Assignment 7 of CISC 2002

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1.1

```
function x=Assignment_7_1_x(t)
    x=2/(1+exp(-2*t));
end
```

Listing 1: Code

```
function y=Assignment_7_1_f(x)
    y=2*x-x^2;
end
```

Listing 2: Code

```
clear
delta=0.1
n=0;
x0=1;
x1=0;
while (n*delta<=1)
    error=x0-Assignment_7_1_x(n*delta);
    x1=x0+delta*Assignment_7_1_f(x0);
    x0=x1;
    n=n+1;
    fprintf('%10.5f,%10.5f\n',x1,error)
end
disp("------------------------------")
clear
delta=0.01
n=0;
x0=1;
x1=0;
while (n*delta<=1)
    error=x0-Assignment_7_1_x(n*delta);
    x1=x0+delta*Assignment_7_1_f(x0);
    x0=x1;
    n=n+1;
    fprintf('%10.5f,%10.5f\n',x1,error)
```

```
end
```

Listing 3: Code

```

delta =

    0.1000

   1.10000,   0.00000
   1.19900,   0.00033
   1.29504,   0.00162
   1.38634,   0.00373
   1.47141,   0.00639
   1.54919,   0.00929
   1.61903,   0.01214
   1.68071,   0.01466
   1.73437,   0.01667
   1.78044,   0.01807
   1.81953,   0.01885
————————————————————

delta =

    0.0100

   1.01000,   0.00000
   1.02000,   0.00000
   1.03000,   0.00000
   1.03999,   0.00000
   1.04997,   0.00001
   1.05995,   0.00001
   1.06991,   0.00002
   1.07986,   0.00002
   1.08980,   0.00003
   1.09972,   0.00004
   1.10962,   0.00005
   1.11950,   0.00006
   1.12935,   0.00007
   1.13919,   0.00008
   1.14899,   0.00009
   1.15877,   0.00011
   1.16852,   0.00012
   1.17823,   0.00014
   1.18792,   0.00015
   1.19756,   0.00017
   1.20717,   0.00019
   1.21674,   0.00021
   1.22627,   0.00023
   1.23576,   0.00025
   1.24521,   0.00027
   1.25461,   0.00029
   1.26396,   0.00031
   1.27326,   0.00033
   1.28251,   0.00036
```

```
   1.29172,   0.00038
   1.30086,   0.00040
   1.30996,   0.00043
   1.31900,   0.00045
   1.32798,   0.00048
   1.33691,   0.00050
   1.34577,   0.00053
   1.35457,   0.00056
   1.36332,   0.00058
   1.37200,   0.00061
   1.38061,   0.00064
   1.38916,   0.00066
   1.39765,   0.00069
   1.40607,   0.00072
   1.41442,   0.00075
   1.42270,   0.00078
   1.43092,   0.00080
   1.43906,   0.00083
   1.44713,   0.00086
   1.45513,   0.00089
   1.46306,   0.00092
   1.47092,   0.00094
   1.47870,   0.00097
   1.48641,   0.00100
   1.49404,   0.00103
   1.50160,   0.00105
   1.50908,   0.00108
   1.51649,   0.00111
   1.52383,   0.00113
   1.53108,   0.00116
   1.53826,   0.00119
   1.54536,   0.00121
   1.55239,   0.00124
   1.55934,   0.00126
   1.56621,   0.00129
   1.57300,   0.00131
   1.57972,   0.00133
   1.58636,   0.00136
   1.59292,   0.00138
   1.59941,   0.00140
   1.60581,   0.00142
   1.61214,   0.00144
   1.61840,   0.00147
   1.62457,   0.00149
   1.63067,   0.00151
   1.63669,   0.00153
   1.64264,   0.00154
   1.64851,   0.00156
   1.65430,   0.00158
   1.66002,   0.00160
   1.66567,   0.00161
   1.67124,   0.00163
   1.67673,   0.00164
   1.68215,   0.00166
   1.68750,   0.00167
```

```
   1.69277,    0.00169
   1.69797,    0.00170
   1.70310,    0.00171
   1.70816,    0.00173
   1.71314,    0.00174
   1.71806,    0.00175
   1.72290,    0.00176
   1.72767,    0.00177
   1.73238,    0.00178
   1.73701,    0.00178
   1.74158,    0.00179
   1.74608,    0.00180
   1.75052,    0.00181
   1.75488,    0.00181
   1.75919,    0.00182
   1.76342,    0.00182
   1.76759,    0.00183
```

