Assignment 1 of CISC 1006

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1	**H** **H** **T** **T**	**H** **T** **H** **T**
2	**H** **T**	
3	**H** **H** **T** **T**	**H** **T** **H** **T**
4	**H** **T**	
5	**H** **H** **T** **T**	**H** **T** **H** **T**
6	**H** **T**	

1.1

$$A = \{1HH, 1HT, 1TH, 1TT, 2H, 2T\}$$

1.2

$$\begin{aligned} B = \{&1HH, 1HT, 1TH, 1TT, \\ &3HH, 3HT, 3TH, 3TT, \\ &5HH, 5HT, 5TH, 5TT\} \end{aligned}$$

1.3

$$\begin{aligned} A' = \{&4H, 4T, 6T, 6H, \\ &3HH, 3HT, 3TH, 3TT, \\ &5HH, 5HT, 5TH, 5TT\} \end{aligned}$$

1.4

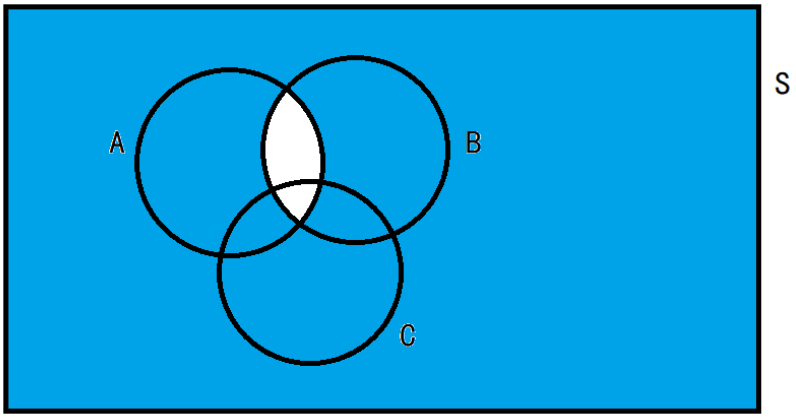
$$\begin{aligned} A' \cap B = \{&3HH, 3HT, 3TH, 3TT, \\ &5HH, 5HT, 5TH, 5TT\} \end{aligned}$$

1.5

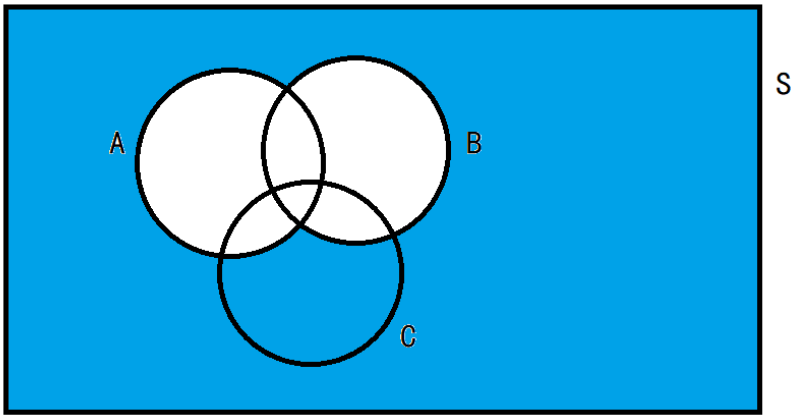
$$\begin{aligned} A' \cup B = \{&1HH, 1HT, 1TH, 1TT, \\ &3HH, 3HT, 3TH, 3TT, \\ &5HH, 5HT, 5TH, 5TT \\ &4H, 4T \\ &6H, 6T\} \end{aligned}$$

2

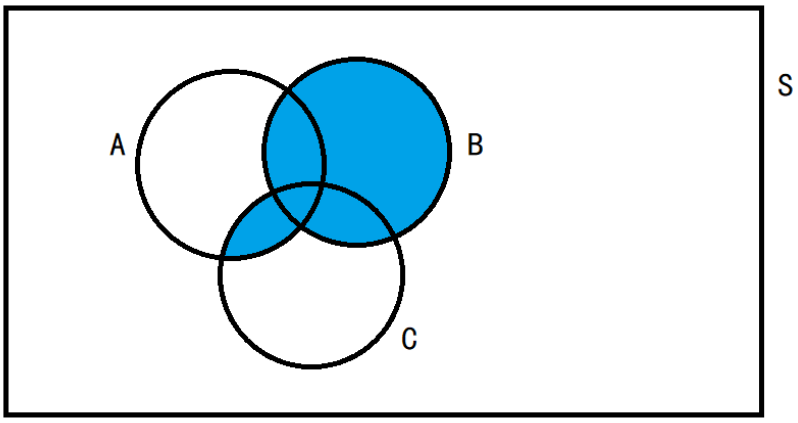
2.1



2.2



2.3



3

3.1

$$P_6^6 = 720$$

3.2

$$P_4^4 P_3^3 = 144$$

3.3

$$P_4^4 P_5^2 = 480$$

3.4

- $[1, 4, 4]$, $C_9^1 C_8^4 C_4^4 = 630$
- $[1, 3, 5]$, $C_9^1 C_8^3 C_5^5 = 504$
- $[2, 2, 5]$, $C_9^2 C_7^2 C_5^5 = 756$

