# Artifical Intelligence Problem Set 6

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**Problem 1.**

- Combining rule 1 with fact 4 with substitution $\{x \to \text{Mary}, y \to \text{Henry}, z \to \text{Catherine}\}$, infer 6: Parent(Henry, Mary).
- Combining rule 2 with fact 4 with substitution $\{x \to \text{Mary}, y \to \text{Henry}, z \to \text{Catherine}\}$, infer 7: Parent(Catherine, Mary).
- Combining rule 1 with fact 5 with substitution $\{x \to \text{Edward}, y \to \text{Jane}, z \to \text{Henry}\}$, infer 8: Parent(Jane, Edward).
- Combining rule 2 with fact 5 with substitution $\{x \to \text{Edward}, y \to \text{Jane}, z \to \text{Henry}\}$, infer 9: Parent(Henry, Edward).
- Combining rule 3 with facts 6 and 9 with substitution $\{x \to \text{Henry}, y \to \text{Mary}, z \to \text{Edward}\}$, infer 10: Sibling(Mary, Edward).

**Problem 2.**

**A.**

$$P(X = a) = 0.1 + 0.2 + 0.05 = 0.35$$
$$P(X = b) = 0.2 + 0.05 + 0 = 0.25$$
$$P(X = c) = 0.1 + 0.2 + 0.1 = 0.4$$
$$P(X = d) = 0.1 + 0.2 + 0.1 = 0.4$$
$$P(X = e) = 0.2 + 0.05 + 0.2 = 0.45$$
$$P(X = f) = 0.05 + 0 + 0.1 = 0.15$$

**B.** X and Y are not independent because the probability in the top right cell (0.1) is not equal to the probability of a times c ($0.35 \cdot 0.4 = 0.14 \neq 0.1$)

**C.** $P(X = a|Y = e) = \frac{0.2}{0.45} = 0.444$

**Problem 3.**

**A.**

$$\frac{2}{5} \cdot 0.2 + \frac{2}{5} \cdot 0.6 + \frac{1}{5} \cdot 0.9 = 0.5$$

**B.** The probability that the coin is category 1 is: $\frac{\frac{2}{5} \cdot 0.2}{0.5} = 0.16$

The probability that the coin is category 2 is: $\frac{\frac{2}{5} \cdot 0.6}{0.5} = 0.48$

The probability that the coin is category 2 is: $\frac{\frac{1}{5} \cdot 0.9}{0.5} = 0.36$

**C.** The probability that a coin randomly picked up from the bag gets heads twice in two tosses is $\frac{2}{5} \cdot 0.2^2 + \frac{2}{5} \cdot 0.6^2 + \frac{1}{5} \cdot 0.9^2 = 0.322$

The probability that the coin is category 1 is: $\frac{\frac{2}{5} \cdot 0.2^2}{0.322} = 0.04969$

The probability that the coin is category 2 is: $\frac{\frac{2}{5} \cdot 0.6^2}{0.322} = 0.44720$

The probability that the coin is category 3 is: $\frac{\frac{1}{5} \cdot 0.9^2}{0.322} = 0.50310$