Artifical Intelligence Problem Set 3

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Problem 1.
A. First we calculate $Error(\{A, E\}) = 0 + 6 = 6$. We can perform three different operations on the set:

1. Add an object to the set. If we add B, then $Error(\{A, E, B\}) = 3 + 0 = 3$. If we add C, then $Error(\{A, E, C\}) = 2 + 0 = 2$. If we add D, then $Error(\{A, E, D\}) = 2 + 0 = 2$.
2. Delete an object from the set. If we delete E, then $Error(\{A\}) = 0 + 10 = 10$. If we delete A, then $Error(\{E\}) = 0 + 16 = 16$. In either case, this is higher error than what we had before.
3. Swap out an object in the set with an object outside the set. If we swap out A and add B, then $Error(\{B, E\}) = 0 + 8 = 8$. If we swap out A and add C, then $Error(\{C, E\}) = 0 + 9 = 9$. If we swap out A and add D, then $Error(\{D, E\}) = 0 + 10 = 10$. If we swap out E and add B, then $Error(\{A, B\}) = 2 + 2 = 4$. If we swap out E and add C, then $Error(\{A, C\}) = 1 + 3 = 4$. If we swap out E and add D, then $Error(\{A, D\}) = 1 + 4 = 5$.

Out of all of these possibilities, the operation with the lowest error after the operation would be to add C or add D to the set. To determine what happens at the next iteration, we must assume that we add C to the set, and not D. Thus, our set as of now is $\{A, C, E\}$. With this, the next iteration will be the set $\{B, C, E\}$ where we have swapped out A and added B to our set. $Error(\{B, C, E\}) = 0 + 1 = 1$.

B. The size of the state space is all combinations of 1, 2, 3, ... , N objects. Thus, the state space is

$$\binom{N}{1} + \binom{N}{2} + \cdots + \binom{N}{N} = \sum_{i=1}^{N} \binom{N}{i}.$$

The maximal number of neighbors of any state can be calculated by the total number of operations that we can perform on a general state. Suppose we have a state with X elements in it. We can add $N - X$ elements to the state space. We can remove any one of X elements from the state space. Or we can replace any of the X elements with one of the $N - X$ elements not in the space. This totals to

$$\begin{aligned} N - X + X + X \cdot (N - X) &= N + X \cdot (N - X) \\ &= N + XN - X^2 \end{aligned}$$

To maximize that, we look where the derivative of $N + XN - X^2 = 0$ with respect to X: $-2X + N = 0$ implies that $X = \frac{N}{2}$. Thus the maximum number of neighbors for any given state is

$$\begin{aligned} N + \frac{N}{2} \cdot N - (\frac{N}{2})^2 &= N + \frac{N^2}{2} - \frac{N^2}{4} \\ &= \frac{N^2 + 4N}{4} \end{aligned}$$

Problem 2. Roots that are pruned (in order):

1. 6, the leaf node
2. 20, the leaf node
3. 7, the leaf node

The best move for the max node at the top level is to go right for a score of 8.