Listing 4: Code

1.2

```
clear
delta=0.1
n=0;
x0=1;
x1=0;
while (n*delta<=1)
    error=x0-Assignment_7_1_x(n*delta);
    k1=delta*Assignment_7_1_f(x0);
    k2=delta*Assignment_7_1_f(x0+k1);
    x1=x0+(k1+k2)/2;
    x0=x1;
    n=n+1;
    fprintf('%10.5f,%10.5f\n',x1,error)
end
disp("------------------------------")
clear
delta=0.01
n=0;
x0=1;
x1=0;
while (n*delta<=1)
    error=x0-Assignment_7_1_x(n*delta);
    k1=delta*Assignment_7_1_f(x0);
    k2=delta*Assignment_7_1_f(x0+k1);
    x1=x0+(k1+k2)/2;
    x0=x1;
    n=n+1;
    fprintf('%10.5f,%10.5f\n',x1,error)
end
```

Listing 5: Code

```
delta =

    0.1000

   1.09950,   0.00000
   1.19703,  -0.00017
   1.29080,  -0.00034
   1.37926,  -0.00052
   1.46126,  -0.00069
   1.53604,  -0.00086
   1.60324,  -0.00100
   1.66281,  -0.00113
   1.71500,  -0.00123
   1.76027,  -0.00130
   1.79917,  -0.00133
-----------------------

delta =

    0.0100

   1.01000,   0.00000
   1.02000,  -0.00000
   1.02999,  -0.00000
   1.03998,  -0.00000
   1.04996,  -0.00000
   1.05993,  -0.00000
   1.06988,  -0.00000
   1.07983,  -0.00000
   1.08976,  -0.00000
   1.09967,  -0.00000
   1.10956,  -0.00000
   1.11943,  -0.00000
   1.12927,  -0.00000
   1.13909,  -0.00000
   1.14888,  -0.00000
   1.15865,  -0.00000
   1.16838,  -0.00000
   1.17808,  -0.00000
   1.18774,  -0.00000
   1.19737,  -0.00000
   1.20696,  -0.00000
   1.21651,  -0.00000
   1.22602,  -0.00000
   1.23549,  -0.00000
   1.24491,  -0.00000
   1.25429,  -0.00000
   1.26362,  -0.00000
   1.27290,  -0.00000
   1.28213,  -0.00000
   1.29131,  -0.00000
   1.30043,  -0.00000
   1.30950,  -0.00001
   1.31852,  -0.00001
```

```
   1.32747,  -0.00001
   1.33637,  -0.00001
   1.34521,  -0.00001
   1.35399,  -0.00001
   1.36270,  -0.00001
   1.37135,  -0.00001
   1.37994,  -0.00001
   1.38847,  -0.00001
   1.39692,  -0.00001
   1.40531,  -0.00001
   1.41364,  -0.00001
   1.42189,  -0.00001
   1.43008,  -0.00001
   1.43819,  -0.00001
   1.44624,  -0.00001
   1.45421,  -0.00001
   1.46211,  -0.00001
   1.46994,  -0.00001
   1.47769,  -0.00001
   1.48537,  -0.00001
   1.49298,  -0.00001
   1.50051,  -0.00001
   1.50797,  -0.00001
   1.51535,  -0.00001
   1.52266,  -0.00001
   1.52989,  -0.00001
   1.53704,  -0.00001
   1.54412,  -0.00001
   1.55112,  -0.00001
   1.55804,  -0.00001
   1.56489,  -0.00001
   1.57166,  -0.00001
   1.57835,  -0.00001
   1.58497,  -0.00001
   1.59151,  -0.00001
   1.59797,  -0.00001
   1.60436,  -0.00001
   1.61067,  -0.00001
   1.61690,  -0.00001
   1.62305,  -0.00001
   1.62913,  -0.00001
   1.63514,  -0.00001
   1.64107,  -0.00001
   1.64692,  -0.00001
   1.65270,  -0.00001
   1.65840,  -0.00001
   1.66403,  -0.00001
   1.66958,  -0.00001
   1.67506,  -0.00001
   1.68046,  -0.00001
   1.68580,  -0.00001
   1.69106,  -0.00001
   1.69625,  -0.00001
   1.70136,  -0.00001
   1.70641,  -0.00001
```

```
   1.71138,   -0.00001
   1.71629,   -0.00001
   1.72112,   -0.00001
   1.72589,   -0.00001
   1.73058,   -0.00001
   1.73521,   -0.00001
   1.73977,   -0.00001
   1.74426,   -0.00001
   1.74869,   -0.00001
   1.75305,   -0.00001
   1.75735,   -0.00001
   1.76158,   -0.00001
   1.76575,   -0.00001
```

