Coventry University

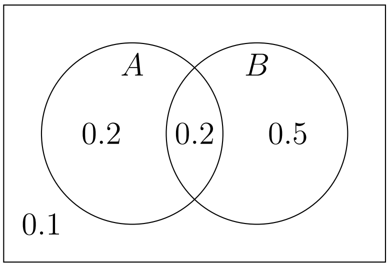
School of Computing, Electronics and Mathematics

5005CEM **Feedback for Probability Problem Sheet 1b** **Week 5**

1 Given

$$P(A) = 0.4 \qquad P(B) = 0.7 \qquad P(A \cap B) = 0.2$$

It is useful to summarise what we know in a Venn diagram.



Definition of conditional probability

$$P(A \mid B) \;=\; \frac{P(A \cap B)}{P(B)}$$

(a) $P(A \mid B) \;=\; \dfrac{P(A \cap B)}{P(B)} \;=\; \dfrac{0.2}{0.7} \;=\; \frac{2}{7}$

(b) $P(\overline{A} \mid B) \;=\; \dfrac{P(\overline{A} \cap B)}{P(B)} \;=\; \dfrac{0.5}{0.7} \;=\; \frac{5}{7}$

(c) $P(A \mid \overline{B}) \;=\; \dfrac{P(A \cap \overline{B})}{P(\overline{B})} \;=\; \dfrac{0.2}{0.3} \;=\; \frac{2}{3}$

(d) $P(\overline{A} \mid \overline{B}) \;=\; \dfrac{\overline{A} \cap \overline{B}}{P(\overline{B})} \;=\; \dfrac{0.1}{0.3} \;=\; \frac{1}{3}$ epr015

2 Solution — (C)

If $P(A \mid B) = P(B \mid A)$ then

$$\frac{P(A \cap B)}{P(B)} \;=\; \frac{P(B \cap A)}{P(A)}$$

but $A \cap B = B \cap A$ so $P(A \cap B) = P(B \cap A)$ and we are left with $P(B) = P(A)$. epr006

3 Given

$$P(A) = 0.8 \qquad P(B) = 0.7 \qquad P(C) = 0.6$$
$$P(A \mid B) = 0.8 \qquad P(C \mid B) = 0.7 \qquad P(A \cap C) = 0.48$$

Definition of independence

$$\begin{array}{lrcl} & P(A \mid B) & = & P(A) \\ \textbf{or equivalently} & P(B \mid A) & = & P(B) \\ \textbf{or equivalently} & P(A \cap B) & = & P(A) \times P(B) \end{array}$$

(a) Are A and B are independent? $\boxed{\text{yes}}$ since $P(A) = P(A \mid B)$

(b) Are A and C are independent? $\boxed{\text{yes}}$ since

- $P(A \cap C) = 0.48$ and $P(A)P(C) = 0.8 \times 0.6 = 0.48$

$\boxed{\text{or}}$

- $P(A) = 0.8$ and $P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{0.48}{0.6} = 0.8$

$\boxed{\text{or}}$

- $P(C) = 0.6$ and $P(C \mid A) = \frac{P(C \cap A)}{P(A)} = \frac{0.48}{0.8} = 0.6$

(c) Are B and C are independent? $\boxed{\text{no}}$ since $P(C \mid B) \neq P(C)$

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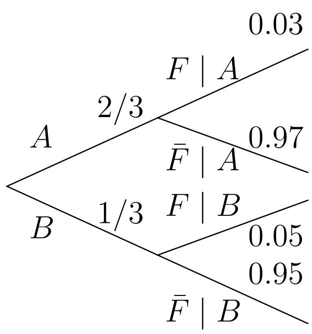
4 (a) *Tree diagram*



(b) (i) $P(B) = \frac{1}{3}$

(ii) $P(A \cap F) = \frac{2}{3} \times 0.03 = 0.02$

(iii) $P(B \cap \overline{F}) = \frac{1}{3} \times 0.95 = \frac{19}{60}$

(iv) $P(F) = P(A \cap F) + P(B \cap F) = \frac{2}{3} \times 0.03 + \frac{1}{3} \times 0.05 = \frac{11}{300}$

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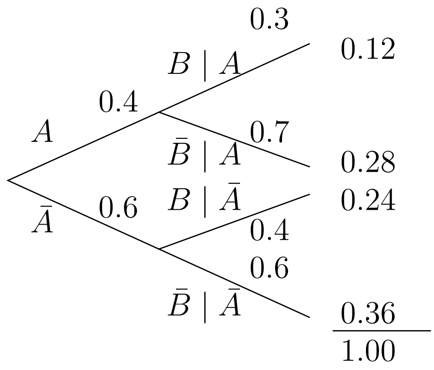
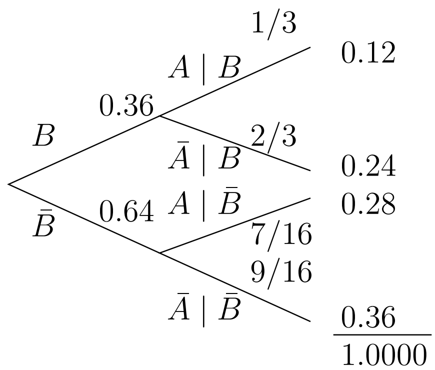
5 (a) Tree diagram



