

Assignment 7 of CISC 2002

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1

1.1

```
1 function x=Assignment_7_1_x(t)
2     x=2/(1+exp(-2*t));
3 end
```

Listing 1: Code

```
1 function y=Assignment_7_1_f(x)
2     y=2*x-x^2;
3 end
```

Listing 2: Code

```
1 clear
2 delta=0.1
3 n=0;
4 x0=1;
5 x1=0;
6 while (n*delta<=1)
7     error=x0-Assignment_7_1_x(n*delta);
8     x1=x0+delta*Assignment_7_1_f(x0);
9     x0=x1;
10    n=n+1;
11    fprintf('%10.5f,%10.5f\n',x1,error)
12 end
13 disp("-----")
14 clear
15 delta=0.01
16 n=0;
17 x0=1;
18 x1=0;
19 while (n*delta<=1)
20     error=x0-Assignment_7_1_x(n*delta);
21     x1=x0+delta*Assignment_7_1_f(x0);
22     x0=x1;
23     n=n+1;
24     fprintf('%10.5f,%10.5f\n',x1,error)
```

25 end

Listing 3: Code

```
1
2 delta =
3
4     0.1000
5
6     1.1000,    0.00000
7     1.19900,    0.00033
8     1.29504,    0.00162
9     1.38634,    0.00373
10    1.47141,    0.00639
11    1.54919,    0.00929
12    1.61903,    0.01214
13    1.68071,    0.01466
14    1.73437,    0.01667
15    1.78044,    0.01807
16    1.81953,    0.01885
17
18
19 delta =
20
21     0.0100
22
23     1.01000,    0.00000
24     1.02000,    0.00000
25     1.03000,    0.00000
26     1.03999,    0.00000
27     1.04997,    0.00001
28     1.05995,    0.00001
29     1.06991,    0.00002
30     1.07986,    0.00002
31     1.08980,    0.00003
32     1.09972,    0.00004
33     1.10962,    0.00005
34     1.11950,    0.00006
35     1.12935,    0.00007
36     1.13919,    0.00008
37     1.14899,    0.00009
38     1.15877,    0.00011
39     1.16852,    0.00012
40     1.17823,    0.00014
41     1.18792,    0.00015
42     1.19756,    0.00017
43     1.20717,    0.00019
44     1.21674,    0.00021
45     1.22627,    0.00023
46     1.23576,    0.00025
47     1.24521,    0.00027
48     1.25461,    0.00029
49     1.26396,    0.00031
50     1.27326,    0.00033
51     1.28251,    0.00036
```

52	1.29172,	0.00038
53	1.30086,	0.00040
54	1.30996,	0.00043
55	1.31900,	0.00045
56	1.32798,	0.00048
57	1.33691,	0.00050
58	1.34577,	0.00053
59	1.35457,	0.00056
60	1.36332,	0.00058
61	1.37200,	0.00061
62	1.38061,	0.00064
63	1.38916,	0.00066
64	1.39765,	0.00069
65	1.40607,	0.00072
66	1.41442,	0.00075
67	1.42270,	0.00078
68	1.43092,	0.00080
69	1.43906,	0.00083
70	1.44713,	0.00086
71	1.45513,	0.00089
72	1.46306,	0.00092
73	1.47092,	0.00094
74	1.47870,	0.00097
75	1.48641,	0.00100
76	1.49404,	0.00103
77	1.50160,	0.00105
78	1.50908,	0.00108
79	1.51649,	0.00111
80	1.52383,	0.00113
81	1.53108,	0.00116
82	1.53826,	0.00119
83	1.54536,	0.00121
84	1.55239,	0.00124
85	1.55934,	0.00126
86	1.56621,	0.00129
87	1.57300,	0.00131
88	1.57972,	0.00133
89	1.58636,	0.00136
90	1.59292,	0.00138
91	1.59941,	0.00140
92	1.60581,	0.00142
93	1.61214,	0.00144
94	1.61840,	0.00147
95	1.62457,	0.00149
96	1.63067,	0.00151
97	1.63669,	0.00153
98	1.64264,	0.00154
99	1.64851,	0.00156
100	1.65430,	0.00158
101	1.66002,	0.00160
102	1.66567,	0.00161
103	1.67124,	0.00163
104	1.67673,	0.00164
105	1.68215,	0.00166
106	1.68750,	0.00167

```

107 1.69277, 0.00169
108 1.69797, 0.00170
109 1.70310, 0.00171
110 1.70816, 0.00173
111 1.71314, 0.00174
112 1.71806, 0.00175
113 1.72290, 0.00176
114 1.72767, 0.00177
115 1.73238, 0.00178
116 1.73701, 0.00178
117 1.74158, 0.00179
118 1.74608, 0.00180
119 1.75052, 0.00181
120 1.75488, 0.00181
121 1.75919, 0.00182
122 1.76342, 0.00182
123 1.76759, 0.00183

