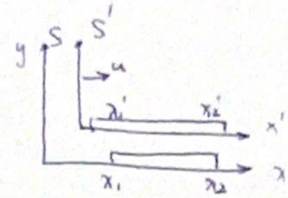
8.3 已知：$x_2 - x_1 = 1m$，$t_2 = t_1$

求：$x_2' - x_1'$

解：洛伦兹变换得



$$x_2' - x_1' = \frac{x_2 - ut_2 - x_1 + ut_1}{\sqrt{1-u^2/c^2}}$$

$$= \frac{(x_2 - x_1) - u(t_2 - t_1)}{\sqrt{1-u^2/c^2}} = \frac{1-0}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-u^2/c^2}} > 1$$

这是由于在 S'系中两枪不是同时打出的。

8.5 已知：S系中 $\Delta x = 1m$，$\Delta t = 0$，S'系中 $\Delta x' = 2m$

求：$\Delta t'$

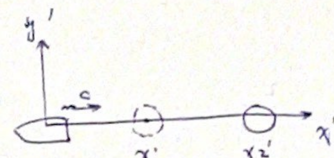
解：由洛伦兹变换：$\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-u^2/c^2}} = \frac{\Delta x}{\sqrt{1-u^2/c^2}} \Rightarrow u = c\sqrt{1-\left(\frac{\Delta x}{\Delta x'}\right)^2}$

$$|\Delta t'| = \left|\frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-u^2/c^2}}\right| = \left|\frac{0-\frac{u}{c^2}\Delta x}{\sqrt{1-u^2/c^2}}\right| = \frac{u}{c^2}\Delta x' = \frac{\Delta x'}{c}\cdot\sqrt{1-\left(\frac{\Delta x}{\Delta x'}\right)^2} = \frac{2}{3\times10^8}\times\sqrt{1-\left(\frac{1}{2}\right)^2} = 5.77\times10^{-9}s$$

8.6 已知：飞船中：$\Delta t' = 60s$，飞船速度 $u = \frac{4}{5}c$

求：(1) 地球与飞船距离 l_0 (2) 飞船收到信号时，地球与飞船距离.

解：



(1) 以飞船为参考系，光速不变，信号到达地球与返回飞船的距离相等，故所用时间亦相等，即信号从飞船到地球用了 30s.

此时地球与飞船相距 $l_0 = 30c = 9\times10^9 m$.

(2) 以飞船为参考系，宇航员发射信号时，两者相距 $l' = (c - \frac{4c}{5})\cdot 30 = 6c$.

在地球参考系中，$l = \frac{l'}{\sqrt{1-(\frac{u}{c})^2}} = \frac{6c}{\sqrt{1-(\frac{4}{5})^2}} = 10c$.

当宇航员收到信号时，地球经过的时间 $\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{60}{\sqrt{1-(\frac{4}{5})^2}} = 100s$.

这段时间中，从地球测量飞船走过的距离为 $l_1 = 100\times\frac{4}{5}c = 80c$.

总距离为 $l + l_1 = 10c + 80c = 90c = 2.7\times10^{10}m$.

8.7 已知：飞船速率 $u = 0.8c$，飞船参考系中 $t' = -6\times10^8 s$，$x' = 1.8\times10^{17}m$，$y' = 1.2\times10^{17}m$，$z' = 0$.

求：地球参考系中 t, x, y, z

解：洛伦兹变换：

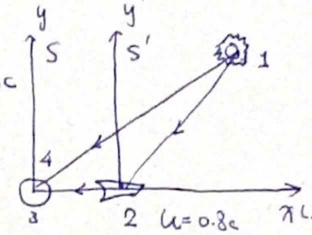
$$t = \frac{t' + \frac{u}{c^2}x'}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{-6\times10^8 + \frac{0.8}{3\times10^8}\times1.8\times10^{17}}{\sqrt{1-0.8^2}} = -2\times10^8 s$$

$$x = \frac{x' + ut'}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1.8\times10^{17} + 0.8\times3\times10^8\times(-6)\times10^8}{\sqrt{1-0.8^2}} = 6\times10^{16}m$$

$y = y' = 1.2\times10^{17}m$，$z = z' = 0$.

