Assignment 6 of CISC 1006

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1

It easy to know that $X \sim Poission$ where $\lambda = 3$.

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

1.1

$$\begin{aligned} P(X = 5) = & \frac{3^5}{5!} e^{-3} \\ \approx & 0.1008 \end{aligned}$$

1.2

$$\begin{aligned} P(X < 3) = & \sum_{x=0}^{2} P(X = x) \\ = & \sum_{x=0}^{2} \frac{\lambda^x}{x!} e^{-\lambda} \\ \approx & 0.4232 \end{aligned}$$

1.3

$$\begin{aligned} P(X \geq 2) = & 1 - P(X \leq 1) \\ = & 1 - \sum_{x=0}^{1} \frac{3^x}{x!} e^{-3} \\ \approx & 0.8008 \end{aligned}$$

2

It easy to know that x $Poission$ **where** $\lambda = 5$**.**

$$P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

2.1

$$\begin{aligned} P(X > 5) =& 1 - P(X \leq 5) \\ =& 1 - \sum_{x=0}^{5} \frac{5^x}{x!} e^{-5} \\ \approx& 0.3840 \end{aligned}$$

2.2

$$\begin{aligned} P(\textbf{3 of next 4 days}) =& C_4^3 P(X > 5)^3 P(X \leq 5) \\ \approx& 0.0349 \end{aligned}$$

2.3

$$\begin{aligned} P(\textbf{The first time in April on April 5th}) =& P(X > 5) P(X \leq 5)^4 \\ \approx& 0.0553 \end{aligned}$$

3

3.1

Using Binomial distribution:

$$\begin{aligned} P(X < 5|\textbf{In 2000 people}) =& \sum_{x=0}^{4} C_{2000}^{x} 0.002^x (1 - 0.002)^{2000-x} \\ \approx& 0.6288 \end{aligned}$$

Using Poission Approximation:

$$\begin{aligned} \lambda =& np \\ =& 2000 \times 0.002 \\ =& 4 \\ P(X = x) =& \frac{4^x}{x!} e^{-4} \\ P(X < 5) =& \sum_{x=0}^{4} \frac{4^x}{x!} e^{-4} \\ \approx& 0.6288 \end{aligned}$$

3.2

Using Binomial distribution:

$$P(X = x) = C_{2000}^{x} 0.002^{x} (1 - 0.002)^{2000-x}$$

Using Poission Approximation:

$$P(X = x) = \frac{4^x}{x!} e^{-4}$$

3.2.1

Using Binomial distribution:

$$\begin{aligned}\mu &= \sum_{x=0}^{2000} x P(X = x) \\ &= \sum_{x=0}^{2000} x C_{2000}^{x} 0.002^{x} (1 - 0.002)^{2000-x} \\ &= 3.99\dot{9} \\ &\approx 4.0000\end{aligned}$$

Using Poission Approximation:

By definition:

$\mu = 4.000$

3.2.2

Chebyshev's inequality:

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

and

$$P(|X - \mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}.$$

$$\begin{aligned}X &\geq 1500 \\ |X - \mu| &> 1496 \\ \sigma^2 &= \lambda \\ &= 4 \\ \sigma &= 2 \\ 1496 &= k\sigma \\ k &= 748\end{aligned}$$

Thus

$$\begin{aligned}P(|X - \mu| \geq 748\sigma) &\leq \frac{1}{748^2} \\ &= \frac{1}{559504} \\ &\approx 1.787 \times 10^{-6}\end{aligned}$$

4

4.1

$$\begin{aligned}P(X=4;\lambda=6)=&\frac{6^4}{4!}e^{-6}\\\approx&0.1339\end{aligned}$$

4.2

$$\begin{aligned}P(X\geq 4;\lambda=6)=&1-P(X<3;\lambda=6)\\=&1-\sum_{x=0}^{3}\frac{6^x}{x!}e^{-6}\\\approx&1-0.1512\\=&0.8488\end{aligned}$$

