

Assignment 3 of CISC 2002

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1

1.1

Let

$$B = \begin{bmatrix} 3 & -2 & 5 & 2 \\ 4 & -7 & -1 & 19 \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$\frac{Row1}{3}$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 4 & -7 & -1 & 19 \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$Row2 - 4Row1$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & -\frac{13}{3} & -11 & \frac{49}{3} \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$Row2 / -\frac{13}{3}$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$$Row3 - 5Row1$$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & -\frac{8}{3} & -\frac{13}{3} & \frac{29}{3} \end{bmatrix}$$

$$Row1 + \frac{2}{3}Row2 \text{ and } Row3 + \frac{8}{3}Row2$$

$$\begin{bmatrix} 1 & 0 & \frac{37}{13} & -\frac{24}{13} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & 0 & -\frac{5}{13} & -\frac{5}{13} \end{bmatrix}$$

$Row3/\frac{5}{13}$

$$\begin{bmatrix} 1 & 0 & \frac{37}{13} & -\frac{24}{13} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$Row1 - \frac{37}{13}Row3$ and $Row2 - \frac{23}{13}Row3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Thus

$$x_1 = 1$$

$$x_2 = -2$$

$$x_3 = -1$$

1.2

Let

$$A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix}$$

From Question 1 we can know that

$$\begin{aligned} A &= \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & -\frac{13}{3} & -11 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ -\frac{5}{3} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 0 & -\frac{8}{3} & -\frac{13}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ -\frac{89}{39} & \frac{8}{13} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 0 & 0 & -\frac{353}{39} \end{bmatrix} \end{aligned}$$

1.3

$$A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix}$$

We can get

$$A^{-1} = \begin{bmatrix} 6.8 & 4.4 & -7.4 \\ 4.2 & 2.6 & -4.6 \\ -2.2 & -1.6 & 2.6 \end{bmatrix}$$

$$A\vec{x}_{k+1} = \vec{x}_k$$

$$\vec{x}_{k+1} = \vec{x}_k A^{-1}$$

$$x_0 = \begin{bmatrix} 2 \\ 19 \\ 13 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 12.6 & 8.8 & -14.8 \\ 79.8 & 49.4 & -87.4 \\ -28.6 & -20.8 & 33.8 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 92.48 & 38.72 & 109.52 \\ 335.160 & 128.44 & 402.04 \\ 62.92 & 33.28 & 87.88 \end{bmatrix}$$

$$x_3 = 10^3 \times \begin{bmatrix} 0.6289 & 0.1704 & -0.8104 \\ 1.4077 & 0.3339 & -1.8494 \\ -0.1384 & -0.0532 & 0.2285 \end{bmatrix}$$

2

2.1

$$T_n = kn^3$$

$$\frac{T_{40}}{T_{10}} = \frac{k40^3}{k10^3}$$

$$T_{40} = 4^3 \times T_{10}$$

$$= 6.4s$$

2.2

$$\begin{aligned}
 C_n &= kn^3 \\
 \frac{C_{256}}{C_{1024}} &= \frac{k256^3}{k1024^3} \\
 C_{256} &= \left(\frac{2^8}{2^{10}}\right)^3 C_{1024} \\
 &= \frac{1}{64} \times 1 \\
 &= \frac{1}{64} s
 \end{aligned}$$

2.3

$$\begin{aligned}
 D_n &= kn^2 \\
 \frac{D_{2048}}{D_{1024}} &= \frac{k2048^2}{k1024^2} \\
 D_{2048} &= 2^2 D_{1024} \\
 &= 4
 \end{aligned}$$