```

Listing 4: Code

1.2

```

1 clear
2 delta=0.1
3 n=0;
4 x0=1;
5 x1=0;
6 while (n*delta<=1)
7     error=x0-Assignment_7_1_x(n*delta);
8     k1=delta*Assignment_7_1_f(x0);
9     k2=delta*Assignment_7_1_f(x0+k1);
10    x1=x0+(k1+k2)/2;
11    x0=x1;
12    n=n+1;
13    fprintf('%10.5f,%10.5f\n',x1,error)
14 end
15 disp('-----')
16 clear
17 delta=0.01
18 n=0;
19 x0=1;
20 x1=0;
21 while (n*delta<=1)
22     error=x0-Assignment_7_1_x(n*delta);
23     k1=delta*Assignment_7_1_f(x0);
24     k2=delta*Assignment_7_1_f(x0+k1);
25     x1=x0+(k1+k2)/2;
26     x0=x1;
27     n=n+1;
28     fprintf('%10.5f,%10.5f\n',x1,error)
29 end

```

Listing 5: Code

```
1
2 delta =
3
4     0.1000
5
6     1.09950,    0.00000
7     1.19703,   -0.00017
8     1.29080,   -0.00034
9     1.37926,   -0.00052
10    1.46126,   -0.00069
11    1.53604,   -0.00086
12    1.60324,   -0.00100
13    1.66281,   -0.00113
14    1.71500,   -0.00123
15    1.76027,   -0.00130
16    1.79917,   -0.00133
17
18
19 delta =
20
21     0.0100
22
23     1.01000,    0.00000
24     1.02000,   -0.00000
25     1.02999,   -0.00000
26     1.03998,   -0.00000
27     1.04996,   -0.00000
28     1.05993,   -0.00000
29     1.06988,   -0.00000
30     1.07983,   -0.00000
31     1.08976,   -0.00000
32     1.09967,   -0.00000
33     1.10956,   -0.00000
34     1.11943,   -0.00000
35     1.12927,   -0.00000
36     1.13909,   -0.00000
37     1.14888,   -0.00000
38     1.15865,   -0.00000
39     1.16838,   -0.00000
40     1.17808,   -0.00000
41     1.18774,   -0.00000
42     1.19737,   -0.00000
43     1.20696,   -0.00000
44     1.21651,   -0.00000
45     1.22602,   -0.00000
46     1.23549,   -0.00000
47     1.24491,   -0.00000
48     1.25429,   -0.00000
49     1.26362,   -0.00000
50     1.27290,   -0.00000
51     1.28213,   -0.00000
52     1.29131,   -0.00000
53     1.30043,   -0.00000
54     1.30950,   -0.00001
55     1.31852,   -0.00001
```