Listing 6: Code

1.3

```
clear
delta=0.1
n=0;
x0=1;
x1=0;
while (n*delta<=1)
    error=x0-Assignment_7_1_x(n*delta);
    k1=delta*Assignment_7_1_f(x0);
    k2=delta*Assignment_7_1_f(x0+k1/2);
    k3=delta*Assignment_7_1_f(x0+k2/2);
    k4=delta*Assignment_7_1_f(x0+k3);
    x1=x0+(k1+2*k2+2*k3+k4)/6;
    x0=x1;
    n=n+1;
    fprintf('%10.5f,%10.5f\n',x1,error)
end
disp("------------------------------")
clear
delta=0.01
n=0;
x0=1;
x1=0;
while (n*delta<=1)
    error=x0-Assignment_7_1_x(n*delta);
    k1=delta*Assignment_7_1_f(x0);
    k2=delta*Assignment_7_1_f(x0+k1/2);
    k3=delta*Assignment_7_1_f(x0+k2/2);
    k4=delta*Assignment_7_1_f(x0+k3);
    x1=x0+(k1+2*k2+2*k3+k4)/6;
    x0=x1;
    n=n+1;
    fprintf('%10.5f,%10.5f\n',x1,error)
end
```

Listing 7: Code

```
delta =

    0.1000

   1.09967,   0.00000
   1.19738,  -0.00000
   1.29131,  -0.00000
   1.37995,  -0.00000
   1.46212,  -0.00000
   1.53705,  -0.00000
   1.60437,  -0.00000
   1.66404,  -0.00000
   1.71630,  -0.00000
   1.76159,  -0.00000
   1.80050,  -0.00000
-----------------------

delta =

    0.0100

   1.01000,   0.00000
   1.02000,  -0.00000
   1.02999,  -0.00000
   1.03998,  -0.00000
   1.04996,  -0.00000
   1.05993,  -0.00000
   1.06989,  -0.00000
   1.07983,  -0.00000
   1.08976,  -0.00000
   1.09967,  -0.00000
   1.10956,  -0.00000
   1.11943,  -0.00000
   1.12927,  -0.00000
   1.13909,  -0.00000
   1.14889,  -0.00000
   1.15865,  -0.00000
   1.16838,  -0.00000
   1.17808,  -0.00000
   1.18775,  -0.00000
   1.19738,  -0.00000
   1.20697,  -0.00000
   1.21652,  -0.00000
   1.22603,  -0.00000
   1.23550,  -0.00000
   1.24492,  -0.00000
   1.25430,  -0.00000
   1.26362,  -0.00000
   1.27291,  -0.00000
   1.28213,  -0.00000
   1.29131,  -0.00000
   1.30044,  -0.00000
   1.30951,  -0.00000
   1.31852,  -0.00000
```

```
   1.32748,  -0.00000
   1.33638,  -0.00000
   1.34521,  -0.00000
   1.35399,  -0.00000
   1.36271,  -0.00000
   1.37136,  -0.00000
   1.37995,  -0.00000
   1.38847,  -0.00000
   1.39693,  -0.00000
   1.40532,  -0.00000
   1.41364,  -0.00000
   1.42190,  -0.00000
   1.43008,  -0.00000
   1.43820,  -0.00000
   1.44624,  -0.00000
   1.45422,  -0.00000
   1.46212,  -0.00000
   1.46995,  -0.00000
   1.47770,  -0.00000
   1.48538,  -0.00000
   1.49299,  -0.00000
   1.50052,  -0.00000
   1.50798,  -0.00000
   1.51536,  -0.00000
   1.52267,  -0.00000
   1.52990,  -0.00000
   1.53705,  -0.00000
   1.54413,  -0.00000
   1.55113,  -0.00000
   1.55805,  -0.00000
   1.56490,  -0.00000
   1.57167,  -0.00000
   1.57836,  -0.00000
   1.58498,  -0.00000
   1.59152,  -0.00000
   1.59798,  -0.00000
   1.60437,  -0.00000
   1.61068,  -0.00000
   1.61691,  -0.00000
   1.62307,  -0.00000
   1.62915,  -0.00000
   1.63515,  -0.00000
   1.64108,  -0.00000
   1.64693,  -0.00000
   1.65271,  -0.00000
   1.65841,  -0.00000
   1.66404,  -0.00000
   1.66959,  -0.00000
   1.67507,  -0.00000
   1.68048,  -0.00000
   1.68581,  -0.00000
   1.69107,  -0.00000
   1.69626,  -0.00000
   1.70137,  -0.00000
   1.70642,  -0.00000
```

```
   1.71139,   -0.00000
   1.71630,   -0.00000
   1.72113,   -0.00000
   1.72590,   -0.00000
   1.73059,   -0.00000
   1.73522,   -0.00000
   1.73978,   -0.00000
   1.74428,   -0.00000
   1.74870,   -0.00000
   1.75307,   -0.00000
   1.75736,   -0.00000
   1.76159,   -0.00000
   1.76576,   -0.00000
```

