1. $\mathcal{F}(T) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$= \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$

$= \int_{0}^{\infty} e^{-t(a+j\omega)} dt$

$= -\frac{e^{-t(a+j\omega)}}{a+j\omega}\Big|_{0}^{\infty}$

$= \frac{1}{a+j\omega}$

2. (1) 考虑 $2\pi\delta(\omega)$ 的逆变换：

$\mathcal{F}^{-1}[2\pi\delta(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega) e^{j\omega t} d\omega$

$= \delta(0) e^{0}$

$= 1.$

故 $\mathcal{F}(1) = 2\pi\delta(\omega)$

(2) $\mathcal{F}^{-1}[E\delta(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} E\delta(\omega) e^{j\omega t} d\omega$

$= \frac{1}{2\pi} \cdot E \cdot \delta(\omega) \cdot e^{0}$

$= \frac{E}{2\pi}$

△