1. a C
b A
c B
d C
e D
f A
g B
h D
i A
j A

2. (a) $f(x,y,z) = \sin(\pi xyz)$

pick $x = t,\ y = 2t,\ z = 1$, we have.

$f(t, 2t, 1) = \sin(2t^2\pi)$

so $f'(t, 2t, 1)\Big|_{t=1} = 4t\pi\cos(2t^2\pi)\Big|_{t=1} = 4\pi$

(b) $\nabla f = \pi yz\cos(\pi xyz)\vec{i} + \pi xz\cos(\pi xyz)\vec{j} + \pi xy\cos(\pi xyz)\vec{k}$

gives $\nabla f(1,2,1) = \pi(2\vec{i} + \vec{j} + 2\vec{k})$

so $\vec{u} = \dfrac{\nabla f(1,2,1)}{|\nabla f(1,2,1)|} = \frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}$ is the direction where f increase most rapidly.

(c) $(x,y,z) = (1,2,1)$ gives $t = 1$.

so $\vec{r}(1) = \vec{i} + 2\vec{j} + \vec{k}$, $\vec{v} = \dfrac{\vec{r}(1)}{|\vec{r}(1)|} = \dfrac{\vec{i}}{\sqrt{6}} + \dfrac{2\vec{j}}{\sqrt{6}} + \dfrac{\vec{k}}{\sqrt{6}}$

and $D_{\vec{v}} f(1,2,1) = \nabla f(1,2,1)\cdot\vec{v} = \dfrac{2}{\sqrt{6}} + \dfrac{2}{\sqrt{6}} + \dfrac{2}{\sqrt{6}} = \sqrt{6}$

3. (a) $f_x(x,y) = e^x(x^2-2x-y^2) + e^x(2x-2) = e^x(x^2-y^2-2)$

$f_y(x,y) = -2y\,e^x$

let $f_x(x,y) = f_y(x,y) = 0$

as $e^x > 0$, so we should have $\begin{cases} x^2-y^2-2=0 \\ -2y=0 \end{cases} \Rightarrow \begin{cases} x=\pm\sqrt{2} \\ y=0 \end{cases}$

So the critical points of f are $(-\sqrt{2},0)$ and $(\sqrt{2},0)$

(b) $f_{xx}(x,y) = e^x(x^2-y^2-2) + e^x \cdot 2x = e^x(x^2+2x-y^2-2)$

$f_{xy}(x,y) = -2ye^x$

$f_{yy}(x,y) = -2e^x$

At point $(-\sqrt{2},0)$: $f_{xx}(-\sqrt{2},0) = -2\sqrt{2}e^{-\sqrt{2}} < 0$

$f_{xy}(-\sqrt{2},0) = 0$

$f_{yy}(-\sqrt{2},0) = -2e^{-\sqrt{2}}$

so $f_{xx}\cdot f_{yy} - f_{xy}^2 = 4\sqrt{2}\cdot e^{-2\sqrt{2}} - 0 > 0$

hence $(-\sqrt{2},0)$ is a local maximum.

At point $(\sqrt{2},0)$: $f_{xx}(\sqrt{2},0) = 2\sqrt{2}\cdot e^{\sqrt{2}} > 0$

$f_{xy}(\sqrt{2},0) = 0$

$f_{yy}(\sqrt{2},0) = -2\cdot e^{\sqrt{2}}$

so $f_{xx}\cdot f_{yy} - f_{xy}^2 = -4\sqrt{2}\cdot e^{2\sqrt{2}} - 0 < 0$

hence $(\sqrt{2},0)$ is a saddle point.

4.

(b) let $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$ and $z = \rho\cos\phi$. $(\rho \geq 0, \phi \geq 0)$.

