1}{e} \\
\approx& 0.6321
\end{aligned}
$$

5

5.1

$$\begin{aligned}\int_{-\infty}^{\infty} f(y)dy =& \int_{0}^{1} f(y)dy + 0 \\ =& -(1-y)^5|_0^1 \\ =& 1-0 \\ =& 1\end{aligned}$$

$\forall y \in \mathbb{R}$,

$$f(y) \geq 0$$

5.2

$$\begin{aligned}P(Y < 10\%) =& \int_{-\infty}^{0.1} f(y)dy \\ =& \int_{0}^{0.1} f(y)dy \\ =& 1-0.9^5+1 \\ \approx& 0.4195\end{aligned}$$

5.3

$$\begin{aligned}P(Y > 50\%) =& \int_{0.5}^{\infty} f(y)dy \\ =& \int_{0.5}^{1} f(y)dy \\ =& 1-0.5^5 \\ =& \frac{31}{32} \\ \approx& 0.9688\end{aligned}$$