Assignment 5 of CISC 2002

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1

1.1

```
clear
x=[2,4,6,8,10,12];
y=[2,4,4,5,5,7];
A=vander(x);
c=A\y'
```

Listing 1: Code

```

c =

     0.0026
    -0.0911
     1.2083
    -7.5104
    21.8750
   -20.0000
```

Listing 2: Output

1.2

$$
\begin{aligned}
P(x) =& \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)} y_0 \\
&+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} y_1 \\
&+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} y_2 \\
&+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} y_3 \\
&+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} y_4 \\
&+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_0)(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} y_5 \\
=& \frac{(x-4)(x-6)(x-8)(x-10)(x-12)}{(2-4)(2-6)(2-8)(2-10)(2-12)} 2 \\
&+ \frac{(x-2)(x-6)(x-8)(x-10)(x-12)}{(4-2)(4-6)(4-8)(4-10)(4-12)} 4 \\
&+ \frac{(x-2)(x-4)(x-8)(x-10)(x-12)}{(6-2)(6-4)(6-8)(6-10)(6-x12)} 4 \\
&+ \frac{(x-2)(x-4)(x-6)(x-10)(x-x12)}{(8-2)(8-4)(8-6)(8-10)(8-12)} 5 \\
&+ \frac{(x-2)(x-4)(x-6)(x-8)(x-12)}{(10-2)(10-4)(10-6)(10-8)(10-12)} 5 \\
&+ \frac{(x-2)(x-4)(x-6)(x-8)(x-10)}{(12-2)(12-4)(12-6)(12-8)(12-10)} 7
\end{aligned}
$$

1.3

$$
A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 4 & 8 & 0 & 0 & 0 \\ 1 & 6 & 24 & 48 & 0 & 0 \\ 1 & 8 & 48 & 192 & 384 & 0 \\ 1 & 10 & 80 & 480 & 1920 & 3840 \end{bmatrix}, \vec{y} = \begin{bmatrix} 2 \\ 4 \\ 4 \\ 5 \\ 5 \\ 7 \end{bmatrix}
$$

$$
A\vec{c} = \vec{y}
$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 4 & 8 & 0 & 0 & 0 \\ 1 & 6 & 24 & 48 & 0 & 0 \\ 1 & 8 & 48 & 192 & 384 & 0 \\ 1 & 10 & 80 & 480 & 1920 & 3840 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \\ 5 \\ 5 \\ 7 \end{bmatrix}$$

We can get the augmented matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 & 0 & 0 & 4 \\ 1 & 4 & 8 & 0 & 0 & 0 & 4 \\ 1 & 6 & 24 & 48 & 0 & 0 & 5 \\ 1 & 8 & 48 & 192 & 384 & 0 & 5 \\ 1 & 10 & 80 & 480 & 1920 & 3840 & 7 \end{bmatrix}$$

$R_n = R_n - R_1, n \in [2, 6]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 4 & 8 & 0 & 0 & 0 & 2 \\ 0 & 6 & 24 & 48 & 0 & 0 & 3 \\ 0 & 8 & 48 & 192 & 384 & 0 & 3 \\ 0 & 10 & 80 & 480 & 1920 & 3840 & 5 \end{bmatrix}$$

$R_n = R_n - kR_2, n \in [3, 6], k \in \mathbb{R}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 8 & 0 & 0 & 0 & -2 \\ 0 & 0 & 24 & 48 & 0 & 0 & -3 \\ 0 & 0 & 48 & 192 & 384 & 0 & -5 \\ 0 & 0 & 80 & 480 & 1920 & 3840 & -5 \end{bmatrix}$$

$R_n = R_n - kR_3, n \in [4, 6], k \in \mathbb{R}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 8 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 48 & 0 & 0 & 3 \\ 0 & 0 & 0 & 192 & 384 & 0 & 7 \\ 0 & 0 & 0 & 480 & 1920 & 3840 & 15 \end{bmatrix}$$