- $[2, 3, 4]$, $C_9^2 C_7^3 C_4^4 = 1260$
- $[2, 4, 3]$, $C_9^2 C_7^4 C_3^3 = 1260$

$$Total = 4410$$

4

4.1

$$C_5^4 = 5$$

4.2

$$C_5^1 C_7^3 = 175$$

4.3

$$C_5^1 C_7^3 + C_5^2 C_7^2 + C_5^3 C_7^1 + C_5^4 C_7^0 = 460$$

4.4

$$C_5^0 C_7^4 + C_5^1 C_7^3 + C_5^2 C_7^2 = 420$$

5

5.1

$$0.19 + 0.38 + 0.29 + 0.15 = 1.01 \neq 1$$

We know that

$$\mathbb{P}(S) = 1$$

where S is a sample space of any random experiment

5.2

$$0.4 + 0.52 = 0.92 \neq 1$$

We know that

$$\mathbb{P}(S) = 1$$

where S is a sample space of any random experiment

5.3

The probability of a random event can equal to -0.25. We know that for any event A, the probability of A should be nonnegative real number, i.e.

$$\mathbb{P}(A) \geq 0$$

5.4

Heart is red. The probability of a card is balck heart is 0

6

Let A be the set of students who somke, B be the set of students who drink alcholic beverages, and C be the set of students who eat between meals. We know that the number of elements in ecah set is that

$$|T| = 500$$
$$|A| = 210$$
$$|B| = 258$$
$$|C| = 216$$

We also know that

$$|A \cap B| = 122$$
$$|B \cap C| = 83$$
$$|A \cap C| = 97$$
$$|A \cap B \cap C| = 52$$

$$
\begin{aligned}
\emptyset =& (A \cap B' \cap C') \cup [(A \cap B) \cup (A \cap C) \cup (A \cap B \cap C) \\
A =& (A \cap B' \cap C') \cap [(A \cap B) \cup (A \cap C) \cup (A \cap B \cap C) \\
|A \cap B' \cap C'| =& |A| - |(A \cap B) \cup (A \cap C) \cup (A \cap B \cap C)| \\
=& |A| - |A \cup B| - |A \cup C| + |A \cap B \cap C| \\
=& 43
\end{aligned}
$$

$$
\begin{aligned}
|B \cap A' \cap C'| =& 105 \\
|C \cap A' \cap B'| =& 88, \\
(A \cap B \cap C) \cup (A \cap B \cap C') =& A \cap B \\
(A \cap B \cap C) \cap (A \cap B \cap C') =& \emptyset \\
|A \cap B \cap C| + |A \cap B \cap C'| =& |A \cap B| \\
|A \cap B \cap C'| =& 70 \\
|A \cap C \cap B'| =& 45 \\
|B \cap C \cap A'| =& 31,
\end{aligned}
$$

and

$$
\begin{aligned}
(A \cup B \cup C) \cup (A' \cap B' \cap C') =& T \\
(A \cup B \cup C) \cap (A' \cap B' \cap C') =& \emptyset \\
|A \cup B \cup C| =& |A| + |B \cup A'| + |C \cup A' \cup B'| \\
=& 433 \\
|A' \cap B' \cap C'| =& |T| - |A \cup B \cup C| \\
=& 67
\end{aligned}
$$

6.1

The probability of a student somkes but does not drink alcholic beverages is

$$
\begin{aligned}
\mathbb{P}_1 =& \frac{|A \cap B'|}{|T|} \\
=& \frac{|A \cap B'| + |C \cap B'| - |A \cap C \cap B'|}{|T|} \\
=& \frac{|A \cap B'| + |C \cap B'| - (|A \cap C| - |A \cap B \cap C|))}{|T|} \\
=& \frac{(|A| - |A \cap B|) + (|C| - |B \cap C|)) - (|A \cap C| - |A \cap B \cap C|))}{|T|} \\
=& \frac{88}{500} \\
=& 0.176
\end{aligned}
$$

6.2

The probability of a student eats between meals and drinks alcoholic beverages but does not smoke is

$$\begin{aligned}\mathbb{P}_1 =& \frac{|B \cap C \cap A'|}{|T|} \\ =& \frac{224}{500} \\ =& 0.448\end{aligned}$$

6.3

The probability of a student neither smokes nor eats between meals is

$$\begin{aligned}\mathbb{P}_1 =& \frac{|A' \cap B'|}{|T|} \\ =& \frac{155}{500} \\ =& 0.31\end{aligned}$$