Chapter 1

Hua Kang

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1 Set

1.1 Definition Part

1.1.1 Proper Subset

We say that a set A is a proper subset of a set B if $A \subseteq B$, but there is at least one element of B that is not in A. In this case we sometimes write

$$A \subset B.$$

In short, If $A \subseteq B$and $\exists b \in B, b \notin A$, then $A \subset B$.

1.1.2 Two set is equal

If $A \in B$ and $B \in A$, then two set are said to be equal, and we write $A = B$.

1.1.3 Set Operations

The union of sets A and B is the set

$$A \cup B = \{x : x \in A\mathbf{or}x \in B\}.$$

The intersection of the sets A and B is the set

$$A \cap B = \{x : x \in A\mathbf{and}x \in B\}.$$

The complement of B relative to A is the set

$$A \backslash B = \{x : x \in A\mathbf{and}x \notin B\}.$$

1.1.4 Empty set and disjoint

The set that has no elements is called the empty set and is denoted by the symbol $\emptyset$. Two set A and B are sasid to be disjoint if they have no elements in common, this can be expressed by writing $A \cap B = \emptyset$

1.1.5 Infinite union or intersection

$$\cup_{n=1}^{\infty} A_n = \{x : x \in A_n, \exists n \in \mathbb{N}\}$$

$$\cap_{n=1}^{\infty} A_n = \{x : x \in A_n, \forall n \in \mathbb{N}\}$$

1.2 Theorem Part

1.2.1 De Morgan Law

If A, B, C are sets, then

$$A \backslash (B \cup C) = (A \backslash B) \cap (A \backslash C)$$

$$A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash C)$$

1.3 Other

2 Function

2.1 Definition Part

2.1.1 Cartesian product

If A and B are noempty sets, then the Cartesian product $A \times B$ of A and B is the set of all ordered pairs (a, b) with $a \in A$ and $b \in B$. That is

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

2.1.2 Function

Let A and B be setd. Then a function from A to B is a set f of ordered pairs in $A \times B$ such that for each $a \in A$there exists a unique $b \in B$ with $(a, b) \in f$.
In other word, if $(a, b) \in f, (a, b') \in f$, then $b = b'$.

2.1.3 Domain and Range

The set A of first elements of a function f is called the domain of f and is often denoted by $D(f)$
The set of all second elements in f is called the range of f and is often denoted by $R(f)$
Note that, although $D(f) = A$, we only have $R(f) \subseteq B$ *B is codomain*

2.1.4 Direct and Inverse Images

Let $f : A \to B$ be a function with domain $D(f) = A$ and range $R(f) \subseteq B$ If E is a subset of A, then the direct image of E unser f is the subset $f(E)$ of B given by

$$f(E) = \{f(x) : x \in E\}$$

If H is s subset of B , then the inverse image of H under f is the subset $f^{-1}(H)$ of A given by

$$f^{-1}(H) = \{x \in A : f(x) \in H\}$$

2.1.5 Injective, surjective and bijective

The function f is said to be injective (or to be one-one) if whenever $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.If f is an injective function, we also say that f is an injective function, we also say that f is an ingetion.

The function f is said to be surjective (or to map A onto B) if $f(A) = B$;that is, if the range $R(f) = B$. If f is a surjective function, we also say that f is a surjection.

If f is both inkective and surjective, the f is said to be bijective. If f is bijective, we also say that f is a bijetion

2.2 Inversen Function

If $f : A \to B$ is a bijection of A onto B, then

$$g := \{(b, a) \in B \times A : (a, b) \in f\}$$

is a function on B into A. This function is called the inverse function of f, and is denoted by f^{-1}

2.3 Composition of Functions

If $f : A \to B$ and $g : B \to C$, and if $R(f) \subseteq D(g) = B$, then the composite function $g \circ f$

2.4 Theorem Part

2.5 Other

A function f from a set A into a set B is a rule of correspondence that assigns to each element x in A a uniquely determined element $f(x)$ in B.
The essential condition that :

$$(a, b) \in f \textbf{ and } (a, b') \in f \textbf{ implies that } b = b'$$

is sometimes called the *vertical line test.*
The notation

$$f : A \to B$$

is often used to indicate that f **is a function from** A **to** B**. We will also say that** f **is a mapping of** A **into** B**, or that** f **maps** A **into** B**.**
If (a, b) **is an element in**f**, it is customary to write**

$$b = f(a)\textbf{, or sometimes } a \to b.$$