3

3.1

```

1 clear all
2 x=2; y=1; z=5;
3 sig=1; r=3; b=1;
4 d=zeros(3,1); rhs=d;
5 A=zeros(3); n=0;
6 for i=1:10
7     A(1,1)=-sig; A(1,2)=sig; A(1,3)=0;
8     A(2,1)=r-z; A(2,2)=-1; A(2,3)=-x;
9     A(3,1)=y; A(3,2)=x; A(3,3)=-b;
10    rhs(1)=-sig*(y-x);
11    rhs(2)=-(r*x-y-x*z);
12    rhs(3)=-(x*y-b*z);
13    d=A\rhs;
14    x=x+d(1); y=y+d(2); z=z+d(3);
15    n=n+1;
16    if abs(d(1))<0.001 || abs(d(2))<0.001||abs(d(3))<0.001
17        break;
18    end
19 end
20 disp([x,y,z]')
21 fprintf('Loop run %d times\n',n)

```

Listing 1: Code after optimization

What I add is

```

1   if abs(d(1))<0.001 || abs(d(2))<0.001||abs(d(3))<0.001
2       break;
3   end

```

Listing 2: Added code

Check the output:

```

1   1.4142
2   1.4142
3   2.0000
4
5 Loop run 10 times

```

Listing 3: Before optimization

```

1
2
3   1.4142
4   1.4142
5   2.0000
6
7 Loop run 4 times

```

Listing 4: After optimization

3.2

```

1 clear all
2 x=0; y=0; z=0;
3 sig=1; r=3; b=1;
4 d=zeros(3,1); rhs=d;
5 A=zeros(3);n=0;
6 for i=1:10
7     A(1,1)=-sig; A(1,2)=sig; A(1,3)=0;
8     A(2,1)=r-z; A(2,2)=-1; A(2,3)=-x;
9     A(3,1)=y; A(3,2)=x; A(3,3)=-b;
10    rhs(1)=-sig*(y-x);
11    rhs(2)=-(r*x-y-x*z);
12    rhs(3)=-(x*y-b*z);
13    d=A\rhs;
14    x=x+d(1); y=y+d(2); z=z+d(3);
15    n=n+1;
16    if abs(d(1))<0.001 || abs(d(2))<0.001||abs(d(3))<0.001
17        break;
18    end
19 end
20 disp([x,y,z]')
21 fprintf("Loop run %d times\n",n)

```

Listing 5: Code after optimization

Output

```

1      0
2      0
3      0
4
5 Loop run 1 times

```

Listing 6: After optimization

4

4.1

$$x_1 = 1$$

$$x_2 = 2$$

4.2

```

1 function z = Assignment_3_4_f(x,y)
2 z=x-y+1;
3 end

```

Listing 7: f

```

1 function z = Assignment_3_4_g(x,y)
2 z=x^2+y^2-4;
3 end

```

Listing 8: g

```

1 function z = Assignment_3_4_fx(x,y)
2 z=1;
3 end

```

Listing 9: fx

```

1 function z = Assignment_3_4_fy(x,y)
2 z=-1;
3 end

```

Listing 10: fy

```

1 function z = Assignment_3_4_gx(x,y)
2 z=2*x;
3 end

```

Listing 11: gx

```

1 function z = Assignment_3_4_gy(x,y)
2 z=2*y;
3 end

```

Listing 12: gy

```
1 x=1;y=2;
2 d=zeros(2,1); rhs=d;
3 answer=[x,y];
4 A=zeros(2);
5
6 for i=1:10
7     fx=Assignment_3_4_fx(answer(1),answer(2));
8     fy=Assignment_3_4_fy(answer(1),answer(2));
9     gx=Assignment_3_4_gx(answer(1),answer(2));
10    gy=Assignment_3_4_gy(answer(1),answer(2));
11
12    f=Assignment_3_4_f(answer(1),answer(2));
13    g=Assignment_3_4_g(answer(1),answer(2));
14
15    A(1,1)=fx; A(1,2)=fy;
16    A(2,1)=gx; A(2,2)=gy;
17
18
19    rhs(1)=f;
20    rhs(2)=g;
21
22    d=A\ rhs;
23    answer(1)=answer(1)+d(1);
24    answer(2)=answer(2)+d(2);
25 end
26 disp([answer(1),answer(2)])
```

Listing 13: Code

Output

```
1 35.0324 36.0324
```

Listing 14: Output