Assignment 6 of CISC 1006

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1

It easy to know that x $Poission$ **where** $\lambda = 3$**.**

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

1.1

$$\begin{aligned} P(X = 5) =& \frac{3^5}{5!} e^{-3} \\ \approx& 0.1008 \end{aligned}$$

1.2

$$\begin{aligned} P(X < 3) =& \sum_{x=0}^{2} P(X = x) \\ =& \sum_{x=0}^{2} \frac{\lambda^x}{x!} e^{-\lambda} \\ \approx& 0.4232 \end{aligned}$$

1.3

$$\begin{aligned} P(X \geq 2) = 1 - P(X \leq 1) \\ =& 1 - \sum_{x=0}^{1} \frac{3^x}{x!} e^{-3} \\ \approx& 0.8008 \end{aligned}$$

2

It easy to know that x *Poission* **where** $\lambda = 5$**.**

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

2.1

$$\begin{aligned} P(X > 5) =& 1 - P(X \leq 5) \\ =& 1 - \sum_{x=0}^{5} \frac{5^x}{x!} e^{-5} \\ \approx& 0.3840 \end{aligned}$$

2.2

$$\begin{aligned} P(\textbf{3 of next 4 days}) =& C_4^3 P(X > 5)^3 P(X \leq 5) \\ \approx& 0.0349 \end{aligned}$$

2.3

$$\begin{aligned} P(\textbf{The first time in April on April 5th}) =& P(X > 5) P(X \leq 5)^4 \\ \approx& 0.0553 \end{aligned}$$

3

3.1

Using Binomial distribution:

$$\begin{aligned} P(X < 5|\textbf{In 2000 people}) =& \sum_{x=0}^{4} C_{2000}^x 0.002^x (1 - 0.002)^{2000-x} \\ \approx& 0.6288 \end{aligned}$$

Using Poission Approximation:

$$\begin{aligned} \lambda =& np \\ =& 2000 \times 0.002 \\ =& 4 \\ P(X = x) =& \frac{4^x}{x!} e^{-4} \\ P(X < 5) =& \sum_{x=0}^{4} \frac{4^x}{x!} e^{-4} \\ \approx& 0.6288 \end{aligned}$$

3.2

Using Binomial distribution:

$$P(X = x) = C_{2000}^{x} 0.002^{x}(1 - 0.002)^{2000-x}$$

Using Poission Approximation:

$$P(X = x) = \frac{4^{x}}{x!}e^{-4}$$

3.2.1

Using Binomial distribution:

$$\begin{aligned}\mu &= \sum_{x=0}^{2000} xP(X = x) \\ &= \sum_{x=0}^{2000} xC_{2000}^{x} 0.002^{x}(1 - 0.002)^{2000-x} \\ &= 3.99\dot{9} \\ &\approx 4.0000\end{aligned}$$

Using Poission Approximation:

By definition:

$\mu = 4.000$

3.2.2

Chebyshev's inequality:

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

and

$$P(|X - \mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}.$$

$$\begin{aligned}X &\geq 1500 \\ |X - \mu| &> 1496 \\ \sigma^2 &= \lambda \\ &= 4 \\ \sigma &= 2 \\ 1496 &= k\sigma \\ k &= 748 \\ &\textbf{Thus} \\ P(|X - \mu| \geq 748\sigma) &\leq \frac{1}{748^2} \\ &= \frac{1}{559504} \\ &\approx 1.787 \times 10^{-6}\end{aligned} \tag{1}$$