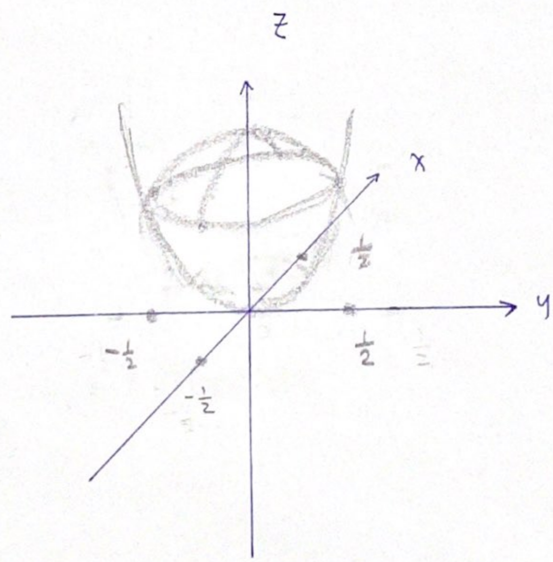
then
$$\sqrt{3\rho^2\sin^2\phi} \leq \rho\cos\phi \leq \sqrt{1-\rho^2\sin^2\phi}$$

gives
$$\begin{cases} \sqrt{3\rho^2\sin^2\phi} \leq \rho\cos\phi \\ \rho\cos\phi \leq \sqrt{1-\rho^2\sin\phi} \\ 0 \leq \rho\cos\phi \end{cases} \Rightarrow \begin{cases} 3\rho^2\sin^2\phi \leq \rho^2\cos^2\phi \\ \rho^2\cos^2\phi \leq 1-\rho^2\sin^2\phi \\ 0 \leq \cos\phi \end{cases}$$

$$\Rightarrow \begin{cases} \tan^2\phi \leq \frac{1}{3} \\ \rho^2 \leq 1 \\ \phi \leq \frac{\pi}{2} \end{cases} \text{(as } \rho \geq 0, \phi \geq 0) \Rightarrow \begin{cases} 0 \leq \phi \leq \frac{\pi}{6} \\ 0 \leq \rho \leq 1 \\ 0 \leq \phi \leq \frac{\pi}{2} \end{cases} \Rightarrow \begin{cases} 0 \leq \phi \leq \frac{\pi}{6} \\ 0 \leq \rho \leq 1 \end{cases}$$

(c)
$$V = \iiint_D \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$
$$= \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{6}} \sin\phi \, d\phi \int_0^1 \rho^2 \, d\rho$$
$$= 2\pi \cdot \left[-\cos\phi\right]_0^{\frac{\pi}{6}} \left[\frac{1}{3}\rho^3\right]_0^1$$
$$= 2\pi \times \left(-\frac{\sqrt{3}}{2}+1\right) \times \frac{1}{3}$$
$$= -\frac{\sqrt{3}}{3}\pi + \frac{2}{3}\pi.$$

(a)



5. (a) $\nabla \cdot \vec{F} = 2x\sin y - 2x\sin y + 0 = 0$

(b) By divergence theroem, we can say.

$$\iint_{S_1} \vec{F} \cdot \vec{n}\, d\sigma = \iiint_D \nabla \cdot \vec{F}\, dV = 0$$

$$\iint_{S_2} \vec{F} \cdot \vec{n}\, d\sigma = \iiint_D \nabla \cdot \vec{F}\, dV = 0$$

So $\iint_{S_1} \vec{F} \cdot \vec{n}\, d\sigma = \iint_{S_2} \vec{F} \cdot \vec{n}\, d\sigma$.

(c) $x = \sin\phi\cos\theta$, $y = \sin\phi\sin\theta$, $z = \rho\cos\phi$

$\vec{r}_\phi = \langle \cos\phi\cos\theta,\ \cos\phi\sin\theta,\ -\sin\phi \rangle$

$\vec{r}_\theta = \langle -\sin\phi\sin\theta,\ \sin\phi\cos\theta,\ 0 \rangle$

So $\vec{r}_\phi \times \vec{r}_\theta = \langle \sin^2\phi\cos\theta,\ \sin^2\phi\sin\theta,\ \sin\phi\cos\phi \rangle$

hence $\iint_S \vec{F} \cdot n\, d\sigma$

$$= \iint_S \vec{F} \cdot (\vec{r}_\phi \times \vec{r}_\theta)\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \vec{F} \cdot$$