Assignment 1 of CISC 3018

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C/S service model

$$d_{CS} = max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\}$$

P2P model

$$d_{P2P} = max\{\frac{F}{u_s}, \frac{F}{min_i(d_i)}, \frac{NF}{u_s + \sum u_i}\}$$

1.1

$N = 5$

$$\begin{aligned} d_{CS} =& max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\} \\ =& max\{\frac{5 \times 20MBits}{40MHz}, \frac{20MBits}{10MHz}\} \\ =& \frac{5}{2}Bits/Hz \end{aligned}$$

$N = 10$

$$\begin{aligned} d_{CS} =& max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\} \\ =& max\{\frac{10 \times 20MBits}{40MHz}, \frac{20MBits}{10MHz}\} \\ =& 5Bits/Hz \end{aligned}$$

$N = 20$

$$\begin{aligned} d_{CS} =& max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\} \\ =& max\{\frac{20 \times 20MBits}{40MHz}, \frac{20MBits}{10MHz}\} \\ =& 10Bits/Hz \end{aligned}$$

$N = 40$

$$\begin{aligned} d_{CS} =& max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\} \\ =& max\{\frac{40 \times 20MBits}{40MHz}, \frac{20MBits}{10MHz}\} \\ =& 20Bits/Hz \end{aligned}$$

$N = 60$

$$\begin{aligned} d_{CS} =& max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\} \\ =& max\{\frac{60 \times 20MBits}{40MHz}, \frac{20MBits}{10MHz}\} \\ =& 30Bits/Hz \end{aligned}$$

1.2

$N = 5$

$$\begin{aligned} d_{P2P} =& max\{\frac{F}{u_s}, \frac{F}{min_i(d_i)}, \frac{NF}{u_s + \sum u_i}\} \\ =& max\{\frac{20MBits}{40MHz}, \frac{20MBits}{10MHz}, \frac{5 \times 20MBits}{40MHz + 5 \times 5MHz}\} \\ =& \frac{20}{13}Bits/Hz \end{aligned}$$

$N = 10$

$$\begin{aligned} d_{P2P} =& max\{\frac{F}{u_s}, \frac{F}{min_i(d_i)}, \frac{NF}{u_s + \sum u_i}\} \\ =& max\{\frac{20MBits}{40MHz}, \frac{20MBits}{10MHz}, \frac{10 \times 20MBits}{40MHz + 10 \times 5MHz}\} \\ =& \frac{20}{9}Bits/Hz \end{aligned}$$

$N = 20$

$$\begin{aligned} d_{P2P} =& max\{\frac{F}{u_s}, \frac{F}{min_i(d_i)}, \frac{NF}{u_s + \sum u_i}\} \\ =& max\{\frac{20MBits}{40MHz}, \frac{20MBits}{10MHz}, \frac{20 \times 20MBits}{40MHz + 20 \times 5MHz}\} \\ =& \frac{20}{7} Bits/Hz \end{aligned}$$

$N = 40$

$$\begin{aligned} d_{P2P} =& max\{\frac{F}{u_s}, \frac{F}{min_i(d_i)}, \frac{NF}{u_s + \sum u_i}\} \\ =& max\{\frac{20MBits}{40MHz}, \frac{20MBits}{10MHz}, \frac{40 \times 20MBits}{40MHz + 40 \times 5MHz}\} \\ =& \frac{10}{3} Bits/Hz \end{aligned}$$

$N = 60$

$$\begin{aligned} d_{P2P} =& max\{\frac{F}{u_s}, \frac{F}{min_i(d_i)}, \frac{NF}{u_s + \sum u_i}\} \\ =& max\{\frac{20MBits}{40MHz}, \frac{20MBits}{10MHz}, \frac{60 \times 20MBits}{40MHz + 60 \times 5MHz}\} \\ =& \frac{60}{17} Bits/Hz \end{aligned}$$

