



班级: 计01 · 姓名: 吴逸朗 编号: 2020010869 科目: 信号 第 1 页

1. 证明: (1)  $e^{ja\omega t}$ ,  $e^{jb\omega t}$ ,  $a, b = 0, \pm 1, \pm 2, \dots$ ,  $a \neq b$  (即  $a-b \in \mathbb{Z} \setminus \{0\}$ ) 正交.

$$\begin{aligned}
 & \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{ja\omega t} \cdot e^{-jb\omega t} dt \\
 &= \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{j(a-b)\omega_0 t} dt \\
 &= \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} \cos[(a-b)\omega_0 t] + j \sin[(a-b)\omega_0 t] dt \\
 &= \frac{1}{(a-b)\omega_0} \cdot \left[ \sin[(a-b)\omega_0 t] - j \cos[(a-b)\omega_0 t] \right] \Big|_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} \\
 &= \frac{1}{(a-b)\omega_0} \cdot \left[ \sin(a-b)\pi - \sin[-(a-b)\pi] - j \cos(a-b)\pi + j \cos[-(a-b)\pi] \right] \\
 &= \frac{2 \sin(a-b)\pi}{(a-b)\omega_0} \\
 &= 0
 \end{aligned}$$

(2)  $e^{ja\omega t}$ ,  $e^{ja\omega t}$ ,  $a = 0, \pm 1, \pm 2, \dots$  不正交.

$$\begin{aligned}
 & \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^{ja\omega t} \cdot e^{-ja\omega t} dt \\
 &= \int_{-\frac{\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} e^0 dt \\
 &= \frac{2\pi}{\omega_0}
 \end{aligned}$$

由此可知,  $\{e^{ja\omega t} \mid a = 0, \pm 1, \pm 2, \dots\}$  在  $[-\frac{\pi}{\omega_0}, \frac{\pi}{\omega_0}]$ ,  $\omega_0 \in \mathbb{R}$  上是正交函数集.