Assignment 3 of CISC 2002

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March 10, 2021

1

1.1

Let

$$B = \begin{bmatrix} 3 & -2 & 5 & 2 \\ 4 & -7 & -1 & 19 \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$\frac{Row1}{3}$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 4 & -7 & -1 & 19 \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$Row2 - 4Row1$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & -\frac{13}{3} & -11 & \frac{49}{3} \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$Row2/-\frac{13}{3}$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 5 & -6 & 4 & 13 \end{bmatrix}$$

$Row3 - 5Row1$

$$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} & \frac{2}{3} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & -\frac{8}{3} & -\frac{13}{3} & \frac{29}{3} \end{bmatrix}$$

$Row1 + \frac{2}{3}Row2$ and $Row3 + \frac{8}{3}Row2$

$$\begin{bmatrix} 1 & 0 & \frac{37}{13} & -\frac{24}{13} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & 0 & -\frac{5}{13} & -\frac{5}{13} \end{bmatrix}$$

$Row3/\frac{5}{13}$

$$\begin{bmatrix} 1 & 0 & \frac{37}{13} & -\frac{24}{13} \\ 0 & 1 & \frac{23}{13} & \frac{49}{13} \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$Row1 - \frac{37}{13}Row3$ and $Row2 - \frac{23}{13}Row3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Thus

$$\begin{aligned} x_1 &= 1 \\ x_2 &= -2 \\ x_3 &= -1 \end{aligned}$$

1.2

Let

$$A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix}$$

From Question 1 we can know that

$$\begin{aligned} A &= \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & -\frac{13}{3} & -11 \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 5 & -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ -\frac{5}{3} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 0 & -\frac{8}{3} & -\frac{13}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{4}{13} & -\frac{3}{13} & 0 \\ -\frac{89}{39} & \frac{8}{13} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -\frac{2}{3} & \frac{5}{2} \\ 0 & 1 & \frac{23}{13} \\ 0 & 0 & -\frac{353}{39} \end{bmatrix} \end{aligned}$$

1.3

$$A = \begin{bmatrix} 3 & -2 & 5 \\ 4 & -7 & -1 \\ 5 & -6 & 4 \end{bmatrix}$$

We can get

$$A^{-1} = \begin{bmatrix} 6.8 & 4.4 & -7.4 \\ 4.2 & 2.6 & -4.6 \\ -2.2 & -1.6 & 2.6 \end{bmatrix}$$

$$\begin{aligned} A\vec{x}_{k+1} =& \vec{x}_k \\ \vec{x}_{k+1} =& \vec{x}_k A^{-1} \\ x_0 =& \begin{bmatrix} 2 \\ 19 \\ 13 \end{bmatrix} \\ x_1 =& \begin{bmatrix} 12.6 & 8.8 & -14.8 \\ 79.8 & 49.4 & -87.4 \\ -28.6 & -20.8 & 33.8 \end{bmatrix} \\ x_2 =& \begin{bmatrix} 92.48 & 38.72 & 109.52 \\ 335.160 & 128.44 & 402.04 \\ 62.92 & 33.28 & 87.88 \end{bmatrix} \\ x_3 =& 10^3 \times \begin{bmatrix} 0.6289 & 0.1704 & -0.8104 \\ 1.4077 & 0.3339 & -1.8494 \\ -0.1384 & -0.0532 & 0.2285 \end{bmatrix} \end{aligned}$$

2

2.1

$$\begin{aligned} T_n =& kn^3 \\ \frac{T_{40}}{T_{10}} =& \frac{k40^3}{k10^3} \\ T_{40} =& 4^3 \times T_{10} \\ =& 6.4s \end{aligned}$$

2.2

$$\begin{aligned} C_n &= kn^3 \\ \frac{C_{256}}{C_{1024}} &= \frac{k256^3}{k1024^3} \\ C_{256} &= (\frac{2^8}{2^10})^3 C_{1024} \\ &= \frac{1}{64} \times 1 \\ &= \frac{1}{64} s \end{aligned}$$

2.3

$$\begin{aligned} D_n &= kn^2 \\ \frac{D_{2048}}{D_{1024}} &= \frac{k2048^2}{k1024^2} \\ D_{2048} &= 2^2 D_{1024} \\ &= 4 \end{aligned}$$

