Responses to the comments from Referee 2

Thank you very much for your thorough reading of the manuscript and for providing valuable and constructive comments. We have addressed all the comments raised. Below are our point-by-point responses.

\*\*Comparison with the U.S. Case\*\*:

The comparison with the U.S. case is intriguing. This paper decomposes GDP per working-age population for Japan; however, it is unclear what variable is used for the U.S. case. If GDP per working-age population is also used for the U.S., then further analysis may not be required. Please confirm that the U.S. comparison uses GDP per working-age population while FHSW uses GDP.

Nevertheless, the U.S. figures presented in the paper appear identical to those in FHSW, raising concerns that the paper might be using GDP instead of GDP per working-age population. If so, the authors should use the same variable when comparing the two countries, as using different variables may affect the share of common factors, particularly through labor force changes. Either GDP or GDP per working-age population is acceptable (with a preference for GDP, as it accounts for demographic effects), but consistency is crucial.

Thank you for this important point. We completely agree on the importance of using consistent data across countries for valid comparisons. In response, we have recalculated the U.S. data using GDP per working-age population to ensure comparability with the Japanese data. As a result, the contribution of the common factor in the U.S. decreased slightly (from approximately 25% to around 20%). There were no other significant changes, and the conclusions of the paper remain the same. The recalculated results have been reflected in Section 4.4.

- \*\*Clarification in Section 4.3.3\*\*:

In Section 4.3.3, the authors should better clarify how the estimation relates to the model. It is recommended to adhere to FHSW's notations unless there are specific reasons for deviation. Specifically:

1. An equation decomposing the growth rate into direct and indirect effects, similar to the equation on p.3320 of FHSW, should be added.

2. The definitions for variables such as \( g\_{z,u,i,t} \), \( g\_{l,u,i,t} \), etc., as shown on p.3299 of FHSW, should be provided.

3. A more detailed explanation is needed for how the direct and indirect sequences are generated from the model.

Following your specific suggestions, we have added a new second paragraph in Section 4.3.3. In this paragraph, we introduce the variables (corresponding to variables such as \( g\_{z,u,i,t} \), \( g\_{l,u,i,t} \), etc., in FHSW, p.3299) mentioned in Section 3.2.2. We also include the equation from p.3320 of FHSW and provide a clearer explanation of how we calculate the direct and indirect effects.

\*\*Empirical Analysis Issues\*\*:

Some inconsistencies or unrealistic results were noted in the empirical analysis. For example:

- In Figure 4, the TFP growth rates for the Petroleum and Real Estate sectors recorded values of -100% and -50%, respectively, at certain points, which seem erroneous.

We have checked the data and code, and confirmed that there were no data handling errors. Additionally, we conducted an additional robustness check to ensure that these noises in data do not affect our main results. Specifically, to examine the influence of short-term noisy fluctuations, we conducted an exercise by adjusting the parameters used in trend extraction. In the baseline analysis, we extracted trends with cycles of 15.5 years or longer (q=8) and used them for analysis. In this additional exercise, we extracted trends with cycles of 20.7 years or longer (q=6) and analyzed them. We confirmed that the main conclusions are maintained. Specifically, the common factor is the major driver of trend growth rate of GDP (R^2=0.75). These findings are mentioned in Footnote 16. Although not included in this footnote, we also confirmed that our main conclusion is robust when using a shorter trend (e.g., a 12.4-year period with a frequency band of q=10).

- In Table 3, the \( R^2 \) value for "Other Manufacturing" is 0.99, which appears unusually high. The authors should verify all the results in this table.

We have rechecked all results and confirmed that they are correct. The \( R^2 \) here reflects the relative explanatory power of common and industry-specific factors, which differs from the classical regression \( R^2 \). In industries where the contribution of industry-specific factors is minimal, it is plausible for the \( R^2 \) to approach 1. To clarify this, we have renamed the column heading in Table 3 to "Relative explanatory power of common factor" and added a note to the table explaining this point.

- In Figure 7, the sum of the white and blue bars does not equal the red lines. The code generating these figures should be reviewed.

This was indeed an error on our part. We sincerely apologize for the confusion, and we have corrected the figure accordingly.

### 3. Minor Issues

- \*\*Terminology Consistency\*\*:

The terms "per working-age population," "per working population," and "per workforce population" are used interchangeably. The authors should standardize this terminology throughout the paper to avoid confusion.

We have revised the manuscript to use consistent terminology throughout. Thank you for your careful reading.

- \*\*Consistency with FHSW's Argument\*\*:

The paper mentions that the share of common factors is "about 30%" or "falls below 30%" for the U.S., whereas FHSW states it is around one-fourth. The authors should maintain consistency with FHSW's findings when discussing the same result.

Thank you for pointing this out. In this revision, we have recalculated the data for the U.S., using consistent definitions across countries. As a result, we found that while FHSW reports a share of around one-fourth, our recalculated results show a share of around 20% after adjusting the data. This has been explained in the second paragraph of Section 4.4 and in Footnote 26.

- \*\*Clarification of Footnote 6\*\*:

The term "per capita productivity" requires clarification.

This was an error. We apologize for the confusion, and we have removed this statement from Footnote 6.

- \*\*Table 5\*\*:

Table 5 does not add information beyond what is provided in Figure 10. Consider omitting Table 5 and instead include figures akin to Panels A and B in Figure 10 of FHSW (p.3321), as they would enhance the reader's understanding of the significance of common factors.

We have removed Table 5 and replaced it with figures similar to Panels A and B in Figure 10 of FHSW.

In addition to the points explained here, we have also corrected typos that were discovered.

Once again, we sincerely appreciate the referee's thorough review and constructive feedback. We have implemented all the suggested changes and recalculated the U.S. data for comparison. As a result, we believe that the quality of the paper has significantly improved. I hope this meets your expectations.