56	1.32747,	-0.00001
57	1.33637,	-0.00001
58	1.34521,	-0.00001
59	1.35399,	-0.00001
60	1.36270,	-0.00001
61	1.37135,	-0.00001
62	1.37994,	-0.00001
63	1.38847,	-0.00001
64	1.39692,	-0.00001
65	1.40531,	-0.00001
66	1.41364,	-0.00001
67	1.42189,	-0.00001
68	1.43008,	-0.00001
69	1.43819,	-0.00001
70	1.44624,	-0.00001
71	1.45421,	-0.00001
72	1.46211,	-0.00001
73	1.46994,	-0.00001
74	1.47769,	-0.00001
75	1.48537,	-0.00001
76	1.49298,	-0.00001
77	1.50051,	-0.00001
78	1.50797,	-0.00001
79	1.51535,	-0.00001
80	1.52266,	-0.00001
81	1.52989,	-0.00001
82	1.53704,	-0.00001
83	1.54412,	-0.00001
84	1.55112,	-0.00001
85	1.55804,	-0.00001
86	1.56489,	-0.00001
87	1.57166,	-0.00001
88	1.57835,	-0.00001
89	1.58497,	-0.00001
90	1.59151,	-0.00001
91	1.59797,	-0.00001
92	1.60436,	-0.00001
93	1.61067,	-0.00001
94	1.61690,	-0.00001
95	1.62305,	-0.00001
96	1.62913,	-0.00001
97	1.63514,	-0.00001
98	1.64107,	-0.00001
99	1.64692,	-0.00001
100	1.65270,	-0.00001
101	1.65840,	-0.00001
102	1.66403,	-0.00001
103	1.66958,	-0.00001
104	1.67506,	-0.00001
105	1.68046,	-0.00001
106	1.68580,	-0.00001
107	1.69106,	-0.00001
108	1.69625,	-0.00001
109	1.70136,	-0.00001
110	1.70641,	-0.00001

```

111 1.71138, -0.00001
112 1.71629, -0.00001
113 1.72112, -0.00001
114 1.72589, -0.00001
115 1.73058, -0.00001
116 1.73521, -0.00001
117 1.73977, -0.00001
118 1.74426, -0.00001
119 1.74869, -0.00001
120 1.75305, -0.00001
121 1.75735, -0.00001
122 1.76158, -0.00001
123 1.76575, -0.00001

```

Listing 6: Code

1.3

```

1 clear
2 delta=0.1
3 n=0;
4 x0=1;
5 x1=0;
6 while (n*delta<=1)
7     error=x0-Assignment_7_1_x(n*delta);
8     k1=delta*Assignment_7_1_f(x0);
9     k2=delta*Assignment_7_1_f(x0+k1/2);
10    k3=delta*Assignment_7_1_f(x0+k2/2);
11    k4=delta*Assignment_7_1_f(x0+k3);
12    x1=x0+(k1+2*k2+2*k3+k4)/6;
13    x0=x1;
14    n=n+1;
15    fprintf('%10.5f,%10.5f\n',x1,error)
16 end
17 disp("-----")
18 clear
19 delta=0.01
20 n=0;
21 x0=1;
22 x1=0;
23 while (n*delta<=1)
24     error=x0-Assignment_7_1_x(n*delta);
25     k1=delta*Assignment_7_1_f(x0);
26     k2=delta*Assignment_7_1_f(x0+k1/2);
27     k3=delta*Assignment_7_1_f(x0+k2/2);
28     k4=delta*Assignment_7_1_f(x0+k3);
29     x1=x0+(k1+2*k2+2*k3+k4)/6;
30     x0=x1;
31     n=n+1;
32     fprintf('%10.5f,%10.5f\n',x1,error)
33 end

```

Listing 7: Code

```
1
2 delta =
3
4     0.1000
5
6     1.09967,    0.00000
7     1.19738,   -0.00000
8     1.29131,   -0.00000
9     1.37995,   -0.00000
10    1.46212,   -0.00000
11    1.53705,   -0.00000
12    1.60437,   -0.00000
13    1.66404,   -0.00000
14    1.71630,   -0.00000
15    1.76159,   -0.00000
16    1.80050,   -0.00000
17
18
19 delta =
20
21     0.0100
22
23     1.01000,    0.00000
24     1.02000,   -0.00000
25     1.02999,   -0.00000
26     1.03998,   -0.00000
27     1.04996,   -0.00000
28     1.05993,   -0.00000
29     1.06989,   -0.00000
30     1.07983,   -0.00000
31     1.08976,   -0.00000
32     1.09967,   -0.00000
33     1.10956,   -0.00000
34     1.11943,   -0.00000
35     1.12927,   -0.00000
36     1.13909,   -0.00000
37     1.14889,   -0.00000
38     1.15865,   -0.00000
39     1.16838,   -0.00000
40     1.17808,   -0.00000
41     1.18775,   -0.00000
42     1.19738,   -0.00000
43     1.20697,   -0.00000
44     1.21652,   -0.00000
45     1.22603,   -0.00000
46     1.23550,   -0.00000
47     1.24492,   -0.00000
48     1.25430,   -0.00000
49     1.26362,   -0.00000
50     1.27291,   -0.00000
51     1.28213,   -0.00000
52     1.29131,   -0.00000
53     1.30044,   -0.00000
54     1.30951,   -0.00000
55     1.31852,   -0.00000
```