1.3

$$\begin{aligned} F =& 20MBits, \\ u_s =& 40MHz, \\ min_i(d_i) =& 10MHz, \\ u_i =& 5Mhz \end{aligned}$$

$$\begin{aligned} d_{CS}(N; F, u_s, d_i, u_i) =& max\{\frac{NF}{u_s}, \frac{F}{min_i(d_i)}\} \\ =& max\{\frac{N \times 20MBits}{40MHz}, \frac{20MBits}{10MHz}\} \\ =& max\{\frac{N}{2}, \frac{1}{2}\} Bits/Hz \\ =& \frac{N}{2} Bits/Hz \end{aligned}$$

$$\begin{aligned} d_{P2P}(N;F,u_s,d_i,u_i) =& max\{\frac{F}{u_s},\frac{F}{min_i(d_i)},\frac{NF}{u_s+\sum u_i}\} \\ =& max\{\frac{20MBits}{40MHz},\frac{20MBits}{10MHz},\frac{N\times 20MBits}{40MHz+N\times 5MHz}\} \\ =& max\{\frac{1}{2},\frac{1}{2},\frac{4N}{8+N}\}Bits/Hz \end{aligned}$$

We can get:

$$\frac{4N}{8+N}-\frac{1}{2}=\frac{3N-8}{16+2n}.$$

When $3N-8>0$, i.e. $N\geq 3>\frac{8}{3}$

$$d_{P2P}(N;F,u_s,d_i,u_i)=\frac{4N}{8+N}Bits/Hz$$

and when $3N-8\leq 0$, i.e. $0\leq N\leq 2<\frac{8}{3}$

$$d_{P2P}(N;F,u_s,d_i,u_i)=\frac{1}{2}Bits/Hz$$

When $N=5,10,20,40,60$,

$$\begin{aligned} \Delta_{CS,P2P}(5) =& \frac{25}{26}Bits/Hz \\ \Delta_{CS,P2P}(10) =& \frac{25}{9}Bits/Hz \\ \Delta_{CS,P2P}(20) =& \frac{50}{7}Bits/Hz \\ \Delta_{CS,P2P}(40) =& \frac{50}{3}Bits/Hz \\ \Delta_{CS,P2P}(60) =& \frac{450}{17}Bits/Hz \end{aligned}$$

We can know that as N becomes larger, the difference between the two will become larger and larger, and the advantages of P2P will become more and more obvious

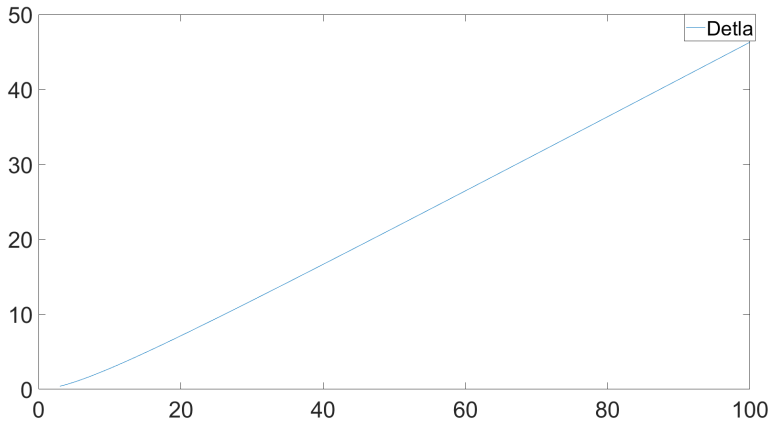
1.4

$$\begin{aligned} \lim_{N\to\infty} d_{CS}(N;F,u_s,d_i,u_i) =& \lim_{N\to\infty}\frac{N}{2}Bits/Hz \\ =& \infty \\ \lim_{N\to\infty} d_{P2P}(N;F,u_s,d_i,u_i) =& \lim_{N\to\infty}\frac{4N}{8+N}Bits/Hz \\ =& \infty \end{aligned}$$

$$\begin{aligned}\lim_{N\to\infty}\frac{d_{P2P}(N;F,u_s,d_i,u_i)}{d_{CS}(N;F,u_s,d_i,u_i)} &= \lim_{N\to\infty}\frac{\frac{4N}{8+N}}{\frac{N}{2}}\\ &= \lim_{N\to\infty}\frac{8}{8+N}\\ &=0\end{aligned}$$

But, when $N > 3$,

$$\begin{aligned}\Delta_{CS,P2P}(N) =&(\frac{N}{2}-\frac{4N}{8+N})Bits/Hz\\ =&\frac{N^2}{16+2N}Bits/Hz\end{aligned}$$



We can know that as N is infty, both of them will need infty time to finish the distribution, but the difference between the two will become larger and larger, and the advantages of P2P will become more and more obvious