Listing 8: Code

1.4

```
clear
tspan=[0 1];
y0=1;
[t,y]=ode45(@(t,y) 2*y-y^2,tspan,y0);
plot(t,y,'-o')
for i=1:41
    error=2/(1+exp(-2*t(i)))-y(i);
    disp("x= "+y(i)+" ,error= "+error)
end
```

Listing 9: Code

```
x= 1 ,error= 0
x= 1.025 ,error= -1.2671e-08
x= 1.05 ,error= -2.113e-09
x= 1.0749 ,error= 1.0199e-08
x= 1.0997 ,error= -2.9569e-11
x= 1.1244 ,error= -1.0619e-08
x= 1.1489 ,error= -1.634e-09
x= 1.1732 ,error= 8.5277e-09
x= 1.1974 ,error= -4.1045e-11
x= 1.2213 ,error= -7.2699e-09
x= 1.2449 ,error= -1.3276e-09
x= 1.2683 ,error= 5.4479e-09
x= 1.2913 ,error= -1.4501e-11
x= 1.314 ,error= -3.6185e-09
x= 1.3364 ,error= -1.5477e-09
x= 1.3584 ,error= 1.5845e-09
x= 1.3799 ,error= 6.1487e-11
x= 1.4011 ,error= -6.1786e-10
x= 1.4219 ,error= -2.5302e-09
x= 1.4422 ,error= -2.3811e-09
x= 1.4621 ,error= 1.875e-10
x= 1.4815 ,error= 1.1338e-09
```

```
x= 1.5005 ,error= -4.2778e-09
x= 1.519 ,error= -5.8795e-09
x= 1.537 ,error= 3.6431e-10
x= 1.5546 ,error= 1.5139e-09
x= 1.5717 ,error= -6.549e-09
x= 1.5883 ,error= -8.5399e-09
x= 1.6044 ,error= 6.021e-10
x= 1.62 ,error= 7.9792e-10
x= 1.6351 ,error= -8.948e-09
x= 1.6498 ,error= -1.0202e-08
x= 1.664 ,error= 9.1778e-10
x= 1.6778 ,error= -5.3161e-10
x= 1.6911 ,error= -1.106e-08
x= 1.7039 ,error= -1.0878e-08
x= 1.7163 ,error= 1.3232e-09
x= 1.7283 ,error= -1.9823e-09
x= 1.7398 ,error= -1.2567e-08
x= 1.7509 ,error= -1.0702e-08
x= 1.7616 ,error= 1.8138e-09
```