4.3

$$\begin{aligned}P(X\geq 75;\lambda=6\times 12)=&1-P(X<75;\lambda=72)\\=&1-\sum_{x=0}^{75}\frac{72^x}{x!}e^{-72}\\\approx&1-0.6227\\=&0.3773\end{aligned}$$

5

5.1

Let X be the number of defective component.

$$\begin{aligned}P(X=15)=&C_{500}^{15}0.01^{15}\times(1-0.01)^{500-15}\\\approx&1.4\times 10^{-4}\\=&0.00014\end{aligned}$$

The probability is too small so that it is impossible that 15 components are defective. Thus the defective rate is not 1%

5.2

$$\begin{aligned}P(X=3)=&C_{500}^{3}0.01^{3}\times(1-0.01)^{500-3}\\\approx&0.1402\end{aligned}$$

5.3

If the defective rate is 1%. Let X be the number of defective component. Then

$$X \sim Poission$$

where $\lambda = np = 5$

$$\begin{aligned} P(X=15) =& \frac{5^{15}}{15!}e^{-5} \\ \approx& 0.00016 \end{aligned}$$

The probability is too small so that it is impossible that 15 **components are defective. Thus the defective rate is not** 1%

5.4

$$\begin{aligned} P(X=3) =& \frac{5^{3}}{3!}e^{-5} \\ \approx& 0.1404 \end{aligned}$$

6

Let X be the numbere of yares that the electrial switch work.

$$P(X=x) = \frac{1}{2}e^{-\frac{x}{2}}, (X \geq 0)$$

6.1

$$\begin{aligned} P(X \leq 1) =& \int_{-\infty}^{1} P(X=x)dx \\ =& 0 + \int_{0}^{1} \frac{1}{2}e^{-\frac{x}{2}}dx \\ =& 1 - e^{-\frac{1}{2}} \\ \approx& 0.3935 \end{aligned}$$

6.2

$$\begin{aligned} P =& \sum_{x=0}^{30} C_{100}^{x} P(X \leq 1)^{x}(1 - P(X \leq 1))^{100-x} \\ \approx& 0.0335 \end{aligned}$$

7

7.1

The response time X is exponential distributions, where it mean is $\theta = 3$.

$$P(X = x) = \frac{1}{3}e^{-\frac{x}{3}}, (X > 0)$$

and $P(X = x) = 0$ elsewhere

$$\begin{aligned} P(X > 5) = &\int_5^\infty \frac{1}{3}e^{-\frac{x}{3}}dx \\ =&e^{-\frac{5}{3}} \\ \approx&0.1889 \end{aligned}$$

7.2

$$\begin{aligned} P(X > 10) = &\int_{10}^\infty \frac{1}{3}e^{-\frac{x}{3}}dx \\ =&e^{-\frac{10}{3}} \\ \approx&0.0357 \end{aligned}$$

8

8.1

Exponrntial Distributions

$$\mu = 2$$

$$var = 4$$

8.2

$$\begin{aligned} P(X \geq 20) = &\int_{20}^\infty P(X = x)dx \\ = &\int_{20}^\infty \frac{1}{2}e^{-\frac{x}{2}}dx \\ =&e^{-10} \\ \approx&4.540 \times 10^{-5} \end{aligned}$$

8.3

$$
\begin{aligned}
P(0 < X < 10) = & \int_0^{10} P(X = x)dx \\
= & \int_0^{10} \frac{1}{2} e^{-\frac{x}{2}} dx \\
= & 1 - e^{-5} \\
\approx & 0.9933
\end{aligned}
$$

8.4

$$
\begin{aligned}
P(X \geq 2) = & \int_2^{\infty} P(X = x)dx \\
= & \int_2^{\infty} \frac{1}{2} e^{-\frac{x}{2}} dx \\
= & e^{-1} \\
\approx & 0.3679
\end{aligned}
$$

8.5

Let the number of messages per hour be X, then we can know that $X \sim Poission$ where

$$\lambda = 60 \times \frac{1}{\beta} = 30$$

Thus,its variance

$$var = 30$$