

Assignment 3 of CISC 2002

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1.1

Let

$$B = \begin{bmatrix} 3 & -2 & 5 & 2 \\ 4 & -7 & -1 & 19 \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$\frac{Row1}{3}$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 4 & -7 & -1 & 19 \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$Row2 - 4Row1$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & -\frac{13}{3} & -11 & \frac{49}{3} \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$Row2 / -\frac{13}{3}$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$Row3 - 5Row1$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & -\frac{8}{3} & -\frac{13}{3} & \frac{29}{3} \end{bmatrix}$$

$$Row1 + \frac{2}{3}Row2 \text{ and } Row3 + \frac{8}{3}Row2$$

$$\begin{bmatrix} 1 & 0 & \frac{37}{13} & -\frac{24}{13} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & 0 & -\frac{5}{13} & -\frac{5}{13} \end{bmatrix}$$

$Row3/\frac{5}{13}$

$$\begin{bmatrix} 1 & 0 & \frac{37}{13} & -\frac{24}{13} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$Row1 - \frac{37}{13}Row3$ and $Row2 - \frac{23}{13}Row3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Thus

$$x_1 = 1$$

$$x_2 = -2$$

$$x_3 = -1$$

1.2

Let

$$A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix}$$

From Question 1 we can know that

$$\begin{aligned} A &= \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & -\frac{13}{3} & -11 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ -\frac{5}{3} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 0 & -\frac{8}{3} & -\frac{13}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ -\frac{89}{39} & \frac{8}{13} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 0 & 0 & -\frac{353}{39} \end{bmatrix} \end{aligned}$$

1.3

$$A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix}$$

We can get

$$A^{-1} = \begin{bmatrix} 6.8 & 4.4 & -7.4 \\ 4.2 & 2.6 & -4.6 \\ -2.2 & -1.6 & 2.6 \end{bmatrix}$$

$$A\vec{x}_{k+1} = \vec{x}_k$$

$$\vec{x}_{k+1} = \vec{x}_k A^{-1}$$

$$x_0 = \begin{bmatrix} 2 \\ 19 \\ 13 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 12.6 & 8.8 & -14.8 \\ 79.8 & 49.4 & -87.4 \\ -28.6 & -20.8 & 33.8 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 92.48 & 38.72 & 109.52 \\ 335.160 & 128.44 & 402.04 \\ 62.92 & 33.28 & 87.88 \end{bmatrix}$$

$$x_3 = 10^3 \times \begin{bmatrix} 0.6289 & 0.1704 & -0.8104 \\ 1.4077 & 0.3339 & -1.8494 \\ -0.1384 & -0.0532 & 0.2285 \end{bmatrix}$$

2

2.1

$$T_n = kn^3$$

$$\frac{T_{40}}{T_{10}} = \frac{k40^3}{k10^3}$$

$$T_{40} = 4^3 \times T_{10} \\ = 6.4s$$

2.2

$$\begin{aligned}C_n &= kn^3 \\ \frac{C_{256}}{C_{1024}} &= \frac{k256^3}{k1024^3} \\ C_{256} &= \left(\frac{2^8}{2^{10}}\right)^3 C_{1024} \\ &= \frac{1}{64} \times 1 \\ &= \frac{1}{64} s\end{aligned}$$