5005CEM **Feedback for Probability Problem Sheet 1a** **Week 5**

1 Given

$$P(C) = 0.6 \qquad P(F) = 0.35 \qquad P(\overline{C \cup F}) = 0.15$$

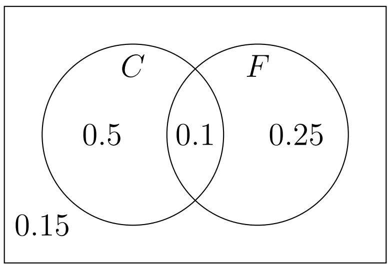
(a)
$$\begin{aligned} P(\text{challenging or fun, or both}) &= P(C \cup F) = 1 - P(\overline{C \cup F}) \\ &= 1 - 0.15 = \boxed{0.85} \end{aligned}$$

(b)
$$\begin{aligned} P(\text{challenging and fun}) &= P(C \cap F) = P(C) + P(F) - P(C \cup F) \\ &= 0.6 + 0.35 - 0.85 = \boxed{0.1} \end{aligned}$$

(c)
$$\begin{aligned} P(\text{fun but not challenging}) &= P(F \cap \overline{C}) = P(F) - P(F \cap C) \\ &= 0.35 - 0.1 = \boxed{0.25} \end{aligned}$$

(d)
$$\begin{aligned} P(\text{challenging or fun but not both}) &= P(C \cap \overline{F}) + P(\overline{C} \cap F) \\ &= (0.6 - 0.1) + (0.35 - 0.1) = \boxed{0.75} \end{aligned}$$

(e) *Venn diagram*



epr023

2 We know

$$P(A) = 0.4 \qquad P(B) = 0.15 \qquad P(\overline{A \cup B}) = 0.55$$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Negation

$$P(\overline{A}) = 1 - P(A)$$

(a)

$$\begin{aligned} P(\text{either condition}) &= P(A \cup B) \;=\; 1 - P(\overline{A \cup B}) \\ &= 1 - 0.55 \;=\; 0.45 \end{aligned}$$

(b)

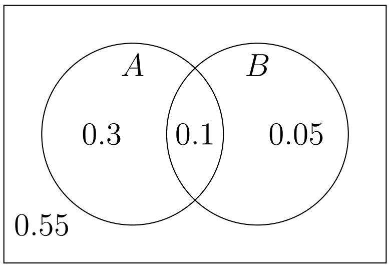
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(\text{both conditions}) &= P(A \cap B) \\ &= P(A) + P(B) - P(A \cup B) \\ &= 0.4 + 0.15 - 0.45 \;=\; 0.1 \end{aligned}$$

(c)

$$\begin{aligned} &P(\text{iron deficient and not ear-nose-throat}) \\ &= P(A \cap \overline{B}) \;=\; P(A) - P(A \cap B) \\ &= 0.5 - 0.1 \;=\; 0.3 \end{aligned}$$

(d)

$$\begin{aligned} P(\overline{A} \cap B) &= P(B) - P(A \cap B) \\ &= 0.15 - 0.1 \;=\; 0.05 \\ P(\text{exactly one condition}) &= P(A \cap \overline{B}) + P(\overline{A} \cap B) \\ &= 0.05 + 0.30 \;=\; 0.35 \end{aligned}$$



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3 Given

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad P(A \cap B) = 0.25$$

Addition Law

$$P(A \cup B) \;=\; P(A) + P(B) - P(A \cap B)$$

Negation

$$P(\overline{A}) \;=\; 1 - P(A)$$

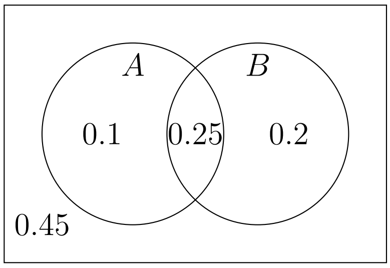
(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35 + 0.45 - 0.25 = 0.55$

(b) $P(\overline{A}) = 1 - P(A) = 1 - 0.35 = 0.65$

(c) $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.55 = 0.45$

(d) $P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.25 = 0.75$

It is useful to summarise what we know in a Venn diagram.



epr001

4 (a) $P(\text{black queen}) \;=\; P(\{Q\clubsuit, Q\spadesuit\}) \;=\; \frac{2}{52} \;=\; \frac{1}{26}$

(b)
$$\begin{aligned} P(7,8,9) &= P(\{7\heartsuit, 8\heartsuit, 9\heartsuit, 7\diamondsuit, 8\diamondsuit, 9\diamondsuit, 7\clubsuit, 8\clubsuit, 9\clubsuit, 7\spadesuit, 8\spadesuit, 9\spadesuit\}) \\ &= \tfrac{12}{52} \;=\; \tfrac{3}{13} \end{aligned}$$

(c) $P(\text{red card}) \;=\; \frac{26}{52} \;=\; \frac{1}{2}$

(d)
$$\begin{aligned} P(\text{black ace or red queen}) &= P(\{A\clubsuit, A\spadesuit, Q\heartsuit, Q\diamondsuit\}) \\ &= \tfrac{4}{52} \;=\; \tfrac{1}{13} \end{aligned}$$

epr009

5

Mutually exclusive $$A \cap B \;=\; \emptyset$$ i.e., they have *no common outcomes*.

(a) $\boxed{\text{no}}$ — not mutually exclusive
$A \cap B = \{(H, H)\}$

(b) $\boxed{\text{yes}}$ — mutually exclusive
$A \cap B = \emptyset$

(c) $\boxed{\text{no}}$ — not mutually exclusive
$A \cap B = \{(2, 3), (3, 2)\}$

(d) $\boxed{\text{no}}$ — not mutually exclusive
$A \cap B =$Any hand of five cards with no aces but at least one spade, e.g.,

$$(2\spadesuit,\ 3\spadesuit,\ 4\spadesuit,\ 5\spadesuit,\ 6\spadesuit)$$

epr002

6 If events A and B were *mutually exclusive* then $P(A \cup B) = P(A) + P(B)$.
We know $P(A \cup B) \leq 1$ (always) but here $P(A) + P(B) = 1.2$.
So $P(A \cap B) \geq 0.2 > 0$ and therefore A and B are not mutually exclusive.

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