27.2 已知：$a = 10cm = 0.1m$

求：(1) E_1，(2) $T = 300K$ 时，平均热运动能量 $\frac{3kT}{2}$，求 n，n 与 $n+1$ 激发态的 ΔE.

解：$m = \frac{32 \times 10^{-3}}{6.02 \times 10^{23}} = 5.3 \times 10^{-26} kg$

(1) $E_1 = \frac{\pi^2 \hbar^2}{2ma^2} = \frac{\pi^2 \times (1.05 \times 10^{-34})^2}{2 \times 5.3 \times 10^{-26} \times 0.1^2} = 1.0 \times 10^{-40} J$

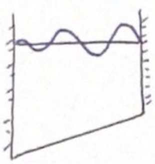
(2) 由 $\frac{3}{2}kT = E_n = \frac{\pi^2 \hbar^2}{2ma^2} n^2 = E_1 n^2$ 得

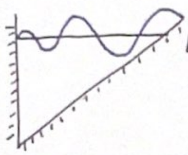
$n = \sqrt{\frac{3kT}{2E_1}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{2 \times 1.0 \times 10^{-40}}} = 7.8 \times 10^9$

$\Delta E = E_1[(n+1)^2 - n^2] = E_1(2n+1) = 1.0 \times 10^{-40} \times (2 \times 7.8 \times 10^9 + 1) = 1.56 \times 10^{-30} J$

27.3 求：$n=5$ 的激发态的波函数曲线

解：





27.6 已知：$\Psi_m(x,t)$，$\Psi_n(x,t)$

求证：$\int_0^a \Psi_m(x,t)\Psi_n(x,t)dt = 0$

证：$\int_0^a \Psi_m(x,t) \cdot \Psi_n(x,t) dt = \int_0^a \sqrt{\frac{2}{a}} e^{i2\pi E_m t/h} \sin(\frac{m\pi}{a}x) \sqrt{\frac{2}{a}} e^{-i2\pi E_n t/h} \sin(\frac{n\pi}{a}x) dx$

$= \frac{2}{a} e^{2\pi i(E_m - E_n)\frac{t}{h}} \int_0^a \sin(\frac{m\pi}{a}x) \cdot \sin(\frac{n\pi}{a}x) dx$

$= \frac{2}{a} e^{2\pi i(E_m - E_n)\cdot\frac{t}{h}} \left[\frac{\sin(\frac{\pi(m-n)x}{a})}{2(m-n)\frac{\pi}{a}} - \frac{\sin(\frac{\pi(m+n)x}{a})}{2(m+n)\frac{\pi}{a}} \right]\Bigg|_0^a$

由于 $m, n \in Z^+$，故上式为 0.

27.8 已知：宽度 a

求：$E_n = \frac{\pi^2 \hbar^2}{2ma^2} n^2$

解：$E_n = \frac{P_n^2}{2m} = \frac{1}{2m} \cdot (\frac{h}{\lambda_n})^2 = \frac{1}{2m}(\frac{hn}{2a})^2 = \frac{1}{2m}(\frac{\pi\hbar n}{a})^2 = \frac{\pi^2 \hbar^2}{2ma^2} n^2$

27.11 已知：$k = 1.13 \times 10^3 N/m$，$m = 1.67 \times 10^{-27} kg$

求：E_n，ΔE，λ

解：$E_n = (n + \frac{1}{2})h\nu = (n + \frac{1}{2}) \frac{h}{2\pi}\sqrt{\frac{k}{m}} = (n + \frac{1}{2}) \times \frac{6.63 \times 10^{-34}}{2\pi} \sqrt{\frac{1.13 \times 10^3}{1.67 \times 10^{-27}}} \times \frac{1}{1.6 \times 10^{-19}} = (n + \frac{1}{2}) \times 0.54 eV$

$\Delta E = E_n - E_{n-1} = \{n + \frac{1}{2} - [(n-1) + \frac{1}{2}]\} h\nu = 0.54 eV$

$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.54 \times 1.6 \times 10^{-19}} = 23 \times 10^{-7} m$