56	1.32748,	-0.00000
57	1.33638,	-0.00000
58	1.34521,	-0.00000
59	1.35399,	-0.00000
60	1.36271,	-0.00000
61	1.37136,	-0.00000
62	1.37995,	-0.00000
63	1.38847,	-0.00000
64	1.39693,	-0.00000
65	1.40532,	-0.00000
66	1.41364,	-0.00000
67	1.42190,	-0.00000
68	1.43008,	-0.00000
69	1.43820,	-0.00000
70	1.44624,	-0.00000
71	1.45422,	-0.00000
72	1.46212,	-0.00000
73	1.46995,	-0.00000
74	1.47770,	-0.00000
75	1.48538,	-0.00000
76	1.49299,	-0.00000
77	1.50052,	-0.00000
78	1.50798,	-0.00000
79	1.51536,	-0.00000
80	1.52267,	-0.00000
81	1.52990,	-0.00000
82	1.53705,	-0.00000
83	1.54413,	-0.00000
84	1.55113,	-0.00000
85	1.55805,	-0.00000
86	1.56490,	-0.00000
87	1.57167,	-0.00000
88	1.57836,	-0.00000
89	1.58498,	-0.00000
90	1.59152,	-0.00000
91	1.59798,	-0.00000
92	1.60437,	-0.00000
93	1.61068,	-0.00000
94	1.61691,	-0.00000
95	1.62307,	-0.00000
96	1.62915,	-0.00000
97	1.63515,	-0.00000
98	1.64108,	-0.00000
99	1.64693,	-0.00000
100	1.65271,	-0.00000
101	1.65841,	-0.00000
102	1.66404,	-0.00000
103	1.66959,	-0.00000
104	1.67507,	-0.00000
105	1.68048,	-0.00000
106	1.68581,	-0.00000
107	1.69107,	-0.00000
108	1.69626,	-0.00000
109	1.70137,	-0.00000
110	1.70642,	-0.00000

```

111 1.71139, -0.00000
112 1.71630, -0.00000
113 1.72113, -0.00000
114 1.72590, -0.00000
115 1.73059, -0.00000
116 1.73522, -0.00000
117 1.73978, -0.00000
118 1.74428, -0.00000
119 1.74870, -0.00000
120 1.75307, -0.00000
121 1.75736, -0.00000
122 1.76159, -0.00000
123 1.76576, -0.00000

```

Listing 8: Code

1.4

```

1 clear
2 tspan=[0 1];
3 y0=1;
4 [t,y]=ode45(@(t,y) 2*y-y^2,tspan,y0);
5 plot(t,y,'-o')
6 for i=1:41
7     error=2/(1+exp(-2*t(i)))-y(i);
8     disp("x= "+y(i)+" ,error= "+error)
9 end

```

Listing 9: Code

```

1 x= 1 ,error= 0
2 x= 1.025 ,error= -1.2671e-08
3 x= 1.05 ,error= -2.113e-09
4 x= 1.0749 ,error= 1.0199e-08
5 x= 1.0997 ,error= -2.9569e-11
6 x= 1.1244 ,error= -1.0619e-08
7 x= 1.1489 ,error= -1.634e-09
8 x= 1.1732 ,error= 8.5277e-09
9 x= 1.1974 ,error= -4.1045e-11
10 x= 1.2213 ,error= -7.2699e-09
11 x= 1.2449 ,error= -1.3276e-09
12 x= 1.2683 ,error= 5.4479e-09
13 x= 1.2913 ,error= -1.4501e-11
14 x= 1.314 ,error= -3.6185e-09
15 x= 1.3364 ,error= -1.5477e-09
16 x= 1.3584 ,error= 1.5845e-09
17 x= 1.3799 ,error= 6.1487e-11
18 x= 1.4011 ,error= -6.1786e-10
19 x= 1.4219 ,error= -2.5302e-09
20 x= 1.4422 ,error= -2.3811e-09
21 x= 1.4621 ,error= 1.875e-10
22 x= 1.4815 ,error= 1.1338e-09

