# Assignment 1

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## 1.

**Let** $u = e^x$**,** $du = e^x dx$

$$\int e^x \sin^4 e^x \cos^2 e^x dx$$

$$= \int \sin^4 u \cos^2 u du$$

$$= \int \sin^4 u(1 - \sin^2 u) du$$

$$= \int \sin^4 u du - \int \sin^6 u du$$

**For** $\int sin^n x dx$**, where** $n = 2k$**,** $k \in \mathbb{Z}$**:**

**We use the substitution:**

$$m = sin^{n-1} x, n' = \sin x$$

$$m' = (n-1) \sin^{n-2} x \cos x, n = -\cos x$$

$$\int \sin^n x dx = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \cos^2 x dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} (1 - \sin^2 x) dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x dx - (n-1) \int \sin^n x dx$$

**We can get:**

$$\int \sin^n x dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx \tag{1}$$

**Hence**

$$\int \sin^4 udu - \int \sin^6 udu$$

$$= \int \sin^4 udx - \frac{5}{6}\int \sin^4 udu + \frac{cosu \sin^5 u}{6}$$

$$= \frac{1}{6}\int \sin^4 udu + \frac{\cos u \sin^5 u}{6}$$

$$= \frac{1}{6}(\frac{\cos u \sin^3 u}{4} + \frac{3}{4}\int \sin^2 dx) + \frac{\cos u \sin^5 u}{6}$$

$$= \frac{\cos u \sin^3 u}{24} + \frac{1}{8}\int \sin^2 xdx + \frac{\cos u \sin^5 u}{6}$$

$$= \frac{\cos u \sin^3 u}{24} + \frac{1}{8}\int \frac{1-\cos 2x}{2}dx + \frac{\cos u \sin^5 u}{6}$$

$$= \frac{3}{8}u - \frac{1}{24}\sin^3 u \cos u - \frac{3}{16}\sin 2u + \frac{1}{6}\sin^5 u \cos u - \frac{5}{32}(2u - \sin 2u)$$

$$= \frac{1}{16}u - \frac{1}{24}\sin^3 u \cos u + \frac{1}{6}\sin^5 u \cos u - \frac{1}{32}\sin 2u$$

$$= \frac{1}{16}e^x - \frac{1}{24}\sin^3 e^x \cos e^x + \frac{1}{6}\sin^5 e^x \cos e^x - \frac{1}{32}\sin 2e^x + C$$

## 2.

### (i)

**We use the substitution:**

$$u = \tan x, du = \sec^2 xdx$$

$$\int \tan^3 xsec^2 xdx = \int u^3 du = \frac{1}{4}u^4 = \frac{1}{4}\tan^4 x + C$$

### (ii)

**We use the substitution:**

$$u = \tan x, du = \sec^2 xdx$$

$$\int \tan^4 x \sec^2 dx = \int u^4 du = \frac{1}{5}u^5 = \frac{1}{5}\tan^5 x + C$$

## 3.

### (i)

$$\int \frac{\sin^5 x}{\cos^2 x} dx = \int \frac{\sin^4 x}{\cos^2 x} \sin x dx$$

$$= \int \frac{(1-\cos^2 x)^2}{\cos^2 x} d(-\cos x) = -\frac{1}{3}\cos^3 x + 2\cos x + \sec x + C$$

### (ii)

**Let** $u = \sin^4 x$**,** $v' = \frac{1}{\cos^2 x} = \sec^2 x$

**Hence,** $u' = 4\sin^3 x \cos x$**,** $v = \tan x$

$$\int \frac{\sin^4 x}{\cos^2 x} dx = \sin^4 x \tan x - 4\int \sin^3 x \cos x \tan x dx$$

$$= \sin^4 x \tan x - 4\int \sin^4 x dx$$

**From (1):**

$$\int \sin^n x dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n}\int \sin^{n-2} x dx$$

$$\int \sin^4 x dx = -\frac{1}{4}\sin^3 x \cos x + \frac{3}{8}(x - \frac{1}{2}\sin 2x)$$

**Hence**

$$\int \frac{\sin^4 x}{\cos^2 x} dx = \sin^4 x \tan x + \sin^3 \cos x - \frac{3}{2}x + \frac{3}{4}\sin 2x + C$$

## 4.

$$\int \frac{x^3 + 4x^2 + 10x + 8}{x^2 + 4x + 3} dx$$

$$= \int x + 1 + \frac{3x+5}{(x+3)(x+1)} dx$$

$$= \frac{x^2}{2} + x + \int \frac{2}{x+3} + \frac{1}{x+1} dx$$

$$= \frac{x^2}{2} + x + 2\ln(x+3) + \ln(x+1) + C$$

## 5.

$$\int \frac{2x+4}{(x+1)^2(x^2+1)}dx = \int \frac{2}{(x+1)(x^2+1)}$$

$$= \int \frac{1}{x+1}dx + \int \frac{1-x}{x^2+1}dx$$

$$= \ln|x+1| + \int \frac{1}{x^2+1}dx - \int \frac{x}{x^2+1}dx$$

$$= \ln|x+1| + \arctan x - \frac{\ln|x^2+1|}{2} + C$$

## 6.

**We use the substitution** $u = e^t + 1$, $du = e^t dt$

$$\int \frac{e^{2t}}{(e^t+1)^3}dt = \int \frac{u-1}{u^3}du$$

$$= \int \frac{1}{u^2}du - \int \frac{1}{u^3}du = -\frac{1}{u} + \frac{1}{2u^2} = -\frac{1}{e^t+1} + \frac{1}{2(e^x+1)^2} + C$$