$R_n = R_n - kR_4, n \in [5,6], k \in \mathbb{R}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 8 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 48 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 384 & 0 & -5 \\ 0 & 0 & 0 & 0 & 1920 & 3840 & -15 \end{bmatrix}$$

$R_6 = R_6 - 5R_5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 8 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 48 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 384 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 & 3840 & 10 \end{bmatrix}$$

We can get

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{16} \\ 0 & 0 & 0 & 0 & 1 & 0 & -\frac{5}{384} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{384} \end{bmatrix}$$

$$\vec{c} = \begin{bmatrix} 2 \\ 1 \\ -\frac{1}{4} \\ \frac{1}{16} \\ -\frac{4}{384} \\ \frac{1}{384} \end{bmatrix}$$

1.4

```
clear
x=[2,4,6,8,10,12];
y=[2,4,4,5,5,7];
A=vander(x);
c=A\y';
xx=0:0.01:12;
yy=polyval(c,xx);
plot(xx,yy,x,y,'ro')
```

Listing 3: Code

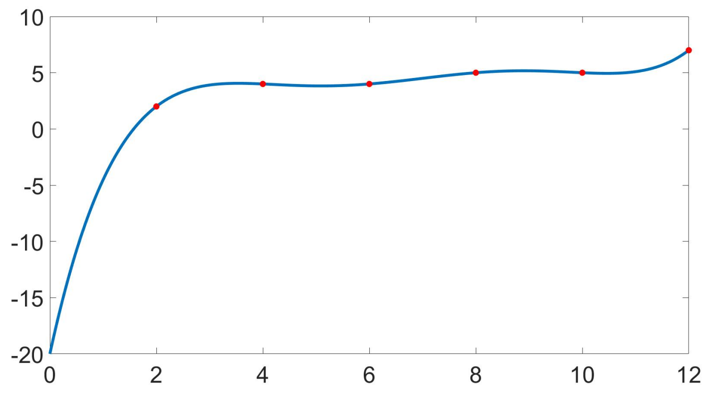

Figure 1: Figure

2

2.1

```
clear
a=0:1:4;
x=-1+(a./2);
y=exp(x);
A=vander(x);
c=A\y';
axis=-1.5:0.01:1.5;
y1=exp(axis);
y2=polyval(c,axis);
plot(axis,y1,'r',axis,y2,'b')
```

Listing 4: Code

```

c =

    0.0434
    0.1773
    0.4996
    0.9979
    1.0000
```

Listing 5: Output

2.2

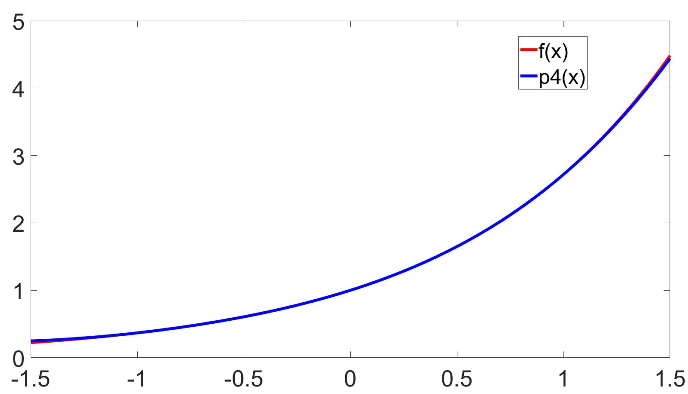

Figure 2: Figure

3

3.1

```
clear
a=0:1:10;
x=-5+a;
y=1./(1+x.^2);
A=vander(x);
c=A\y';
axis=-5:0.01:5;
y1=1./(1+axis.^2);
y2=polyval(c,axis);
plot(axis,y1,'b',axis,y2,'r')
```

Listing 6: Code

```
c =

   -0.0000
   -0.0000
    0.0013
    0.0000
   -0.0244
   -0.0000
    0.1974
    0.0000
   -0.6742
   -0.0000
```

```
      1.0000
```