```

```

23 x= 1.5005 ,error= -4.2778e-09
24 x= 1.519 ,error= -5.8795e-09
25 x= 1.537 ,error= 3.6431e-10
26 x= 1.5546 ,error= 1.5139e-09
27 x= 1.5717 ,error= -6.549e-09
28 x= 1.5883 ,error= -8.5399e-09
29 x= 1.6044 ,error= 6.021e-10
30 x= 1.62 ,error= 7.9792e-10
31 x= 1.6351 ,error= -8.948e-09
32 x= 1.6498 ,error= -1.0202e-08
33 x= 1.664 ,error= 9.1778e-10
34 x= 1.6778 ,error= -5.3161e-10
35 x= 1.6911 ,error= -1.106e-08
36 x= 1.7039 ,error= -1.0878e-08
37 x= 1.7163 ,error= 1.3232e-09
38 x= 1.7283 ,error= -1.9823e-09
39 x= 1.7398 ,error= -1.2567e-08
40 x= 1.7509 ,error= -1.0702e-08
41 x= 1.7616 ,error= 1.8138e-09

```

Listing 10: Code

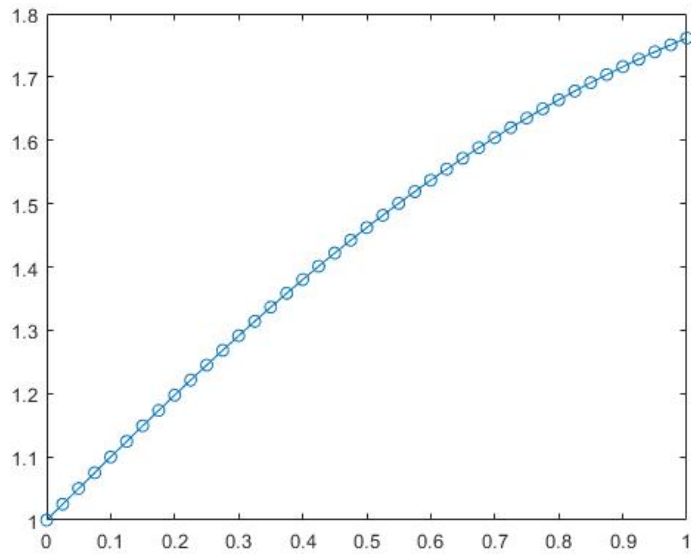


Figure 1: Figure

2.1

Let $\frac{dx}{dt} = x_1$

$$\begin{aligned}x(0) &= 0 \\ \frac{dx}{dt}(0) &= -1 \\ x_1(t) &= \frac{dx}{dt} \\ \frac{dx_1}{dt}(t) &= -x\end{aligned}$$

2.2

$$\begin{aligned}x_{n+1} &= x_n + \Delta t x_1(t_n, y_{1n}, y_{2n}) \\ &= x_n + \Delta t x_1(t_n) \\ x_{1n+1} &= x_{1n} + \Delta t(-x_n) \\ \Delta t &= \frac{\pi}{10} \\ x_{11} &= x_{10} + \Delta t(-x_0) \\ &= -1 + \frac{\pi}{10}(-0) \\ &= -1 \\ x_1 &= x_0 + \Delta t x_{11} \\ &= -\frac{\pi}{10} \\ x_{12} &= x_{11} + \Delta t(-x_1) \\ &= -1 + \frac{\pi^2}{100} \\ x_2 &= x_1 + \Delta t x_{12} \\ &= -\frac{\pi}{5} + \frac{\pi^3}{1000}\end{aligned}$$