(b) "Flipping the tree" (and doing all the calculations on the tree) gives

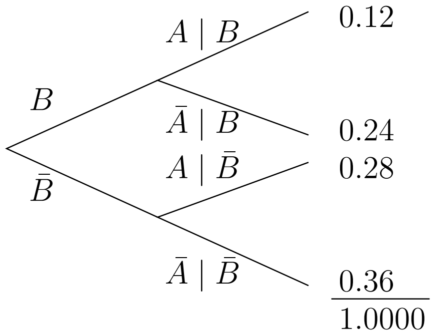


Conclusion —

(i) $P(A \mid B) = \frac{1}{3}$ (ii) $P(\overline{A} \mid B) = \frac{2}{3}$ 7 (iii) $P(A \mid \overline{B}) = \frac{7}{16}$ (iv) $P(\overline{A} \mid \overline{B}) = \frac{9}{16}$

Step by step —

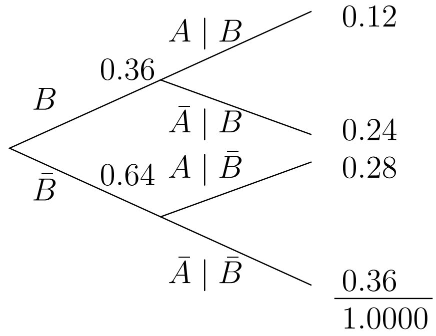
Step 1. Flip the tree structure and copy the outcome probabilities of $P(A \cap B)$, $P(A \cap \overline{B})$, $P(\overline{A} \cap B)$ and $P(\overline{A} \cap \overline{B})$ to the **correct** places on the flipped tree.



Step 2. Calculate the probabilities $P(B)$ and $P(\overline{B})$ by adding, e.g.,

$$P(B) \quad + \quad P(A \cap B) + P(\overline{A} \cap B) \;=\; 0.12 + 0.24 \;=\; 0.36$$

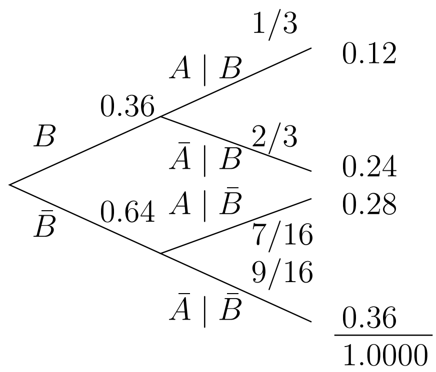
and write them on the tree.



Step 3. Calculate the conditional probabilities using the **<u>definition</u>** of conditional probability, e.g.

$$P(A \mid B) \quad = \quad \frac{P(A \cap B)}{P(B)} \quad = \quad \frac{0.12}{0.36} \quad = \quad \tfrac{1}{3}$$

and write them on the tree.



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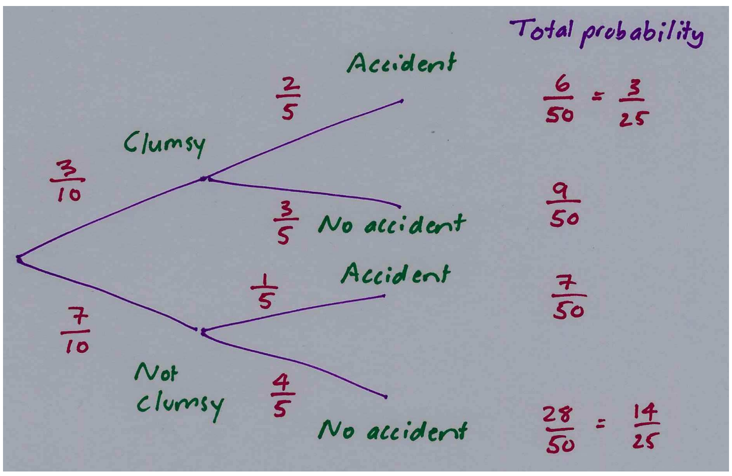
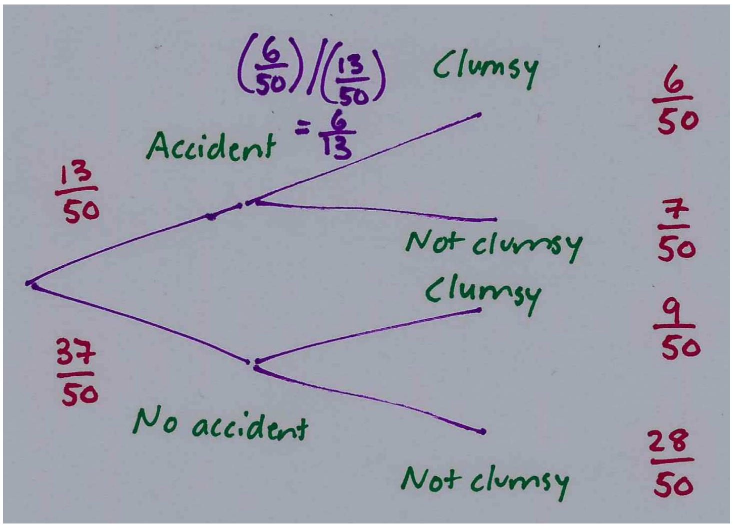
6 (a)

