Assignment 1 of MATH 2005

ZHANG Huakang/DB92760

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1

1.1

$$Total\ number = \sum_{i=1}^{n_1} n_{2i}$$

1.2

In first day, if he studies 0 hour, he can study 0, 1 ,2 or 3 hours in the next day.
If he studies 1 hour in the first day, he can study 0,1,2 or 3 hours in the next day.
If he studies 2 hours in the first day, he can study 0,1 or 2 hours in the next day.
If he studies 3 hours in the first day, he can study 0 or 1 hour in the next day.
We can use the formula in **1.1** where $n_1 = 4$

$$Total\ number = \sum_{i=1}^{4} n_{2i} = 13$$

2

2.1

The total number of games in a whole basketball play-off $n \in [m, 2m-1]$, for the winner, they should win m games and the last game. For the loser, they will win $n-m$ games and the will lose the last game.
Hence,

$$Total\ number = \sum_{i=0}^{m-1} C_{m+i-1}^{i} \times 2$$

2.2

2 out of 3 paly-off:

$m = 2, T = \sum_{i=0}^{2-1} C_{2+i-1}^{i} \times 2 = 6$

3 out of 5 paly-off:

$m = 3, T = \sum_{i=0}^{3-1} C_{3+i-1}^{i} \times 2 = 20$

4 out of 7 paly-off:

$m = 4, T = \sum_{i=0}^{4-1} C_{4+i-1}^{i} \times 2 = 70$

3

If we give this three books to three students, each student can get one book,

$$T_1 = P_{12}^{3}$$

If we give this three books to two students, one of students can get one book, and another can get two books,

$$T_2 = C_3^1 P_{12}^{2}$$

If we give this three books to only one student,

$$T_3 = P_{12}^{1}$$

Hence,

$$T = T_1 + T_2 + T_3 = 1728$$

4

$$Total\ number = C_4^2 = 6$$

5

Proposition 1.4.1

$$(x+y)^n = \sum_{k=0}^{n} C_n^k x^{n-k} y^k$$

5.1

$$\sum_{k=1}^{n} C_n^k = (1+1)^n = 2^n$$

5.2

$$\sum_{k=1}^{n}(-1)^n C_n^k = (1-1)^n = 0$$

5.3

$$\sum_{k=1}^{n}(a-1)^n C_n^k = (a-1+1)^n = a^n$$

6

6.1

$$P_5^2 = 20$$

6.2

$$P_5^3 = 60$$

7

$$T = 4 \times 5 \times 2 = 40$$

8

Let A be the event that two cards are both greater than 3 and less than 8. The card can be $4, 5, 6$ or 7, the number of total cards is 16

$$P(A) = \frac{C_{16}^2}{C_{52}^2} = \frac{20}{221} \approx 0.0905$$

9

9.1

$$P = \frac{C_{13}^1 C_4^2 C_{12}^1 C_4^2 C_{11}^1 C_4^1}{C_{52}^5 P_2^2} = \frac{198}{4165} \approx 0.0475$$

9.2

$$P = \frac{C_{13}^1 C_{48}^1}{C_{52}^5} = \frac{1}{4165} \approx 2.401 \times 10^{-4}$$

10

10.1

$$P = \frac{C_6^3}{C_{10}^3} = \frac{1}{6}$$

10.2

If the first ball is white,

$$P_w = \frac{C_3^1}{C_9^1} \times \frac{C_4^1}{C_{10}^1},$$

if the first ball is red,

$$P_r = \frac{C_4^1}{C_9^1} \times \frac{C_6^1}{C_{10}^1}.$$

$$P = P_w + P_r = \frac{2}{5}$$

10.3

$$P = \frac{C_6^2 C_4^1}{C_{10}^3} = \frac{1}{2}$$

11

The numbers which is divisible by 4 are 4, 8 ,12 and 16. The event can be $(1,7),(1,3),(2,10),(2,6),(3,9),(3,5),(4,8),(5,7),(6,10),(7,9)$
Hence,

$$P = \frac{10}{C_{10}^2} = \frac{2}{9} \approx 0.2222$$

12

12.1

We have letters 'M','A', 'T','H','E','I','C' and 'S', if there are three different letters in the combination,

$$T_1 = C_8^3 = 56$$

But we have two 'M's , two 'A's and two 'T's, if there are two same letters in the combination,

$$T_2 = 3 \times C_7^1 = 21$$

Therefore,

$$T = T_1 + T_2 = 78$$

12.2

$$P = \frac{C_2^1 C_1^1 C_1^1}{C_{11}^3} = \frac{2}{165} \approx 0.0121$$