8.8. 已知: $t_1' = -6\times10^8 s$, $x_1' = 1.8\times10^{17} m$, $y_1' = 1.2\times10^{17} m$, $z_1' = 0$

$t_1 = -2\times10^8 s$, $x_1 = 6\times10^{16} m$, $y_1 = 1.2\times10^{17} m$, $z_1 = 0$, $u=0.8c$



求: t_2' , t_3 , t_4

解: (1) 光到达飞船: $t_2' = t_1' + \Delta t_{12} = t_1' + \frac{\sqrt{x_1'^2+y_1'^2}}{c}$

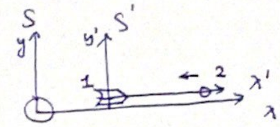
$$= -6\times10^8 + \frac{\sqrt{(1.8\times10^{17})^2+(1.2\times10^{17})^2}}{3\times10^8} = 1.21\times10^8 S.$$

(2) 地球收到报告 $t_3 = t_2 + \Delta t_{23} = \frac{t_2' + \frac{u}{c^2}x_2'}{\sqrt{1-u^2/c^2}} + \frac{x_2}{c} = \frac{t_2'}{\sqrt{1-u^2/c^2}} + \frac{x_2' + u\cdot t_2'}{c\sqrt{1-u^2/c^2}}$ ($x_2' = 0$)

$$= \frac{t_2'}{\sqrt{1-u^2/c^2}}\cdot\left(1+\frac{u}{c}\right) = \frac{1.21\times10^8}{\sqrt{1-0.8^2}}\times(1+0.8) = 3.63\times10^8 s$$

(3) 地球看见超新星 $t_4 = t_1 + \Delta t_{3,4} = t_1 + \frac{\sqrt{x_1^2+y_1^2}}{c} = -2\times10^8 + \frac{\sqrt{(6\times10^{16})^2+(1.2\times10^{17})^2}}{3\times10^8} = 2.47\times10^8 S$

8.9 已知: $V_1 = 0.6c$, $V_2 = -0.8c$, $\Delta t = 5s$

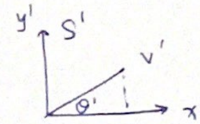


求: V_2' , $\Delta t'$

解: 洛伦兹变换: $V_2' = \frac{V_2 - u}{1-\frac{uV}{c^2}} = \frac{-0.8c-0.6c}{1-\frac{(-0.8c)(0.6c)}{c^2}} = \frac{-1.4c}{1+0.48} = -0.950c \doteq -2.84\times10^8 m/s$

地球观察者发现飞行物存在和他们即将相撞时，飞船在飞船参考系中位置不变，故 $\Delta t'$ 为固有时，$\Delta t' = \Delta t\sqrt{1-\frac{u^2}{c^2}} = \Delta t\sqrt{1-\frac{V_1^2}{c^2}} = 5\times\sqrt{1-0.6^2} = 4s$

8.10. 已知: θ' , $v'=c$



求: θ , v

解: 分解光速，在S系中有: $V_x = \frac{V_x'+u}{1+\frac{u\cdot V_x'}{c^2}} = \frac{c\cdot\cos\theta' + u}{1+\frac{u}{c}\cos\theta'}$

$$V_y = \frac{V_y'}{1+\frac{V_x'u}{c}}\sqrt{1-\frac{u^2}{c^2}} = \frac{c\sin\theta'}{1+\frac{u}{c}\cos\theta'}\cdot\sqrt{1-\frac{u^2}{c^2}}$$

$$\theta = \arctan\frac{V_y}{V_x} = \arctan\left(\frac{\sin\theta'}{u+\cos\theta'}\sqrt{1-\frac{u^2}{c^2}}\right)$$

$$V = \sqrt{V_x^2+V_y^2} = \frac{1}{1+\frac{u}{c}\cos\theta'}\sqrt{u^2\cos^2\theta' + 2uc\cdot\cos\theta' + c^2} = c.$$

8.12. 已知：静质量 m_0，时间 t_0，力 $\vec{F}=F\vec{i}$

求：$t \ll \frac{m_0c}{F}$ 时 v, x 及 $t \gg \frac{m_0c}{F}$ 时 v, x.

解：

F方向不变，物体做直线运动，故 $v=\int dv=\int_0^t \frac{F}{m}dt=\int_0^t \frac{F\sqrt{1-v^2/c^2}}{m_0}\cdot dt=\frac{Ft\sqrt{1-v^2/c^2}}{m_0}$

解得 $v=\frac{Ft}{m_0}\cdot\frac{1}{\sqrt{1+\left(\frac{Ft}{m_0c}\right)^2}}$

位移 $x=\int_0^t v\,dt=\int_0^t \frac{Ft}{m_0\sqrt{1+\left(\frac{Ft}{m_0c}\right)^2}}dt=\frac{m_0c^2}{F}\left(\sqrt{1+\left(\frac{Ft}{m_0c}\right)^2}-1\right)$

当 $t\ll\frac{m_0c}{F}$ 时，$\frac{Ft}{m_0c}\ll 1$，此时 $v\approx\frac{Ft}{m_0}=at$，$x\approx\frac{m_0c^2}{F}\left(1+\frac{1}{2}\left(\frac{Ft}{m_0c}\right)^2-1\right)$

$=\frac{Ft^2}{2m_0}=\frac{1}{2}at^2$.