$$\begin{aligned}
P(\text{accident}) \quad &= \quad P(\text{accident} \mid \text{clumsy}) \times P(\text{clumsy}) \\
&\qquad + \quad P(\text{accident} \mid \text{not clumsy}) \times P(\text{not clumsy}) \\
&= \quad \tfrac{2}{5} \times \tfrac{3}{10} \quad + \quad \tfrac{1}{5} \times \tfrac{7}{10} \\
&= \quad \tfrac{6}{50} + \tfrac{7}{50} \\
&= \quad \tfrac{13}{50} \quad = \quad 0.26
\end{aligned}$$



(b) $$\begin{aligned} P(\text{clumsy} \mid \text{accident}) &= P(\text{clumsy and accident})/P(\text{accident}) \\ &= (\tfrac{2}{5} \times \tfrac{3}{10})/(\tfrac{13}{50}) \\ &= \tfrac{6}{13} \end{aligned}$$



epr045

7 Solution — (B)

If A and B are mutually exclusive then $A \cap B = \emptyset$ so $P(A \cap B) = 0$.
Since $P(A) > 0$ and $P(B) > 0$ we must have $P(A) \times P(B) > 0$.
Therefore A and B are **not** independent. epr005

8 Given

$$P(A) = 0.8 \qquad P(A \mid B) = 0.8 \qquad P(A \cap B) = 0.5$$

Definition of conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Negation

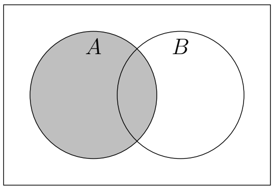
$$P(\overline{A}) = 1 - P(A)$$

(a) $P(B) = \dfrac{P(A \cap B)}{P(A \mid B)} = \dfrac{0.5}{0.8} = \frac{5}{8}$

(b) $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.5}{0.8} = \frac{5}{8}$

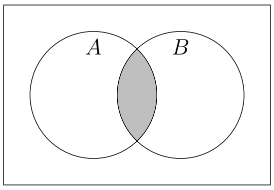
(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + \frac{5}{8} - 0.5 = \frac{37}{40}$

(d) $P(A \mid A \cup B) \;=\; \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} \;=\; \dfrac{P(A)}{P(A \cup B)} \;=\; \dfrac{0.8}{(\frac{37}{40})} \;=\; \frac{32}{37}$



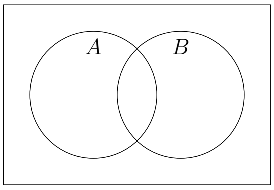
Note that $A \cap (A \cup B) = A$

(e) $P(A \cap B \mid A \cup B) \;=\; \dfrac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} \;=\; \dfrac{P(A \cap B)}{P(A \cup B)} \;=\; \dfrac{0.5}{(\frac{37}{40})} \;=\; \frac{20}{37}$



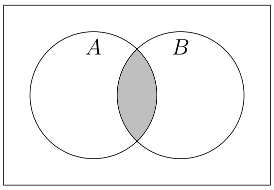
Note that $(A \cap B) \cap (A \cup B) = A \cap B$

(f) $P(A \cap B \mid \overline{B}) \;=\; \dfrac{P((A \cap B) \cap \overline{B})}{P(\overline{B})} \;=\; \dfrac{0}{1 - \frac{5}{8}} \;=\; 0$



Note that $(A \cap B) \cap \overline{B} = \emptyset$

(g) $P(A \cap B \mid A) \;=\; \dfrac{P((A \cap B) \cap A)}{P(A)} \;=\; \dfrac{P(A \cap B)}{P(A)} \;=\; \dfrac{0.5}{0.8} \;=\; \frac{5}{8}$



Note that $(A \cap B) \cap A = A \cap B$

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