Listing 7: Output

3.2

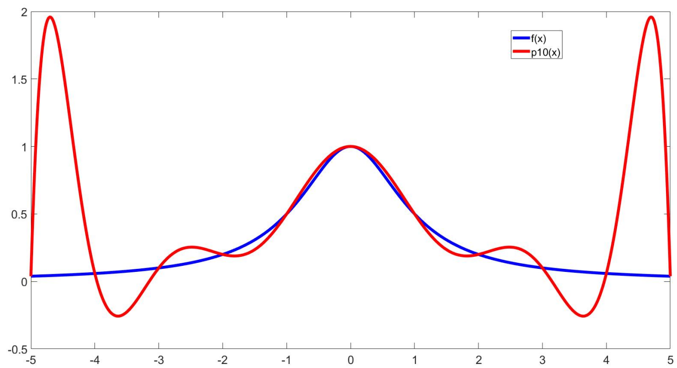

Figure 3: Figure

4

4.1

```
clear
a=0:1:10;
x=-5+(10.*a)./10;
y=1./(1+x.^2);
t=4.8;
res=interp1(x,y,t)
```

Listing 8: Code

```
res =

      0.0425
```

Listing 9: Output

4.2

```
clear
a=0:1:10;
x=-5+(10.*a)./10;
y=1./(1+x.^2);
xx=-5:0.01:5;
y1=1./(1+xx.^2);
y2 = spline(x,y,xx);
plot(xx,y1,xx,y2)
```

Listing 10: Code

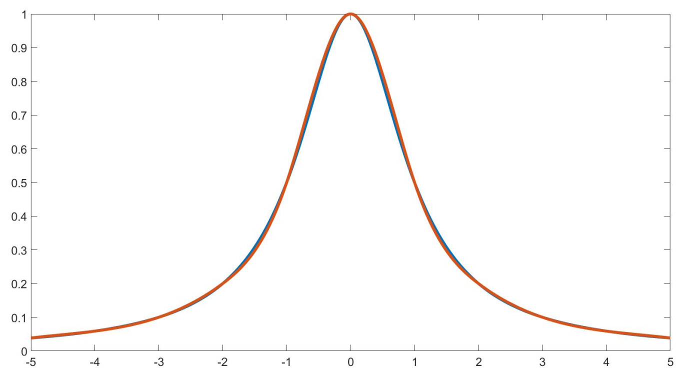

Figure 4: Figure

4.3

Let

$$g_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i$$

where

$$\begin{aligned} g_i(x_i) &= y_i \\ g_i(x_{i+1}) &= y_{i+1} \\ g_i'(x_{i+1}) &= g_{i+1}'(x_{i+1}) \\ g_i''(x_{i+1}) &= g_{i+1}''(x_{i+1}) \end{aligned}$$

We get

$$
\begin{aligned}
g_0(-5) =& \frac{1}{26} \\
g_0(0) =& 1 \\
g_0'(0) =& g_1'(0) \\
g_0''(0) =& g_1''(0) \\
g_1(0) =& 0 \\
g_1(5) =& \frac{1}{26} \\
g_0'(-5) =& 0 \\
g_1'(5) =& 0
\end{aligned}
$$

Find the solution

$$
\begin{aligned}
d_0 =& \frac{1}{26} \\
d_1 =& 1 \\
c_0 =& 0 \\
b_1 =& 0 \\
a_0 =& -\frac{1}{260} \\
b_0 =& \frac{3}{52} \\
c_1 =& \frac{15}{52} \\
a_1 =& -\frac{1}{100}
\end{aligned}
$$

$$g(x) = -\frac{1}{260}(x+5)^3 + \frac{1}{26}$$

when $x \in [-5, 0]$

$$g(x) = -\frac{1}{100}x^3 + \frac{15}{52}x + 1$$

when $x \in (0, 5]$

5

```
clear
x=-1:1;
y=-1:1;
z=1./(1+x.^2+y'.^2);
```

```
res1=interp2(x,y',z,1,0.5)
res2=interp2(x,y',z,0.5,0.5)
```

Listing 11: Code

```

res1 =

    0.4167


res2 =

    0.5833
```

Listing 12: Output