



$$\begin{aligned}
 1. \quad \tilde{F}(T) &= \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \\
 &= \int_0^{\infty} e^{-\alpha t} e^{-j\omega t} dt \\
 &= \int_0^{\infty} e^{-t(\alpha + j\omega)} dt \\
 &= -\frac{e^{-t(\alpha + j\omega)}}{\alpha + j\omega} \Big|_0^{\infty} \\
 &= \frac{1}{\alpha + j\omega}
 \end{aligned}$$

2. (1) 考虑  $2\pi\delta(\omega)$  的逆变换:

$$\begin{aligned}
 \mathcal{F}^{-1}[2\pi\delta(\omega)] &= \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega) e^{j\omega t} d\omega \\
 &= \delta(0) e^0 \\
 &= 1.
 \end{aligned}$$

$$\text{故 } \mathcal{F}(1) = 2\pi\delta(\omega)$$

$$\begin{aligned}
 (2) \quad \mathcal{F}^{-1}[E\delta(\omega)] &= \frac{1}{2\pi} \int_{-\infty}^{\infty} E\delta(\omega) e^{j\omega t} d\omega \\
 &= \frac{1}{2\pi} \cdot E \cdot \delta(0) \cdot e^0 \\
 &= \frac{E}{2\pi}
 \end{aligned}$$