Listing 10: Code

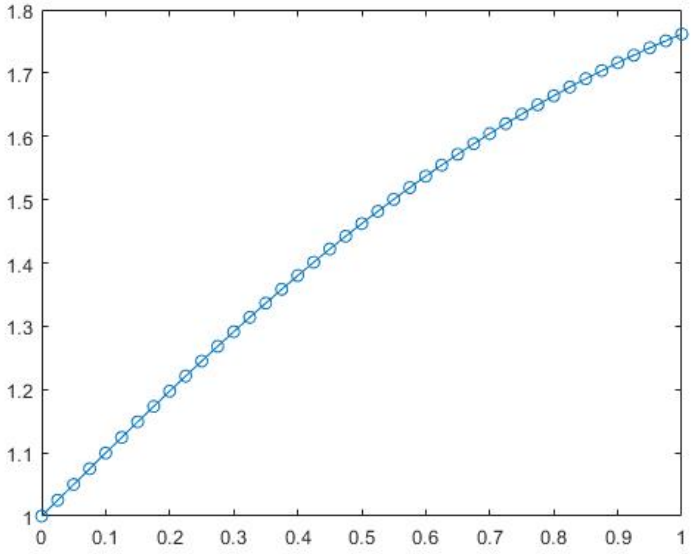

Figure 1: Figure

2

2.1

Let $\frac{dx}{dt} = x_1$

$$\begin{aligned} x(0) =& 0 \\ \frac{dx}{dt}(0) =& -1 \\ x_1(t) =& \frac{dx}{dt} \\ \frac{dx_1}{dt}(t) =& -x \end{aligned}$$

2.2

$$\begin{aligned} x_{n+1} =& x_n + \Delta t x_1(t_n, y_{1n}, y_{2n}) \\ =& x_n + \Delta t x_1(t_n) \\ x_{1n+1} =& x_{1n} + \Delta t(-x_n) \end{aligned}$$

$$\Delta t = \frac{\pi}{10}$$

$$
\begin{aligned}
x_{11} =& x_{10} + \Delta t(-x_0) \\
=& -1 + \frac{\pi}{10}(-0) \\
=& -1 \\
x_1 =& x_0 + \Delta t x_{11} \\
=& -\frac{\pi}{10} \\
x_{12} =& x_{11} + \Delta t(-x_1) \\
=& -1 + \frac{\pi^2}{100} \\
x_2 =& x_1 + \Delta t x_{12} \\
=& -\frac{\pi}{5} + \frac{\pi^3}{1000} \\
x_{13} =& x_{12} + \Delta t(-x_2) \\
=& -1 + \frac{\pi^2}{100} + \frac{\pi}{10}(\frac{\pi}{5} - \frac{\pi^3}{1000}) \\
=& -1 + \frac{3\pi^2}{100} - \frac{\pi^4}{1000} \\
x_3 =& x_2 + \Delta t x_{13} \\
=& -\frac{\pi}{5} + \frac{\pi^3}{1000} + \frac{\pi}{10}(-1 + \frac{3\pi^2}{100} - \frac{\pi^4}{1000}) \\
& \dots \\
x_5 \approx& -1.0125
\end{aligned}
$$

```
x0=0;
x10=-1;
x1=0;
x11=0;
for i=1:5
    x11=x10+(pi/10)*(-x0);
    x1=x0+(pi/10)*x11;
    x0=x1;
    x10=x11;
    error=-sin(i*(pi/10))-x1;
    disp("x"+i+": "+x1+", error:"+error)
end
```

Listing 11: Code

```
x1: -0.31416, error:0.0051423
x2: -0.59731, error:0.009527
x3: -0.82151, error:0.012496
x4: -0.96463, error:0.013577
x5: -1.0125, error:0.012549
```

