13 MULTIPLE INTEGRALS

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1 Double Integrals

The simplest sort of mutiple integrals is the *double integrals*

$$\iint_R f(x,y)dA$$

of a continuous function $f(x,y)$ over the *rectangle*

$$R = [a,b] \times [c,d] = \{(x,y), a \leq x \leq b, c \leq y \leq d\}$$

in the xy-plane

To define the *value*

$$V = \iint_R f(x,y)dA$$

of such a double integral, we begin with an approximation to V. To obtain this approximation, the first step is to construct a **partition** $\mathcal{P}$of R into subrectangles $R_1, R_2, R_3, ..., R_k$determined by points

$$a = x_0 < x_1 < x_2 < ... < x_m = b$$

of $[a,b]$and,

$$c = y_0 < y_1 < y_2 < ... < y_n = d$$

of $[c,d]$.

Next we choose an arbitrary point$(x_i^*, y_i^*)$of the ith rectangle R_i for each i(where $1 \leq i \leq k$). The collection of points $S = \{(x_i^*, y_i^*)|1 \leq i \leq k\}$ is called a **selection** for the partition $\mathcal{P} = \{R_i|1 \leq i \leq k\}$

As a measure of the size of the rectangles of the partition $\mathcal{P}$, we define its **norm** $|\mathcal{P}|$ to be the maximum of the lengths of the diagonals of the rectangles $\{R_i\}$.

Now consider a rectangular column that rises straight up from the xy-plane. Its base is the rectangle R_i and its height is the value $f(x_i^*, y_i^*)$ of f at the selected point (x_i^*, y_i^*) of R_i If ΔA denotes the area of R_i,then the volume of the ith column is $f(x_i^*, y_i^*)\Delta A_i$

The sum of the volumes of all such columns is the **Ruemann sum**

$$\sum_{i=1}^{k} f(x_i^*, y_i^*)\Delta A_i$$

an approximationto the volume V of the solid region that lies above the rectangle R and under the graph $z = f(x, y)$

1.1 Definition of Double Integral

$$\iint_R f(x,y)dA = \lim_{|\mathcal{P}|\to 0} \sum_{i=1}^{k} f(x_i^*, y_i^*)\Delta A_i$$

1.2 Iterated Integrals

1.2.1 THEOREM 1

Support that $f(x, y)$ is a continuous on the rectangle $R = [a, b] \times [c, d]$. Then

$$\iint_R f(x,y)dA = \int_a^b \left(\int_c^d f(x,y)dy \right) dx$$