当 $t\gg\frac{m_0c}{F}$ 时，$\frac{Ft}{m_0c}\gg 1$，$v=\frac{1}{\frac{m_0c}{Ft}}\cdot\frac{c}{\sqrt{1+\left(\frac{Ft}{m_0c}\right)^2}}=\frac{c}{\sqrt{\left(\frac{m_0c}{Ft}\right)^2+1}}\approx c$

$x\approx\frac{m_0c^2}{F}\cdot\frac{Ft}{m_0c}=ct$

8.14. 已知：动能 $E_k=2.8\times10^9 eV$，$m_0=9.11\times10^{-31}kg$

求：(1) $c-v$ (2) 动量 p (3) $2\pi R=240m$ 圆环向心力 F，偏转磁场 B.

解：(1) $E_k=mc^2-m_0c^2=m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)\Rightarrow c^2-v^2=\left(\frac{m_0c^3}{E_k+m_0c^2}\right)^2$

由于 $c\approx v$，故 $c+v\approx 2c$，所以 $c-v=\frac{m_0^2c^5}{2(E_k+m_0c^2)^2}=\frac{(9.11\times10^{-31})^2\times(3\times10^8)^5}{2(2.8\times10^9\times1.6\times10^{-19}+9.11\times10^{-31}\times(3\times10^8)^2)^2}$

$=5.02\ m/s$

(2) $p=\sqrt{\frac{E^2-m_0c^4}{c^2}}=\frac{\sqrt{(E+m_0c^2)(E-m_0c^2)}}{c}=\frac{\sqrt{(E_k+2m_0c^2)\cdot E_k}}{c}=\frac{\sqrt{(2.8\times10^9\times1.6\times10^{-19})^2}}{3\times10^8}$ $(E_k\gg m_0c^2)$

$=1.49\times10^{-18}\ m/s$

(3) $F=\frac{mv^2}{R}\approx\frac{mc^2}{R}=\frac{E_k+m_0c^2}{R}\approx\frac{E_k}{R}=\frac{2.8\times10^9\times1.6\times10^{-19}}{\frac{240}{2\pi}}=1.17\times10^{-11}N.$

$B=\frac{F}{e\cdot v}\approx\frac{F}{e\cdot c}=\frac{1.17\times10^{-11}}{1.6\times10^{-19}\times3\times10^8}=0.244(T)$

8.19 已知：$\beta = 0.5$　$m_0 = 1.67 \times 10^{-27}$ kg

求：(1) 质子相对共同点的动量 P_1 和能量 E_1 (2) 质子相对令一质子为原点的参考系动量 P_2 和能量 E_2

解：

(1) $P_1 = m_1 V_1 = \frac{m_0}{\sqrt{1-\beta^2}} \cdot \beta c = \frac{1.67 \times 10^{-27}}{\sqrt{1-0.5^2}} \times 0.5 \times 3 \times 10^8 = 2.89 \times 10^{-19}$ kg·m/s

$E_1 = m_1 c^2 = \frac{m_0}{\sqrt{1-\beta^2}} \cdot c^2 = \frac{1.67 \times 10^{-27}}{\sqrt{1-0.5^2}} \times (3 \times 10^8)^2 = 1.74 \times 10^{-10}$ J.

(2) 质子之间相对速度 $\beta_2 = \frac{\beta - (-\beta)}{1 - \beta \cdot (-\beta)} = \frac{0.5 + 0.5}{1 + 0.5^2} = 0.8$

$P_2 = m_2 V_2 = \frac{m_0}{\sqrt{1-\beta_2^2}} \cdot \beta_2 c = \frac{1.67 \times 10^{-27}}{\sqrt{1-0.8^2}} \times 0.8 \times (3 \times 10^8) = 6.68 \times 10^{-19}$ kg·m/s

$E_2 = m_2 c^2 = \frac{m_0}{\sqrt{1-\beta_2^2}} \cdot c^2 = \frac{1.67 \times 10^{-27}}{\sqrt{1-0.8^2}} \times (3 \times 10^8)^2 = 2.51 \times 10^{-10}$ J