Listing 12: Code

2.3

$$
\begin{aligned}
k_1 &= \Delta t x_1(t_n) \\
l_1 &= \Delta t - x \\
k_2 &= \Delta t x_1(t_n + k1) \\
l2 &= \Delta t - (x + l2) \\
x_{n+1} &= x_n + \frac{1}{2}(k_1 + k_2) \\
x_{1n+1} &= x_{1n} + \frac{1}{2}(l_1 + l_2)
\end{aligned}
$$

2.4

$$
\begin{aligned}
k_1 &= \Delta t x_1(t_n) \\
l_1 &= \Delta t - x \\
k_2 &= \Delta t x_1(t_n + \frac{1}{2}\Delta t) \\
l_2 &= \Delta t - (x + \frac{1}{2}k_2) \\
k_3 &= \Delta t x_1(t_n + \frac{1}{2}\Delta t) \\
l_3 &= \Delta t - (x + \frac{1}{2}k_3) \\
k_4 &= \Delta t x_1(t_n + k3) \\
l4 &= \Delta t - (x + l3) \\
x_{n+1} &= x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\
x_{1n+1} &= x_{1n} + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)
\end{aligned}
$$

2.5

```
clear
clc
f=@(t,y) [y(2);-y(1)];
[t,y]=ode45(f,[0 pi/2],[0 -1]);
plot(t,y(:,1),'-o',t,y(:,2),'-.')
grid on
xlabel('t')
ylabel('y(x)')
legend('y','y1')
```

Listing 13: Code

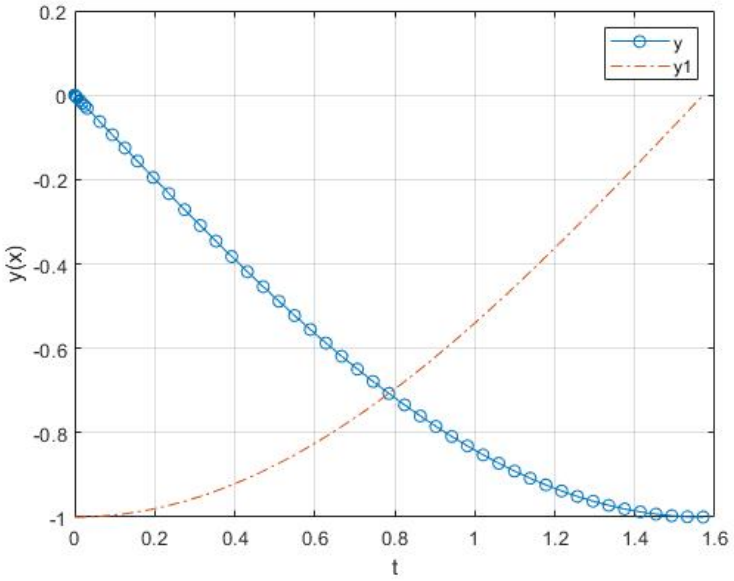

Figure 2: Figure

3

3.1

```
clear
clc
f=@(t,y) -1000*(y-sin(t));
[t,y]=ode45(f,[0,1],0);
plot(t,y,'-ro')
```

Listing 14: Code

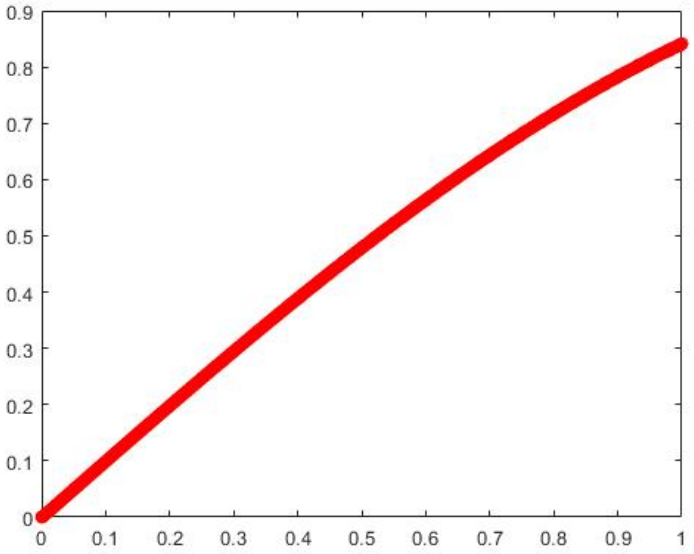

Figure 3: Figure

3.2

```
clear
clc
f=@(t,y) -1000*(y-sin(t));
[t,y]=ode23t(f,[0,1],0);
plot(t,y,'-ro')
```

