Assignment 6 of MATH 2005

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$$\begin{aligned}\mathbb{P}[X < -\theta\log(1-p)] =& \int_{-\infty}^{-\theta\log(1-p)} f(y)dy \\ =& \int_{0}^{-\theta\log(1-p)} \frac{1}{\theta}e^{-\frac{y}{\theta}}dy \\ =& -e^{-\frac{y}{\theta}}|_0^{-\theta\log(1-p)} \\ =& 1-(1-p) \\ =& p\end{aligned}$$

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$$\begin{aligned}\mathbb{E}[X] =& \int_{-\infty}^{\infty} xf(x)dx \\ =& \int_{0}^{\infty} x2\alpha xe^{-\alpha x^2}dx \\ =& 2\alpha\int_{0}^{\infty} x^2e^{-\alpha x^2}dx\end{aligned}$$

Let $u = x$ and $v' = xe^{-\alpha x^2}$, thus $u' = 1$ and $v = -\frac{1}{2\alpha}e^{-\alpha x^2}$

$$\begin{aligned}\int x^2e^{-\alpha x^2}dx =& -\frac{x}{2\alpha}e^{-\alpha x^2} + \int\frac{1}{2\alpha}e^{-\alpha x^2}dx \\ =& -\frac{x}{2\alpha}e^{-\alpha x^2} + \frac{\sqrt{\pi}erf(\sqrt{\alpha}x)}{2\sqrt{\alpha}}\end{aligned}$$

Thus

$$\begin{aligned}\mathbb{E}[X] =& [-\frac{x}{2\alpha}e^{-\alpha x^2} + \frac{\sqrt{\pi}erf(\sqrt{\alpha}x)}{2\sqrt{\alpha}}]|_0^{\infty} \\ =& \frac{\sqrt{\pi}}{2\sqrt{\alpha}}\end{aligned}$$

And

$$\begin{aligned}\mathbb{E}[X^2] &= \int_{-\infty}^{\infty} x^2 f(x) dx \\ &= \int_{0}^{\infty} x^2 2\alpha x e^{-\alpha x^2} dx \\ &= 2\alpha \int_{0}^{\infty} x^3 e^{-\alpha x^2} dx\end{aligned}$$

Let $u = x^2$ and $v' = xe^{\alpha x^2}$, thus $u' = 2x$ and $v = -\frac{1}{2\alpha} e^{-\alpha x^2}$

$$\int x^3 e^{-\alpha x^2} dx = -\frac{x^2}{2\alpha} e^{-\alpha x^2} + \int \frac{x}{\alpha} e^{-\alpha x^2}$$