Assignment 1

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1.

(i)

i	$f(x_i)$
0	1
1	$\frac{1}{2}$
2	$\frac{1}{5}$
3	$\frac{1}{10}$
4	$\frac{1}{17}$
5	$\frac{1}{26}$
6	$\frac{1}{37}$
7	$\frac{1}{50}$
8	$\frac{1}{65}$
9	$\frac{1}{82}$
10	$\frac{1}{101}$

$$\int_0^1 \frac{1}{1+x^2}dx \approx \sum_{i=1}^{10} \frac{f(x_{i-1})+f(x_i)}{2}\frac{1}{10} = 0.7849814972 \approx 0.7850$$

(ii)

$$f(x) = \frac{1}{1+x^2}$$

$$f'(x) = -\frac{2x}{(1+x^2)^2}$$

$$f''(x) = \frac{6x^2 - 2}{(1+x^2)^3}$$

$$K_2 \geq f''(x)_{max} = \frac{1}{2}$$

$$|Error\ of\ Trapezoidal\ Approximation| \leq \frac{K_2(b-a)^3}{12n^2} \leq 0.0001$$

$$n \geq \frac{25\sqrt{6}}{3} \approx 20.41$$

$$n \geq 21$$

2.

(i)

$$f(x) = \frac{\sin x}{x}$$

$$T_n = \sum_{i=1}^{10} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x$$

$$M_n = \sum_{i=1}^{10} f(\frac{x_{i-1} + x_i}{2}) \Delta x$$

$$S_n = \frac{2}{3} M_n + \frac{1}{3} T_n$$

$$S_n \approx 1.8519$$

(ii)

$$|Error\ of\ Simpson's\ Approximation| \leq \frac{K_4(b-a)^5}{180n^4} \leq 5 \times 10^{-8}$$

$$n \geq 53.99$$

$$n \geq 54$$

3.

$Proof:$

$$Left\ side = A = \int_a^b f(x) = \int_a^b px^2 + qx + r = \frac{p}{3}x^3 + \frac{q}{2}x^2 + rx|_a^b$$

$$= \frac{p}{3}(b^3 - a^3) + \frac{q}{2}(b^2 - a^2) + r(b-a) = (b-a)[\frac{p}{3}(b^2 + ab + a^2) + \frac{q}{2}(b+a) + r]$$

$$Right\ side = \frac{1}{3} \times \frac{b-a}{2}[f(a) + 4f(\frac{b+a}{2}) + f(b)] =$$

$$= \frac{b-a}{6}\left\{pa^2 + qa + r + 4[\frac{p}{4}(b+a)^2 + \frac{q}{2}(b-a) + r] + pb^2 + qb + r\right\}$$

$$= (b-a)[\frac{p}{3}(b^2 + ab + a^2) + \frac{q}{2}(b+a) + r]$$

$$Left\ side = Right\ side$$

$$A = \frac{1}{3} \times \frac{b-a}{2}[f(a) + 4f(\frac{b+a}{2}) + f(b)]$$

4.

(i)

Let $t = \ln x$,$dt = \frac{dx}{x}$**, and** $x = e^t$

Hence,

$$\int \sin(\ln x)dx = \int \sin t\ e^t dt$$

Let

$$u = \sin t, v' = e^t$$

$$u' = \cos t, v = e^t$$

$$\int \sin t\ e^t dt = \sin t\ e^t - \int \cos t\ e^t dt$$

Let

$$m = \cos t, n' = e^t$$

$$m' = -\sin t, n = e^t$$

Hence,

$$\int \cos t\ e^t dt = \cos t\ e^t + \int \sin t\ e^t dt$$

$$\int \sin(\ln x)dx = \int \sin t\, e^t dt = \sin t\, e^t - \int \cos t\, e^t dt$$

$$= sint\, e^t - (\cos t\, e^t + \int \sin t\, e^t dt)$$

$$\int \sin t\, e^t dt = \frac{\sin t\, e^t - \cos t\, e^t}{2}$$

$$\int \sin(\ln x) = \frac{x\sin(\ln x) - x\cos(\ln x)}{2} + C$$

(ii)