Listing 15: Code

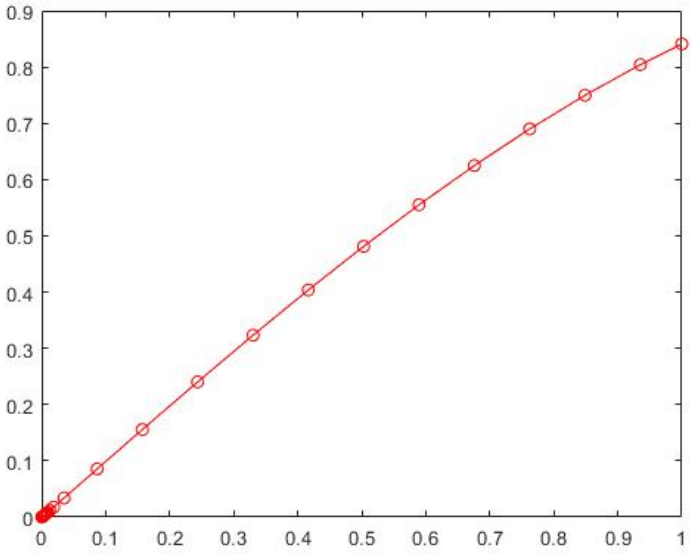

Figure 4: Figure

4

4.1

```
n=10;
M=diag(-ones(n-1,1),-1)+diag(2*ones(n,1),0)+diag(-ones(n-1,1),1);
f=ones(n,1);
h=1/n;
b=h^2.*f;
b(1)=b(1)+0;
b(n)=b(n)+0;
x1=(1/n:1/n:1)';
y1=M\b;
n=100;
M=diag(-ones(n-1,1),-1)+diag(2*ones(n,1),0)+diag(-ones(n-1,1),1);
f=ones(n,1);
h=1/n;
b=h^2.*f;
b(1)=b(1)+0;
b(n)=b(n)+0;
x2=(1/n:1/n:1)';
y2=M\b;
n=1000;
M=diag(-ones(n-1,1),-1)+diag(2*ones(n,1),0)+diag(-ones(n-1,1),1);
f=ones(n,1);
h=1/n;
```

```
b=h^2.*f;
b(1)=b(1)+0;
b(n)=b(n)+0;
x3=(1/n:1/n:1)';
y3=M\b;
plot(x1,y1,'-',x2,y2,'o',x3,y3,'.')
```

Listing 16: Code

4.2

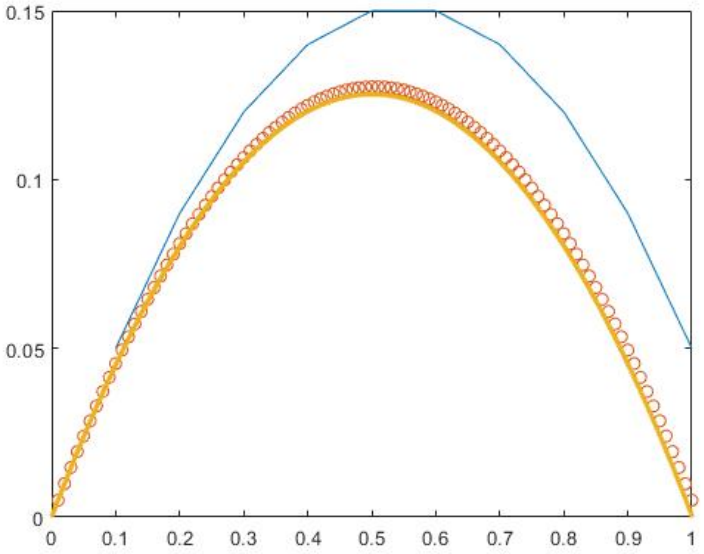

Figure 5: Figure