Assignment 4 of MATH 2005

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1

$$
\begin{aligned}
E(X) =& \sum_{x=-1}^{3} x f(x) \\
=& -1 \times \frac{3}{7} + 0 \times \frac{2}{7} + 1 \times \frac{1}{7} + 2 \times \frac{0}{7} + 3 \times \frac{1}{7} \\
=& \frac{1}{7}
\end{aligned}
$$

2

$$
\begin{aligned}
E(X) =& \int_{-\infty}^{\infty} y h(y) dy \\
=& \int_{2}^{4} \frac{1}{8}(y+1) y dy + 0 \\
=& \frac{79}{3}
\end{aligned}
$$

3

3.1

$$
\begin{aligned}
E(X) =& \int_{-\infty}^{\infty} x f(x) dx \\
=& \int_{1}^{3} x \frac{1}{x \log 3} dx + 0 \\
=& \frac{2}{\log 3}
\end{aligned}
$$

$$
\begin{aligned}
E(X^2) &= \int_{-\infty}^{\infty} x^2 f(x) dx \\
&= \int_1^3 \frac{x}{\log 3} dx \\
&= \frac{4}{\log 3}
\end{aligned}
$$

$$
\begin{aligned}
E(X^3) &= \int_{-\infty}^{\infty} x^3 f(x) dx \\
&= \int_1^3 \frac{x^2}{\log 3} dx \\
&= \frac{26}{3\log 3}
\end{aligned}
$$

$$
\begin{aligned}
E(X^3 + 2X^2 - 3X + 1) &= E(X^3) + 2E(X^2) - 3E(X) + E(1) \\
&= \frac{26}{3\log 3} + 2 \times \frac{4}{\log 3} - 3 \times \frac{2}{\log 3} + 1 \\
&= \frac{35}{3\log 3}
\end{aligned}
$$

4

$$
\begin{aligned}
E(\frac{X}{Y}) &= \int_0^1 \int_0^y \frac{x}{y} f(x,y) dx dy \\
&= \int_0^1 \int_0^y \frac{x}{y^2} dx dy \\
&= \int_0^1 \frac{x^2}{2y^2} |_0^y dy \\
&= \int_0^1 \frac{1}{2} dy \\
&= \frac{1}{2} y|_0^1 \\
&= \frac{1}{2}
\end{aligned}
$$

5

Let $\varphi(x)$ is the money he should pay us where x is the number we get from a balanced die.

$$
\begin{aligned}
E(\varphi(X)) =& \sum_{x=1}^{6} \varphi(x) f(x) \\
=& \frac{1}{6} \sum_{x=1}^{4} \varphi(x) + \frac{5}{3} \\
=& 0
\end{aligned}
$$

Thus, we can get,

$$\sum_{x=1}^{4} \varphi(x) = -10$$

which means that the total money we should pay that person when we roll a $1, 2, 3,$ or 4 is equal to \$10. Hence, there are many solutions for this equation. For example,

$$
\begin{aligned}
\varphi(1) = \varphi(2) = \varphi(3) = 0, \\
\varphi(4) = -10.
\end{aligned}
$$

6

6.1

Let

$$\varphi(x; n) = \begin{cases} x - (n - x) \times 0.4 & (0 \le x \le n) \\ n & (n \le x). \end{cases}$$

be the profit when produce n cake(s) a day.

(a) one of the cakes

$$
\begin{aligned}
E(\varphi(X; 1)) =& \sum_{n=0}^{5} \varphi(x; 1) f(x) \\
=& \frac{1}{6}(-0.4 + 1 + 1 + 1 + 1 + 1) \\
=& \frac{23}{30}
\end{aligned}
$$

(b) two of the cakes

$$\begin{aligned}E(\varphi(X;2)) =& \sum_{n=0}^{5} \varphi(x;2)f(x)\\ =& \frac{1}{6}(-0.8+0.6+2+2+2+2)\\ =& 1.2\end{aligned}$$

(c) three of the cakes

$$\begin{aligned}E(\varphi(X;3)) =& \sum_{n=0}^{5} \varphi(x;3)f(x)\\ =& \frac{1}{6}(-1.2+0.2+1.6+3+3+3)\\ =& 1.6\end{aligned}$$

(d) four of the cakes

$$\begin{aligned}E(\varphi(X;4)) =& \sum_{n=0}^{5} \varphi(x;4)f(x)\\ =& \frac{1}{6}(-1.6-0.2+1.2+2.6+4+4)\\ =& \frac{5}{3}\end{aligned}$$

(e) five of the cakes

$$\begin{aligned}E(\varphi(X;5)) =& \sum_{n=0}^{5} \varphi(x;5)f(x)\\ =& \frac{1}{6}(-2-1.6+0.8+2.2+3.6+5)\\ =& \frac{4}{3}\end{aligned}$$

6.2

By 6.1 **we can know that he should bake 3 cakes a day to maximize his expected profit.**

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