Lösungen von Übungsblatt 12
Funktionale Programmierung (Prof. Dr. Margarita Esponda)

Tutorium: Zachrau, Alexande; Dienstag; 12:00 - 14:00

Boyan Hristov und Luis Herrmann

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Aufgabe 1	Aufgabe 2	Aufgabe 3	Aufgabe 4
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Aufgabe 2

Zu zeigen: $length\ (powset\ xs) \equiv 2^{length\ xs}$

Beweis per strukturelle Induktion

IA:

$length\ (powset\ [])$	$2^{(length\ [])}$
$\|\|powset\ 1$	$\|\|length1$
$length\ [[]]$	2^0
$\|\|$	$\|\|$
1	1

IV:
IV 1: $length\ (powset\ xs) \equiv 2^{length\ xs}$
IV 2: $length\ xs \equiv length\ [z|x <- xs]$

IS:

$length\ (powset\ x : xs)$

$||powset1$

$length\ (powset' ++[x : ys|ys <- powset'])$

$||2.Induktion$

$(length\ powset') + (length\ [x : ys|ys <- powset'])$

$||powset'$

$(length\ xs) + (length[x : ys|ys <- xs])$

$||IV2$

$(length\ xs) + (length\ xs)$

$2.(length\ xs)$

$||IV1$

$2.2^{length\ xs}$

$||2.2^n = 2^{n+1}$

$2^{(length\ xs)+1}$

$||length2$

$2^{length\ (x:xs)}$

Zu zeigen: $length\ (xs + +ys) \equiv (length\ xs) + (length\ ys)$
Beweis per strukturellen Induktion
IA:
$length\ ([] + +ys) \equiv length\ ys \equiv length\ ys + 0 \overset{length\ 1}{\equiv} legth[] + length\ ys$
IV:
$length\ (xs + +ys) \equiv (length\ xs) + (length\ ys)$
IS: $xs- > x : xs$

$length\ ((x : xs) + +ys)$
$||(:)\ Operator$
$length\ ([x] + +xs + +ys)$
$||(:)\ Operator$
$length\ (x : (xs + +ys))$
$||length2$
$1 + length\ (xs + +ys)$
$||IV$
$1 + length\ xs + length\ ys$
$||length2$
$length\ (x : xs) + length\ ys$

Aufgabe 3

Zu zeigen: Leaves (Node lt rt) $\equiv$ SumLeaves (Node lt rt)
Beweis per strukturellen Induktion:

IA:
sum Leaves (Leaf x) $\overset{?}{\equiv}$ (sumNodes lt) + 1

||sumNodes 1

||sumLeaves 1 0 + 1

1 ||

1

IV:
sumLeaves lt $\equiv$ (sumNodes lt) + 1
sumLeaves rt $\equiv$ (sumNodes rt) + 1
IS:

sumLeaves (Node lt rt)
||sumLeaves 2
(sumLeaves lt) + (sumLeaves rt)
||IV. 1, IV. 2
(sumNodes lt) + 1 + (sumNodes rt) + 1
||sumNodes 2
sumNodes (Node lt rt) + 1