Assignment 5 of CISC 1006

ZHANG HUAKANG
DB92760

Computer Science,
Faculty of Science and Technology

March 26, 2021

1

1.1

We can konw the probability that x trucks are fail

$$P(X = x) = f(x) = C_1 5^x 0.25^x 0.75^{15-x}$$

1.1.1

$$\begin{aligned} P(3 \leq X \leq 6) =& \sum_{x=3}^{6} C_1 5^x 0.25^x 0.75^{15-x} \\ \approx& 0.225199065 + 0.225199065 + 0.165145981 + 0.091747767 \\ \approx& 0.7073 \end{aligned}$$

1.1.2

$$\begin{aligned} P(X < 4) =& \sum_{x=0}^{3} C_1 5^x 0.25^x 0.75^{15-x} \\ \approx& 0.4613 \end{aligned}$$

1.1.3

$$\begin{aligned} P(X > 5) =& 1 - P(X \leq 5) \\ \approx& 1 - 0.851631923 \\ \approx& 0.1484 \end{aligned}$$

1.2

1.2.1

$$\begin{aligned}\mathbb{E}[X] =& \sum_{x=0}^{1} 5xf(x)\\ =& 0 + 0.066817305 + 0.31181409 + 0.675597196+\\ & 0.900796261 + 0.825729906 + 0.550486604+\\ & 0.275243302 + 0.104854591 + 0.030582589+\\ & 0.006796131 + 0.001132688 + 0.000137296+\\ & 1.14413E-05 + 5.86733E-07 + 1.39698E-08\\ =& 3.75\end{aligned}$$

1.2.2

$$\begin{aligned}\mathbb{E}[X^2] =& \sum_{x=0}^{1} 5x^2f(x)\\ =& 0 + 0.066817305 + 0.62362818+\\ & 2.026791587 + 3.603185043 + 4.128649528+\\ & 3.302919623 + 1.926703113+\\ & 0.83883673 + 0.275243302 + 0.067961309+\\ & 0.012459573 + 0.001647547 + 0.000148737+\\ & 8.21427E-06 + 2.09548E-07\\ =& 16.875\\ var(X) =& \mathbb{E}[X^2] - \mathbb{E}[X]^2\\ =& 2.8125\end{aligned}$$

2

2.1

2.1.1

$$\begin{aligned}P_a =& \frac{C_{17}^3 C_3^0}{C_{20}^3}\\ =& \frac{34}{57}\\ \approx& 0.5965\end{aligned}$$

2.1.2

$$\begin{aligned}P_b =& \frac{C_{19}^2 C_1^1}{C_{20}^3}\\ =& \frac{17}{20}\\ =& 0.8500\end{aligned}$$

2.2

2.2.1

$$\begin{aligned} P_a =& C_3^3 \frac{17}{20}^3 \\ =& \frac{4913}{8000} \\ \approx& 0.6141 \end{aligned}$$

2.2.2

$$\begin{aligned} P_b =& C_3^1 \frac{1}{20} \\ =& \frac{3}{20} \\ =& 0.1500 \end{aligned}$$

3

Hypergeometric

$$P_H(X) = \frac{C_{4000}^x C_{6000}^{15-x}}{C_{10000}^1 5}$$

Binomial Approximation

We can use Binomial to approximate Hypergeometric, where $\theta = \frac{4000}{10000} = 0.4$

$$\begin{aligned} P(X \leq 7) =& \sum_{x=0}^{7} C_{15}^x \theta^x (1-\theta)^{15-x} \\ \approx& 0.7869 \end{aligned}$$

4

4.1

4.1.1

$$\begin{aligned} P_i =& C_3^0 0.8^0 \times 0.2^3 \\ =& \frac{1}{125} \\ =& 0.0080 \end{aligned}$$

4.1.2

$$\begin{aligned} P_{ii} =& C_3^1 0.8^1 \times 0.2^2 \\ =& \frac{4}{125} \\ =& 0.0960 \end{aligned}$$

4.1.3

$$\begin{aligned} P_{iii} =& C_3^2 0.8^2 \times 0.2^1 + C_3^3 0.8^3 \times 0.2^0 \\ =& \frac{112}{125} \\ =& 0.896 \end{aligned}$$

4.2

4.2.1

$$\begin{aligned} P_{Undetected} =& C_n^0 \times 0.2^n \\ =& 0.2^n \\ =& 0.0001 \\ 0.2^n =& 0.0001 \\ n =& \log_{0.2} 0.0001 \\ n \approx& 5.7 \\ n =& 6 \end{aligned}$$

4.2.2

$$\begin{aligned} P_{Undetected} =& C_3^0 \times p^n \\ =& p^3 \\ =& 0.0001 \\ p^3 =& 0.0001 \\ p \approx& 0.0464 \\ 1 - p \approx& 0.9536 \end{aligned}$$

5

5.1

$$\begin{aligned} P_a =& C_{15}^5 0.05^5 \times 0.95^{10} \\ \approx& 0.0006 \end{aligned}$$

5.2

My recation: WTF??? I'm so unfortunate.