12.6 已知：$\vec{p} = q\vec{l}$ ，$r >> l$.

求证：$\vec{E} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$

解：

$$E = E_+ - E_- = \frac{q}{4\pi\varepsilon_0 (r-\frac{l}{2})^2} - \frac{q}{4\pi\varepsilon_0 (r+\frac{l}{2})^2} = \frac{2qrl}{4\pi\varepsilon_0 (r^2-\frac{l^2}{4})^2} \approx \frac{2pr}{4\pi\varepsilon_0 \cdot r^4}$$，故 $\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$.

12.8. 已知：$r = 2a$，线电荷密度 $\pm\lambda$.

求：F

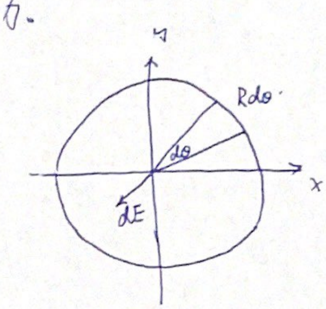
解：其中一条带电直线在另一带电直线的电场为

$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\lambda}{4\pi\varepsilon_0 a}$，（方向垂直于直线）

故另一直线受到 $F = E\lambda = \frac{\lambda^2}{4\pi\varepsilon_0 a}$（垂直于直线）的相互吸引力。

12.10. 已知：圆环半径 R，电荷线密度 λ 和 θ，$\lambda = \lambda_0 \sin\theta$.

求：圆心处电场 $\vec{E}$.



解：圆周上 $dq = \lambda \cdot R d\theta = \lambda_0 \sin\theta \, R d\theta$

故 $dE = \frac{dq}{4\pi\varepsilon_0 \cdot r^2} = \frac{\lambda_0 \sin\theta \cdot d\theta}{4\pi\varepsilon_0 \cdot R}$

此时 x轴上的电场分量为 $dE_x = \frac{\lambda_0 \sin\theta\cos\theta \, d\theta}{4\pi\varepsilon_0 \cdot R}$.

y轴电场分量 $dE_y = -\frac{\lambda_0 \sin^2\theta \cdot d\theta}{4\pi\varepsilon_0 R}$.

故 $E_x = \int dE_x = \int_0^{2\pi} \frac{\lambda_0 \cdot \sin 2\theta \, d\theta}{8\pi\varepsilon_0 \cdot R} = 0$.

$E_y = \int dE_y = \int_0^{2\pi} \frac{-\lambda_0 \sin^2\theta \, d\theta}{4\pi\varepsilon_0 R} = -\frac{\lambda_0}{4\varepsilon_0 R}$

因此 $\vec{E} = E_x \cdot \vec{i} + E_y \cdot \vec{j} = -\frac{\lambda_0}{4\varepsilon_0 R}\vec{j}$

12.16. 已知：$h_1 = 100m$，$E_1 = 150N/C$，$h_2 = 300m$，$E_2 = 100N/C$.

求：体电荷密度

解：设有一个底面大小为S，高度由100m到300m的封闭面，那么面内电荷

$q = \varepsilon_0 \oint E \cdot dS = \varepsilon_0 \cdot S \cdot (E_1 - E_2)$

故每单位体积内有电子

$n = \frac{q}{S \cdot h \cdot e} = \frac{\varepsilon_0 \cdot S \cdot (E_1 - E_2)}{S \cdot (h_2 - h_1) \cdot e} = \frac{8.85 \times 10^{-12} \times (150-100)}{(300-100) \times 1.6 \times 10^{-19}} = 1.38 \times 10^{7}$ $(1/m^3)$

面内电子带正电荷，即缺少电子.

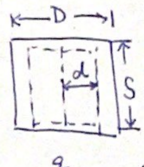
密度 $\rho = \frac{q}{Sh} = \frac{\varepsilon_0 S \cdot (E_1 - E_2)}{S(h_2 - h_1)} = \frac{8.85 \times 10^{-12} \times (150-100)}{300-100} = 2.2125 \times 10^{-12}$ (C/m^3).

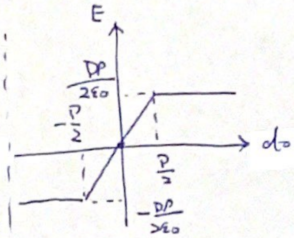
清華大學 TSINGHUA UNIVERSITY 数学作业纸

班级：计01　姓名：岑逸朗　编号：2020010869　科目：物理　第 2 页

12.20 已知：厚度 D，体电荷密度 ρ，

求：电场分布，E-d 曲线





解：如右图所示，取距离厚壁中心为 d 的高斯面，有

$$q = \varepsilon_0 \cdot E \cdot 2S$$

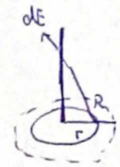
当高斯面仍在壁内，即 $d < \frac{D}{2}$ 时，$q = 2d \cdot S \cdot \rho$，$E = \frac{q}{2\varepsilon_0 S} = \frac{d\rho}{\varepsilon_0}$

否则，$q = D \cdot S \cdot \rho$，$E = \frac{D \cdot \rho}{2\varepsilon_0}$

故 E-d 曲线如右图

12.21 已知：小孔半径为 R，面电荷密度 σ.

求：通过小孔中心并与平面垂直的直线上场强分布.



解：由电场叠加原理，圆盘内电场相当于无限大平面加上带相反电荷的圆盘的电场.

