CS 240
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Problem 1)

a)

	a	b	c	d	e
a	x				x
b		x	x	x	
c		x	x	x	
d		x	x	x	
e	x				x

$\therefore$ (a,a), (a,e), (b,b), (b,c), (b,d), (c,b), (c,c), (c,d), (d,b), (d,c), (d,d), (e,a), (e,e).

b) the equivalence classes are
$[a] = [e] = \{a, e\}$. $[b] = [c] = [d] = \{b, c, d\}$.

c) i) S is not an equivalence relation. Since equivalence relation is Symmetric, which indicates that (X, Y), $|X| \le |Y|$ and $|Y| \le |X|$. It is not true unless $|X| = |Y|$, but a, b, c, d, e are different in A, hence S is not symmetric.

ii) S might be an order relation. Since it is anisymmetric and transitive. For sets of all subsets of A, X, Y, Z, if $|X| \le |Y|$ and $|Y| \le |Z|$, it is true that $|X| \le |Z|$. For antisymmetric, we know that if $|X| \le |Y|$ and $|Y| \le |X|$ it indicates that $|X| = |Y|$. But we can't know if $X = Y$ or $X = -Y$. if a, b, c, d, e are all same sign, then $X = Y$. then S is an order relation.

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Problem 2)

a) In this case if $a \le c \land ab \le c \cdot d$ and $c \le a \land c \cdot d \le a \cdot b$. it indicates that $a = c \land b = d$. So it is antisymmetric. if $a \le c \land a \cdot b \le c \cdot d$ and $c \le e \land c \cdot d \le e \cdot f$. we can know that it is true that $a \le e \land a \cdot b \le e \cdot f$ hence it is transitive. hence it is an order relation on $(\mathbb{Z}^+)^2$.

b) R is not a total order, since when $a = 1, c = 2, b = 10, d = 3$. $(a \le c \land a \cdot b \le c \cdot d)$ and $(c \le a \land c \cdot d \le a \cdot b)$ are both wrong For the $(a, b), (c, d)$ that $(a, b) \ne (c, d)$ hence it is not a total order.

c) Yes, R is a partial order; Since for all (a, b), $(a \le a \land a \cdot b \le a \cdot b)$. hence it is reflexive

d) R is not a strict order Because R is reflexive (from above), is it not antireflexive

Problem 3)

a) $R_1 \cap R_2$ is an equivalence relation on S. Following are the proof.
Reflexive: Let $x \in S$, we have $(x, x) \in R_1$ and $(x, x) \in R_2$. Since R_1, R_2 are reflexive on S, $(x, x) \in R_1 \cap R_2$. hence $(x, x) \in R_1 \cap R_2$ for all $x \in S$ and hence $R_1 \cap R_2$ is reflexive.
symmetric: Let $x, y \in S$, $(x, y) \in R_1 \cap R_2$. we have $(x, y) \in R_1$ and $(x, y) \in R_2$, so $(y, x) \in R_1$ and $(y, x) \in R_2$ when R_1 and R_2 are symmetric So $(y, x) \in R_1 \cap R_2$, hence $R_1 \cap R_2$ is symmetric
transitive: $x, y, z \in S$, $(x, y), (y, z) \in R_1 \cap R_2$, we have $(x, y), (y, z) \in R_1$ and $(x, y), (y, z) \in R_2$ hence $(x, y) \in R_1$ and $(x, z) \in R_2$ Since R_1, R_2 are transitive. hence $(x, z) \in R_1 \cap R_2$ hence $R_1 \cap R_2$ is transitive.
$\therefore$ $R_1 \cap R_2$ is an equivalence relation on S.

b) $R_1 \cup R_2$ need not be equivalence relation on S.
$R_1 = \{(3,3), (4,4), (5,5), (3,4), (4,3)\}$.
$R_2 = \{(3,3), (4,4), (5,5), (4,5), (5,4)\}$ Both R_1, R_2 are equivalent relations on $S = \{3, 4, 5\}$. But $R_1 \cup R_2 = \{(3,3), (4,4), (5,5), (3,4), (4,3), (4,5), (5,4)\}$ is not a equivalence relation because it is not transitive. $(3,4)(4,5) \in R_1 \cup R_2$. But $(3,5) \notin R_1 \cup R_2$.