Coventry University
School of Computing, Electronics and Mathematics

5005CEM **Feedback for Probability Problem Sheet 2b** **Week 6**

1

> *Uniform distribution*: $X \sim U(a,b)$
>
> $$E(X) = \frac{a+b}{2}$$
> $$\text{var}(X) = \frac{(b-a)^2}{12}$$

(a)
$$E(X) = \tfrac{(-2)+(8)}{2} = 3$$
$$\text{var}(X) = \tfrac{(8-(-2))^2}{12} = \tfrac{100}{12} = \tfrac{25}{3}$$

(b)
$$P(-3 < X < 5) = \tfrac{5-(-2)}{8-(-2)} - 0 = \tfrac{7}{10} - 0 = 0.7$$

(c)
$$P(X > 6) = 1 - P(X \le 6) = 1 - \tfrac{6-(-2)}{8-(-2)} = 1 - \tfrac{8}{10} = 0.2$$

epr050

2 Let X be the time waiting for the bus.

(a) $P(X > 10) = \tfrac{20}{30} = \tfrac{2}{3}$

(b) We wish to find $P(X > 25 \mid X > 15)$.

> *Definition of conditional probability*
>
> $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(X > 25 \mid X > 15) = \frac{P(X > 25 \;\cap\; X > 15)}{P(X > 15)}$$
$$= \frac{P(X > 25)}{P(X > 15)} = \frac{(\frac{5}{30})}{(\frac{15}{30})} = \tfrac{1}{3}$$

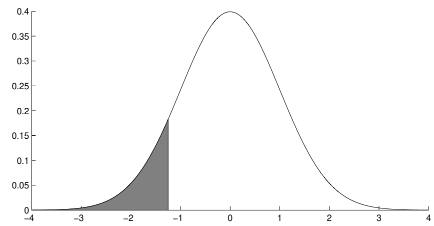
epr053

3 $c = 1$ and $d = 9$

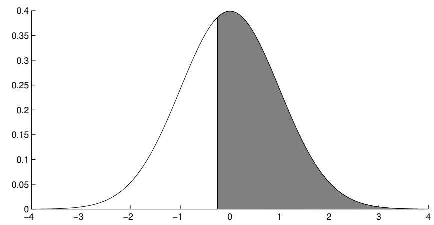
epr051

4 Let X be the mass of the small load of bread.

(a) `pnorm(475,500,20)`
gives $P(X \leq 475) = 0.1056$ (4dp).

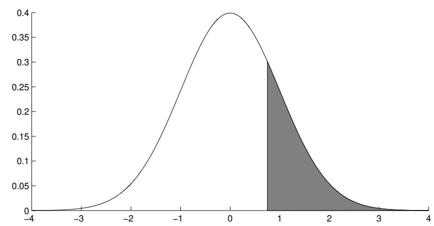


(b) `1-pnorm(495,500,20)`
gives $P(X \geq 495) = 0.5987$ (4dp).



(c) `pnorm(510,500,20)`
gives $P(X \leq 510) = 0.6915$ (4dp).

(d) `1-pnorm(515,500,20)`
gives $P(X \geq 515) = 0.2266$ (4dp)



epr043a

5 (a) `pnorm(70,75,12) - pnorm(60,75,12)`
gives probability 0.2328113 (or 0.2328 (4dp)).

(b) `qnorm(0.85,75,12)`
gives value 87.4372, i.e., the cutoff introvert-extrovert score would be 87.4 (1dp). epr046

6 (a) `1-pnorm(75,60,15)`
gives probability 0.1586553 (or 0.1587 (4dp)).

(b) `qnorm(0.4,60,15)`
gives value 56.19979, i.e., the cutoff between small and medium eggs would be 56.2g (1dp).

epr047

7 (a) `1-pnorm(127,100,15)`

gives probability 0.03593032 (or 0.0359 (4dp)).

(b) `qnorm(0.10,100,15)`

gives value 80.77673 (or 80.8 (1dp))

(c) We wish to find the conditional probability $P(L > 133 \mid L > 127)$.

Definition of conditional probability $$P(A \mid B) \;=\; \frac{P(A \cap B)}{P(B)}$$

$$P(L > 133 \mid L > 127) \;=\; \frac{P(L > 133 \;\cap\; L > 127)}{P(L > 127)} \;=\; \frac{P(L > 133)}{P(L > 127)}$$

$P(L > 133)$ is calculated by `1-pnorm(133,100,15)`

so the answer is calculated using

`(1-pnorm(133,100,15))/(1-pnorm(127,100,15))`

giving probability 0.3869559 (or 0.3870 (4dp)).

epr048

8 (a) `pnorm(224,232,5)`

gives probability 0.05479929 (or 0.0548 (4dp)).

(b) `qnorm(0.7,232,5)`

gives value $f = 234.622$ (or $f = 234.6$ (1dp)).

(c) Suppose the jars are labelled "jar A" and "jar B". Let A be the event that the weight of coffee in jar A is between 232 and f grams. Let B be the event that the weight of coffee in jar B is between 232 and f grams.

For exactly one of the jars to contain between 232 and f grams of coffee, it must either be jar A or jar B but not both. Since the jars are selected at random, the events A and B are **independent**.

$$\begin{aligned} P(\text{exactly one}) &= P(A \cap \overline{B}) + P(\overline{A} \cap B) \\ &= 0.2 \times 0.8 + 0.8 \times 0.2 \\ &= 0.32 \end{aligned}$$

epr049

9

Uniform distribution: $X \sim U(a,b)$

$$E(X) = \frac{a+b}{2}$$
$$E(X^2) = \frac{b^3-a^3}{3(b-a)}$$

(a) Here $R \sim U(1,3)$ so $E(R) = 2$.

(b) Here $A = \pi R^2$

$$E(A) = \pi E(R^2) = \pi \left(\frac{(3)^3 - (1)^3}{3((3)-(1))} \right) = \tfrac{13\pi}{3}$$

Expected area of the circle is $\frac{13\pi}{3}$ cm^2.

Alternatively, find $E(R)$ and var(R) in order to calculate $E(R^2)$.

Note that $E(A) \neq \pi(E(R))^2$. epr052