1. (a) 由于 $DTFT[x(n)] = X(\omega)$，$DTFT[x^*(-n)] = X^*(\omega)$

故 $DTFT[x(n) * x^*(-n)] = X(\omega) \cdot X^*(\omega)$

(b) $$DTFT[x(2n+1)] = \sum_{n=-\infty}^{\infty} x(2n+1) e^{-jn\omega}$$

(取 $m=2n+1$) $$= \sum_{m=-\infty}^{\infty} x(m) e^{-j\frac{m-1}{2}\omega}$$

$$= e^{\frac{j\omega}{2}} \cdot \sum_{m=-\infty}^{\infty} \frac{1}{2} x(m) \left[1+(-1)^{m+1}\right] e^{-\frac{j\omega m}{2}}$$

$$= e^{\frac{j\omega}{2}} \cdot \frac{1}{2} \left[\sum_{m=-\infty}^{\infty} x(m) e^{-\frac{jm\omega}{2}} - \sum_{m=-\infty}^{\infty} x(m) e^{-\frac{jm\omega}{2}} \cdot e^{j\pi m} \right]$$

$$= \frac{1}{2} e^{\frac{j\omega}{2}} \left[X\left(\frac{\omega}{2}\right) - X\left(\frac{\omega}{2} - \pi\right) \right]$$

(c) $DTFT[x(n-2)] = e^{-2j\omega} X(\omega)$

故 $DTFT[x(n) - x(n-2)] = (1 - e^{-2j\omega}) X(\omega)$

(d) $DTFT[x(n+1)] = e^{-j\omega} X(\omega)$

故 $DTFT[x(n) * x(n+1)] = X(\omega) \cdot e^{j\omega} X(\omega) = e^{j\omega} X^2(\omega)$

2. $$r_{xy}(n) = IDTFT[R_{xy}(\omega)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(\omega') Y^*(\omega' - \omega) \, d\omega' \, e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(\omega') Y^*(\omega' - \omega) e^{j(\omega-\omega')n} \cdot e^{j\omega' n} d\omega \, d\omega'$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega') \cdot e^{j\omega' n} \int_{-\pi}^{\pi} Y^*(\omega' - \omega) e^{j(\omega-\omega')n} d\omega \, d\omega'$$

(令 $\omega_1 = \omega - \omega'$) $$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega') e^{j\omega' n} d\omega' \cdot \int_{-\pi}^{\pi} Y^*(-\omega_1) e^{j\omega_1 n} d\omega_1$$

$$= 2\pi \, x(n) \, y^*(n)$$

3. (a) $N=4$ 时，$W_4 = e^{-j\frac{2\pi}{4}} = \cos(-\frac{\pi}{2}) + j\sin\frac{\pi}{2}$

$= -j$

而 $X(k) = \sum_{n=0}^{4-1} x(n) W_4^{nk}$，$k=0,1,2,3$

故 $X(0) = \sum_{n=0}^{3} x(n) \cdot 1 = 10$

$X(1) = \sum_{n=0}^{3} x(n)(-j)^n = -2+2j$

$X(2) = \sum_{n=0}^{3} x(n)(-j)^{2n} = -2$

$X(3) = \sum_{n=0}^{3} x(n)(-j)^{3n} = -2-2j$

(b) $N=8$ 时，$X(k) = \sum_{n=0}^{3} x(n) e^{-j\cdot\frac{2\pi}{8}\cdot nk}$，$k=0,1,\dots,7$

由此知：

$X(0) = \sum_{n=0}^{3} x(n) \cdot e^{-j\cdot\frac{2\pi}{8}\cdot n\cdot 0} = 10$

$X(1) = \sum_{n=0}^{3} x(n) \cdot e^{-j\cdot\frac{2\pi}{8}\cdot n} = 1-\sqrt{2} - 3(1+\sqrt{2})j$

其余同理得：

$X(2) = -2+2j$

$X(3) = 1+\sqrt{2} + 3(1-\sqrt{2})j$

$X(4) = -2$

$X(5) = 1+\sqrt{2} + 3(\sqrt{2}-1)j$

$X(6) = -2-2j$

$X(7) = 1-\sqrt{2} + 3(1+\sqrt{2})j$