直线上距板 x 处由圆盘提供的电场中，$dq = \sigma \cdot 2\pi r \cdot dr$，$dE_{盘} = \frac{x \cdot dq}{4\pi\varepsilon_0 (r^2+x^2)^{\frac{3}{2}}} = \frac{\sigma x \cdot r dr}{2\varepsilon_0 (r^2+x^2)^{3/2}}$

可 $E_{盘} = \int dE_{盘} = \frac{\sigma x}{2\varepsilon_0} \cdot \int_0^R \frac{r dr}{(r^2+x^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0} \cdot \left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$

故 x 处合电场为 $E = E_{板} - E_{盘} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right) = \frac{\sigma x}{2\varepsilon_0 \sqrt{R^2+x^2}}$

12.23. 已知：$\rho(r) = -Ce^{-2r/a_0}$，$a_0 = 0.53 \times 10^{-10}$ m

求：半径 a_0 球内净电荷，距核 a_0 处场强 E.

解：负电荷总量 $e = \int_0^\infty \rho(r) \cdot 4\pi r^2 dr = \int_0^\infty 4\pi C e^{-\frac{2r}{a_0}} \cdot r^2 dr = \pi C a_0^3 \Rightarrow C = \frac{e}{\pi a_0^3}$

半径 a_0 球内净电量为

$$q = e - \int_0^{a_0} \rho(r) \cdot 4\pi r^2 dr = e - \int_0^{a_0} \frac{4e}{a_0^3} \cdot e^{-\frac{2r}{a_0}} \cdot r^2 dr = 5e \cdot e^{-2} = 5 \times 1.6 \times 10^{-19} \times e^{-2} = 1.08 \times 10^{-19} C$$

距核为 a_0 处场强 E.

$$E = \frac{q}{4\pi\varepsilon_0 \cdot a_0^2} = \frac{5e \cdot e^{-2}}{4\pi\varepsilon_0 \cdot a_0^2} = \frac{1.08 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (0.53 \times 10^{-10})^2} = 3.46 \times 10^{11} \ V/m.$$

12.26. 已知：$m = 3.17 \times 10^{-27}$ kg，轨道半径 $r = 2.9 \times 10^{-15}$ m，铀核球半径 $r_0 = 7.4 \times 10^{-15}$ m，其内有 92e 电荷.

求：速率 v，动能 E_k，角动量 L，频率 ν.

解：

$$\rho = \frac{92e}{\frac{4}{3}\pi r_0^3} = \frac{92 \times 1.6 \times 10^{-19}}{\frac{4}{3} \times \pi \times (7.4 \times 10^{-15})^3} = 8.67 \times 10^{24} \ (C/m^3)$$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{8.67 \times 10^{24} \times 2.9 \times 10^{-15}}{3 \times 8.85 \times 10^{-12}} = 9.47 \times 10^{20} \ (V/m)$$

$$v = \sqrt{\frac{E e \cdot r}{m}} = \sqrt{\frac{9.47 \times 10^{20} \times 1.6 \times 10^{-19} \times 2.9 \times 10^{-15}}{3.17 \times 10^{-27}}} = 1.18 \times 10^7 \ (m/s)$$

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 3.17 \times 10^{-27} \times (1.18 \times 10^7)^2 = 2.20 \times 10^{-13} \ (J)$$

$$L = m \cdot v \cdot r = 3.17 \times 10^{-27} \times 1.18 \times 10^7 \times 2.9 \times 10^{-15} = 1.08 \times 10^{-34} \ (kg \cdot m^2/s)$$

$$\nu = \frac{v}{2\pi r} = \frac{1.18 \times 10^7}{2\pi \times 2.9 \times 10^{-15}} = 6.48 \times 10^{20} \ (Hz)$$

12.27. 已知：$r_0 = 0.53\times10^{-10}\,m$，总电量 $-e$，$E = 3\times10^6\,V/m$.

求：球心质子距 r，感生电偶极矩 p.

解：氢原子负电荷密度 $\rho = \dfrac{-e}{\frac{4}{3}\pi r_0^3} = \dfrac{-1.6\times10^{-19}}{\frac{4}{3}\pi\times(0.5\times10^{-10})^3} = -2.57\times10^{11}\ (C/m^3)$

正负电荷平衡，故 $\dfrac{\rho r}{3\varepsilon_0}e + Ee = 0$

即 $r = \dfrac{3\varepsilon_0 E}{-\rho} = \dfrac{3\times8.85\times10^{-12}\times3\times10^6}{2.57\times10^{11}} = 3.10\times10^{-16}\ (m)$

故电偶极矩 $p = e\cdot r = 1.6\times10^{-19}\times3.10\times10^{-16} = 4.96\times10^{-35}\ (C\cdot m)$

12.29. 已知：$E_x = bx^{1/2}$，$E_y = E_z = 0$，$b = 800\ N\cdot m^{-1/2}/C$，$a = 10\,cm = 0.1\,m$

求：过正立方体电通量 Φ_c，总电荷 q.

解：$\Phi_c = E_{2a}\cdot a^2 - E_a\cdot a^2 = b\cdot(2a)^{1/2}\cdot a^2 - b\cdot a^{1/2}\cdot a^2 = (\sqrt{2}-1)b\cdot a^{5/2} = (\sqrt{2}-1)\times800\times(0.1)^{5/2} = 1.05\ (N\cdot m^2/C)$

$q = \varepsilon_0\Phi_c = 8.85\times10^{-12}\times1.05 = 9.29\times10^{-12}\ C$.

12.31 已知：电矩 $\vec{p}$，场强 $\vec{E}$

求证：从电场方向垂直到成 θ 角位置的过程中，做功 $pE\cos\theta = \vec{p}\cdot\vec{E}$.

解：电偶极子受电力矩 $M = pE\sin\theta$，转动 $d\theta$ 角时做功 $-Md\theta = -pE\cdot\sin\theta d\theta$.

故 $W = \int_{\pi/2}^{\theta} -Md\theta = \int_{\pi/2}^{\theta} -pE\sin\theta d\theta = pE\cos\theta = \vec{p}\cdot\vec{E}$.