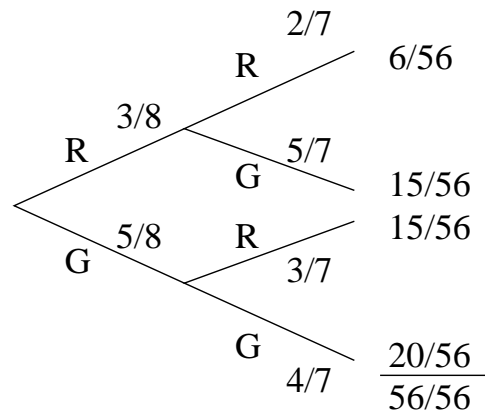


- 1 (a) Sample space $S = \{RR, RG, GR, GG\}$

*Note that these outcomes are **not** equally likely.*

Therefore, a tree diagram will be helpful here.



- (b) Possible values of X are $\{0, 1, 2\}$ green marbles.

$$P(X = 0) = P(RR) = \frac{6}{56}$$

$$P(X = 1) = P(RG) + P(GR) = \frac{15}{56} + \frac{15}{56} = \frac{30}{56}$$

$$P(X = 2) = P(GG) = \frac{20}{56}$$

We can summarise the probability distribution in the table below.

x_i	0	1	2
$P(X = x_i)$	$\frac{6}{56}$	$\frac{30}{56}$	$\frac{20}{56}$

- (c)

$$E(X) = 0 \times \frac{6}{56} + 1 \times \frac{30}{56} + 2 \times \frac{20}{56}$$

$$= \frac{0 + 30 + 40}{56}$$

$$= \frac{70}{56} = \frac{5}{4}$$

$$E(X^2) = 0^2 \times \frac{6}{56} + 1^2 \times \frac{30}{56} + 2^2 \times \frac{20}{56}$$

$$= \frac{0 + 30 + 80}{56}$$

$$= \frac{110}{56} = \frac{55}{28}$$

$$\begin{aligned}\text{var}(X) &= E(X^2) - (E(X))^2 \\ &= \frac{55}{28} - \left(\frac{5}{4}\right)^2 = \frac{45}{112}\end{aligned}$$

epr028

2 (a) Value of X for each outcome

red	green					
	1	2	3	4	5	6
1	0	-1	-2	-3	-4	-5
2	1	0	-1	-2	-3	-4
3	2	1	0	-1	-2	-3
4	3	2	1	0	-1	-2
5	4	3	2	1	0	-1
6	5	4	3	2	1	0

(b) Probability distribution

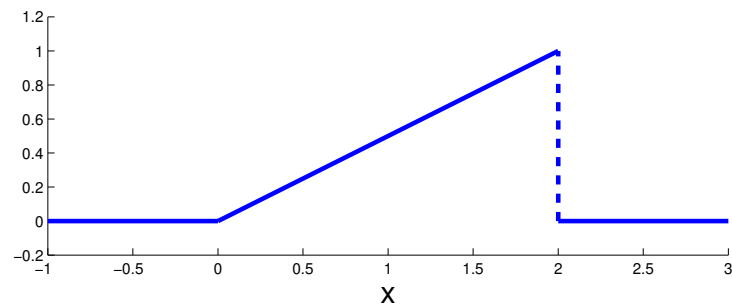
x_i	-5	-4	-3	-2	-1	0	1	2	3	4	5
$P(X = x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(c)

$$\begin{aligned}E(X) &= (-5) \times \frac{1}{36} + (-4) \times \frac{2}{36} + (-3) \times \frac{3}{36} + (-2) \times \frac{4}{36} \\ &\quad + (-1) \times \frac{5}{36} + (0) \times \frac{6}{36} + (1) \times \frac{5}{36} + (2) \times \frac{4}{36} \\ &\quad + (3) \times \frac{3}{36} + (4) \times \frac{2}{36} + (5) \times \frac{1}{36} \\ &= 0 \\ E(X^2) &= (-5)^2 \left(\frac{1}{36}\right) + (-4)^2 \left(\frac{2}{36}\right) + (-3)^2 \left(\frac{3}{36}\right) + (-2)^2 \left(\frac{4}{36}\right) \\ &\quad + (-1)^2 \left(\frac{5}{36}\right) + 0^2 \left(\frac{6}{36}\right) + 1^2 \left(\frac{5}{36}\right) + 2^2 \left(\frac{4}{36}\right) \\ &\quad + 3^2 \left(\frac{3}{36}\right) + 4^2 \left(\frac{2}{36}\right) + 5^2 \left(\frac{1}{36}\right) \\ &= \frac{25 + 32 + 27 + 16 + 5 + 0 + 5 + 16 + 27 + 32 + 25}{36} \\ &= \frac{210}{36} = \frac{35}{6} \\ \text{var}(X) &= E(X^2) - (E(X))^2 = \frac{35}{6} - (0)^2 = \frac{35}{6}\end{aligned}$$

epr031

3 (a) Graph of $f(x)$.



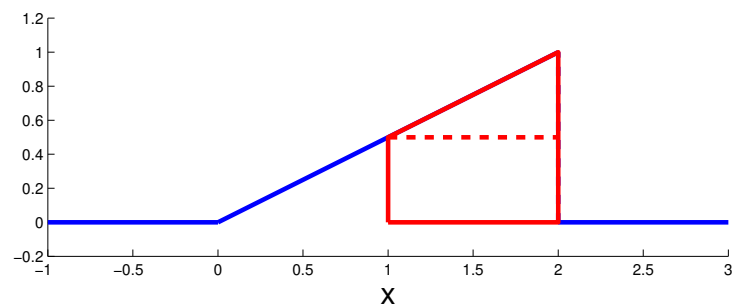
(b)

$$\begin{aligned}
 \text{Area under } f(x) &= \text{Area of the triangle} \\
 &= \frac{1}{2} \times \text{base} \times \text{height} \\
 &= \frac{1}{2} \times 2 \times 1 \\
 &= 1
 \end{aligned}$$

(c) Checklist for pdf:

- Show $f(x) \geq 0$ for all x
☒ Yes our $f(x) \geq 0$ for all x
- Show $\int_{-\infty}^{\infty} f(x) dx = 1$
☒ Yes for our $f(x)$ we have shown that the area under $f(x)$ is 1

(d) $P(X \geq 1)$ is the area of the red trapezium below.



$$\begin{aligned}
 P(X \geq 1) &= \text{Area red trapezium} \\
 &= \text{Area rectangle part} + \text{Area triangle part} \\
 &= 1 \times \frac{1}{2} + \frac{1}{2} \times 1 \times \frac{1}{2} \\
 &= \frac{3}{4}
 \end{aligned}$$

epr036a