Let $u = xe^x$,$v' = \cos x$, **so,** $u' = e^x(x+1), v = \sin x$

$$\int xe^x \cos x dx = xe^x \sin x - \int e^x(x+1)\sin x dx$$

$$= xe^x \sin x - \int e^x x \sin x dx - \int e^x \sin x dx$$

Let $u = e^x x$,$v' = \sin x$, **so,** $u' = e^x(x+1)$,$v = -\cos x$

$$\int e^x x \sin x dx = -e^x x \cos x + \int e^x(x+1)\cos x dx$$

$$\int xe^x \cos x dx = xe^x \sin x + e^x x \cos x - \int xe^x \cos x dx - \int e^x \cos x dx - \int e^x \sin x dx$$

$$2\int xe^x \cos x dx = xe^x \sin x + xe^x \cos x - \int e^x \cos x dx - \int e^x \sin x dx$$

Let $u = e^x$,$v' = \cos x$, **so,** $u' = e^x$,$v = \sin x$

$$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx$$

Hence,

$$\int xe^x \cos x dx = \frac{e^x \sin x(x-1) + xe^x \cos x}{2}$$

5.

(i)

We know that

$$(f^{-1})' = \frac{1}{f'(f^{-1}(x))}$$

Let $u = f^{-1}(x)$,$v' = 1$, **so,**$u' = \frac{1}{f'(f^{-1}(x))}$,$v = x$

$$\int f^{-1}(x)dx = xf^{-1}(x) - \int \frac{x}{f'(f^{-1}(x))}dx$$

Because$\frac{dy}{dx} = \frac{1}{f'(f^{-1}(x))}$**, and** $y = f^{-1}(x)$**, so** $x = f(x)$

Hence

$$\int f^{-1}(x)dx = xf^{-1}(x) - \int f(y)dy$$

(ii)

$$\int \cos^{-1}(x)dx = x\cos^{-1}x - \int \cos y dy$$

where $y = \cos^{-1}x$

$$\int \cos^{-1}(x)dx = x\cos x - \sin y = x\cos^{-1}x - \sin\cos^{-1}x$$

Because $\sin^2\theta + \cos^2\theta = 1$**, let** $\theta = \cos^{-1}x$

Hence $\sin\theta = \sqrt{1-x^2}$

$$\int \cos^{-1}(x)dx = x\cos x - \sqrt{1-x^2}$$

6.

(a)

Let $u = x^n$**,** $v' = \cos x$**, so,** $u' = nx^{n-1}$**,** $v = \sin x$

$$\int x^n \cos x dx = x^n \sin x - \int nx^{n-1}\sin x dx$$

Let $u = nx^{n-1}$**,** $v' = \sin x$**, so,** $u' = n(n-1)x^{n-2}$**,** $v = -\cos x$

$$\int nx^{n-1}\sin x dx = -nx^{n-1}\cos x + \int n(n-1)x^{n-2}\cos x dx$$

$$\int x^n \cos x dx = x^n \sin x + nx^{n-1}\cos x - \int n(n-1)x^{n-2}\cos x dx$$

(b)

Let $u = x$, $v' = \cos x$, **so,** $u' = 1$, $v = \sin x$

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x$$

(c)

$$\int x^5 \cos x dx = x^5 \sin x + 5x^4 \cos x - 20 \int \cos x dx$$

$$= x^5 \sin x + 5x^4 \cos x - 20(x^3 \sin x + 3x^2 \cos x - 6 \int x \cos x dx)$$

$$= x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x - 60x^2 \cos x + 120x \sin x + 120 \cos x$$

$$\int_0^\pi x^5 \cos x dx = x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x - 60x^2 \cos x + 120x \sin x + 120 \cos x \Big|_0^\pi$$

$$= -5\pi^4 + 60\pi^2 - 240$$