



1. 考虑 $f(t)$ 的采样: $\hat{f}(t) = f(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$

$$= \sum_{k=-\infty}^{\infty} f(kT) \delta(t-kT) \quad (1)$$

由于 $f(t) \Leftrightarrow F(\omega)$, 故在频域上的采样为:

$$\hat{F}(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega - n\omega_0), \quad \omega_0 = \frac{2\pi}{T} \quad (2)$$

考虑对 (1) 式进行 FT, 有: $\hat{F}(\omega) = \sum_{k=-\infty}^{\infty} f(kT) \mathcal{F}[\delta(t-kT)]$

$$= \sum_{k=-\infty}^{\infty} f(kT) \cdot e^{-j\omega kT} \quad (3)$$

由于 (2) = (3) = $\hat{F}(\omega)$, 故: $\sum_{k=-\infty}^{\infty} f(kT) \cdot e^{-j\omega kT} = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$

令 $\omega=0$ 并整理得: $T \sum_{k=-\infty}^{\infty} f(kT) = \sum_{n=-\infty}^{\infty} F(n\omega_0), \quad \omega_0 = \frac{2\pi}{T}$