清華大學 TSINGHUA UNIVERSITY 数学作业纸

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1. $\mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

$= \int_0^{\tau} t e^{-j\omega t} dt + \int_{\tau}^{2\tau} \tau e^{-j\omega t} dt$

$= -\frac{t}{j\omega} e^{-j\omega t}\Big|_0^{\tau} - \tau \int_0^{\tau} -\frac{e^{-j\omega t}}{j\omega} dt \quad -\frac{\tau}{j\omega} e^{-j\omega t}\Big|_{\tau}^{2\tau}$

$= -\frac{\tau}{j\omega} e^{-j\tau\omega} + \frac{e^{-j\omega t}}{\omega^2}\Big|_0^{\tau} + \frac{j\tau}{\omega} e^{-2j\tau\omega} + \frac{\tau}{j\omega} e^{-j\tau\omega}$

$= \frac{j\tau}{\omega} e^{-2j\tau\omega} + \frac{1}{\omega^2} e^{-j\tau\omega} - \frac{1}{\omega^2}$

2. a. $\mathcal{F}(f(t)) = \int_{-\infty}^{\infty} e^{-\frac{t^2}{20}} e^{-j\omega t} dt = e^{-5\omega^2} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{20}(t+10j\omega)^2} dt = e^{-5\omega^2} \cdot \int_{-\infty}^{\infty} e^{-(\frac{t}{2\sqrt{5}} + \sqrt{5}j\omega)^2} dt.$

由提示：$\int_{-\infty}^{\infty} e^{-(x+jc)^2} dx = \sqrt{\pi}$，$c, x \in R$，考虑 $\int_{-\infty}^{\infty} e^{-(kx+jc)^2} dx = \frac{\sqrt{\pi}}{k}$，取 $k = \frac{1}{2\sqrt{5}}$，$c = \sqrt{5}\omega$ 有：

$\mathcal{F}(f(t)) = 2\sqrt{5\pi} \cdot e^{-5\omega^2}$

b. $\mathcal{F}[f_w(t,0)] = \int_{-\infty}^{\infty} f(t) \cdot w(t,0)\, e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} e^{-\frac{t^2}{20}} \cdot e^{-\frac{t^2}{2}} e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} e^{-\frac{11}{20}(t-\frac{10}{11}j\omega)^2} \cdot e^{-\frac{5}{11}\omega^2} dt$

$= \frac{2\sqrt{5\pi}}{11} e^{-\frac{5}{11}\omega^2}$

c.