3

3.1

```
clear all
x=2; y=1; z=5;
sig=1; r=3; b=1;
d=zeros(3,1); rhs=d;
A=zeros(3);n=0;
for i=1:10
    A(1,1)=-sig;  A(1,2)=sig;     A(1,3)=0;
    A(2,1)=r-z;     A(2,2)=-1;      A(2,3)=-x;
    A(3,1)=y;       A(3,2)=x;       A(3,3)=-b;
    rhs(1)=-sig*(y-x);
    rhs(2)=-(r*x-y-x*z);
    rhs(3)=-(x*y-b*z);
    d=A\rhs;
    x=x+d(1); y=y+d(2); z=z+d(3);
    n=n+1;
    if abs(d(1))<0.001 || abs(d(2))<0.001||abs(d(3))<0.001
        break;
    end
end
disp([x,y,z]')
fprintf("Loop run %d times\n",n)
```

Listing 1: Code after optimization

What I add is

```
    if abs(d(1))<0.001 || abs(d(2))<0.001||abs(d(3))<0.001
        break;
    end
```

Listing 2: Added code

Check the output:

```
    1.4142
    1.4142
    2.0000

Loop run 10 times
```

Listing 3: Before optimization

```


    1.4142
    1.4142
    2.0000

Loop run 4 times
```

Listing 4: After optimization

3.2

Set

$$x = 0$$
$$y = 0$$
$$z = 0$$

```
clear all
x=0; y=0; z=0;
sig=1; r=3; b=1;
d=zeros(3,1); rhs=d;
A=zeros(3);n=0;
for i=1:10
    A(1,1)=-sig;  A(1,2)=sig;    A(1,3)=0;
    A(2,1)=r-z;    A(2,2)=-1;     A(2,3)=-x;
    A(3,1)=y;       A(3,2)=x;      A(3,3)=-b;
    rhs(1)=-sig*(y-x);
    rhs(2)=-(r*x-y-x*z);
    rhs(3)=-(x*y-b*z);
    d=A\rhs;
    x=x+d(1); y=y+d(2); z=z+d(3);
    n=n+1;
    if abs(d(1))<0.001 || abs(d(2))<0.001||abs(d(3))<0.001
        break;
    end
```

```
end
disp([x,y,z]')
fprintf("Loop run %d times\n",n)
```

Listing 5: Code after optimization

Output

```
     0
     0
     0

Loop run 1 times
```

Listing 6: After optimization

4

4.1

$$x_1 = 1$$

$$x_2 = 1$$

4.2

```
function z = Assignment_3_4_f(x,y)
z=x-y+1;
end
```

Listing 7: f

```
function z = Assignment_3_4_g(x,y)
z=x^2+y^2-4;
end
```

Listing 8: g

```
function z = Assignment_3_4_fx(x,y)
z=1;
end
```

Listing 9: fx

```
function z = Assignment_3_4_fy(x,y)
z=-1;
end
```

Listing 10: fy

```
function z = Assignment_3_4_gx(x,y)
z=2*x;
end
```

Listing 11: gx

```
function z = Assignment_3_4_gy(x,y)
z=2*y;
end
```

Listing 12: gy

```
x=1;y=1;
d=zeros(2,1); rhs=d;
answer=[x,y];
A=zeros(2);

for i=1:10
    fx=Assignment_3_4_fx(answer(1),answer(2));
    fy=Assignment_3_4_fy(answer(1),answer(2));
    gx=Assignment_3_4_gx(answer(1),answer(2));
    gy=Assignment_3_4_gy(answer(1),answer(2));

    f=Assignment_3_4_f(answer(1),answer(2));
    g=Assignment_3_4_g(answer(1),answer(2));

    A(1,1)=fx; A(1,2)=fy;
    A(2,1)=gx; A(2,2)=gy;


    rhs(1)=-f;
    rhs(2)=-g;

    d=A\rhs;
    answer(1)=answer(1)+d(1);
    answer(2)=answer(2)+d(2);
end
disp([answer(1),answer(2)])
```

Listing 13: Code

Output

```
    0.8229    1.8229
```

Listing 14: Output