Assignment 4 of CISC 1006

ZHANG HUAKANG
DB92760

Computer Science,
Faculty of Science and Technology

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The Mathematical Expectation of loos

$$\begin{aligned}\mathbb{E}[X] =& 200000 \times (1 \times 0.02 + 0.5 \times 0.01 + 0.25 \times 0.1) \\ =& \$19,000\end{aligned}$$

Thus the insurance company should charge

$$\mathbb{E}[X] + \$500 = \$19,500$$

2

2.1

$$\begin{aligned}\mathbb{E}[X] =& \int_{-\infty}^{\infty} x f(x) dx \\ =& \int_0^1 x^2 dx + \int_1^2 x(2-x) dx \\ =& 1\end{aligned}$$

2.2

$$\begin{aligned}var(X) =& \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\ =& \int_0^1 x^3 dx + \int_1^2 x^2(2-x) dx - 1 \\ =& \frac{7}{6} - 1 \\ =& \frac{1}{6}\end{aligned}$$

3

$$\begin{aligned}
\mathbb{E}[X] &= \int_{-\infty}^{\infty} xf(x)dx \\
&= \int_{0}^{\infty} x\frac{1}{2000}e^{-\frac{x}{2000}}dx \\
&= \lim_{t\to\infty} -e^{-\frac{x}{2000}}(x+2000)|_0^t \\
&=2000
\end{aligned}$$

4

4.1

$$\begin{aligned}
\mathbb{E}[Y] &= \int_{-\infty}^{\infty} yf(y)dy \\
&= \int_{0}^{1} 5y(1-y)^4 \\
&=\frac{1}{6}
\end{aligned}$$

4.2

$$\begin{aligned}
\mathbb{P}(Y > \frac{1}{6}) &= \int_{\frac{1}{6}}^{\infty} f(y)dy \\
&= \int_{\frac{1}{6}}^{1} 5(1-y)^4 dy \\
&=\frac{3125}{7776} \\
&\approx 0.4019
\end{aligned}$$

5

5.1

$$
\begin{aligned}
\mathbb{E}[X] &= \sum_{x=-\infty}^{\infty} xf(x) \\
&= \sum_{x=2}^{6} xf(x) \\
&= 2 \times 0.01 + 3 \times 0.25 \\
&\quad + 4 \times 0.4 + 5 \times 0.3 + 6 \times 0.04 \\
&= \frac{411}{100} \\
&= 4.11 \\
var(x) &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\
&= \sum_{x=-\infty}^{\infty} x^2 f(x) - \sum_{x=-\infty}^{\infty} xf(x) \\
&= \sum_{x=2}^{6} x^2 f(x) - \sum_{x=2}^{6} xf(x) \\
&= \frac{1763}{100} - (\frac{411}{100})^2 \\
&= \frac{7379}{10000} \\
&= 0.7379
\end{aligned}
$$

5.2

$$
\begin{aligned}
\mathbb{E}[Z] &= \mathbb{E}[3X - 2] \\
&= 3\mathbb{E}[X] - 2 \\
&= \frac{1033}{100} \\
&= 10.33 \\
var(Z) &= var(3X - 2) \\
&= 9var(X) \\
&= 6.6411
\end{aligned}
$$

6

6.1

$$\begin{aligned}
\mathbb{E}[Y] =& \mathbb{E}[3X-2] \\
=& \int_{-\infty}^{\infty} (3x-2) f(x) dx \\
=& \int_{0}^{\infty} (3x-2)(\frac{1}{4} e^{-\frac{x}{4}}) \\
=& -e^{-\frac{x}{4}}(3x+10)|_0^\infty \\
=& 10 \\
var(Y) =& \mathbb{E}[Y^2] - \mathbb{E}[Y]^2 \\
=& \int_{0}^{\infty} (3x-2)^2 (\frac{1}{4} e^{-\frac{x}{4}}) - 100 \\
=& -e^{-\frac{x}{4}}(9x^2+60x+244)_0^\infty - 100 \\
=& 244 - 100 \\
=& 144
\end{aligned}$$

6.2

$$\begin{aligned}
\mathbb{E}[X] =& \int_{\infty}^{\infty} x f(x) dx \\
=& \int_{0}^{\infty} \frac{x}{4} e^{-\frac{x}{4}} \\
=& -e^{-\frac{x}{4}}(x+4)_0^\infty \\
=& 4 \\
var(X) =& \mathbb{E}[X^2] = \mathbb{E}[X]^2 \\
=& \int_{0}^{\infty} \frac{x^2}{4} e^{-\frac{x}{4}} - 16 \\
=& -e^{-\frac{x}{4}}(x^2+8x+32)_0^\infty - 16 \\
=& 32 - 16 \\
=& 16 \\
\mathbb{E}[Y] =& 3\mathbb{E}[X] - 2 \\
var(Y) =& 3^2 var(X)
\end{aligned}$$