国外优秀数学教材选评

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目录

1. 序言 ¶

1.1 数学与数学教材

数学是科学的一个重要工具，这已经是老生常谈的一个常识了。从中小学、大学直到研究生，数学课程始终占据显著的位置。数学学科是庞大的，包含的分支很多，而且随着时间的推移，人类对数学的认识越来越深刻，数学的内容也越来越丰富，新的数学分支也时常产生。然而，尽管数学学科在不断的发展，它的基本原理是相对稳定的。如果把现在的大学数学和 50年前的大学数学作比较，就会发现基础性的内容是差不多的，那时的很多优秀数学书籍现在仍然奉为经典。这是数学和一些新兴学科的一个显著区别。

数学大致分作两类：基础数学和应用数学。基础数学也叫做纯数学或理论数学，它是根据数学本身的需要而发展的。应用数学是在纯数学的基础上产生的各种具有不同程度的应用性的各种学科，这是数学和其他学科如物理、化学、计算机科学、经济学等的桥梁。

大学数学课程按学生的专业可以分成两大类：数学专业的和非数学专业的。按程度又分本科生课程和研究生课程两大类。数学专业本科生有低年级的基础课程和高年级的专业课程选修课程。低年级的基础课程主要包括数学分析、线性代数、复分析、微分方程、抽象代数、复变函数、实变函数、泛函分析等。非数学专业的本科生数学基础课程通常称‘高等数学’，内容以微积分、线性代数和微分方程为主，只是比数学专业学生学习的内容要浅些。非数学专业的学生在数学课程中接受的训练主要是计算和应用的能力，而数学专业的学生主要接受数学推理的能力训练。

学习数学的最主要的途径是看书。数学书籍大凡可以分教科书、学术专著和通俗读物三种。差不多所有数学分支都有一些不同深度的教科书。

1.2 如何选择合适的教材

对于在读的大学生或研究生，不需要化太多心思选择教材，只要用老师指定的教材就可以了。特别是如果在所修的课程中你感觉学到比较扎实，习题基本会做，那也不一定去看太多其他同类的教材。对于学生来说，参考书是双刃剑，一方面它可以开拓视野，加深对所学知识认识的深度，另一方面，由于不同的作者写书的构思不同，内容安排的次序也可能不同，甚至所用的术语也有区别，同时看几本同类的书会造成混乱。所以建议在下面两种情形下去寻找合适的参考书：1）觉得课堂上用的教材太难，大部分习题不会做，这时可找一本浅一点的或者对基本概念解释得更仔细一点的书。2）能轻松对付课堂内容，又对该课程有浓厚兴趣，这时可请老师推荐更深一些的教材。

前面我们提到过数学教材分数学专业和非数学专业教材两大类。它们的差异是巨大的，例如同样书名是 Linear Algebra的教材，工科的学生觉得数学系的教材的叙述太简洁，例子太少，图形少，计算题少，证明太多，习题太难。而数学系的学生觉得工科线性代数教材不严格，解释性的话太多，应用性的例子太简单。所以非数学专业的读者选择数学专业的数学教材要格外慎重，首先要判断其内容和深度是否是你确实需要或你感兴趣的，然后再估量一下你的数学基础够不够，千万不可勉强。

对于自学数学的同志，选择合适的教材是十分关键的，千万不要随便抓起一本书就念。选错书是会走很多弯路的。对于初学者，光看书名、目录和序言是很难准确地判断这本书是不是适合于你，需要仔细看看里面的内容。可以到书店去浏览，有些书店有很多品种的数学书，但是有很

多最好的书籍在书架上是没有的，因此图书馆是一个更好的选择。也可以在互联网上搜索，当然身边有高手指点就再好不过了。

1.3 外国数学教材

中国国内有不少好的数学教材，为什么还需要外国的教材呢？从中学数学教材到大学低年级的教材来看，光用国内的教材已经够了，但是越到高的层次，对国外教材的倚靠就越明显了。不仅要使用翻译的教材，还要使用原版的。从语种来看，英语最为重要。世界上的数学大国是美国、俄罗斯、德国、法国、英国，这五个国家堪称数学超级大国。意大利、日本、印度和东欧诸国的数学也很强。中国虽然出些数学人才，但是和数学五大强国比差距仍不小，我们得摆正位置，老老实实学习人家先进的东西。日本和印度的数学家历来用英文写作。前苏联的数学教科书在 60年代对我国起很大影响，当时会俄文对学数学很有利。几十年前，非英语国家的数学教科书都用本国文字写。在当今的信息时代，英语几乎成了世界语，在数学中也不例外，连法国德国的数学家也经常用英文写作，在加上美国的数学界化了相当大的人力物力翻译数学名著。对于数学工作者来说，只懂英语一种外语也够了。

国外有几家著名的出版商如德国的 Springer Verlag, 美国的 Academic Press, 美国数学会 (AMS) 和一些名校如英国的牛津、剑桥，美国的 Princeton大学的出版社都是数学教材大户。非数学专业用的数学书的出版商不象基础数学那样集中，多数由一些综合性的出版社如 John Wiley, Prentice Hall, McGraw-Hill 等出版。

数学类书籍的领头羊当数 Springer Verlag，它有很多系列丛书，主要给数学专业使用，最有名的几种是

1）GTM, 即 Graduate Texts in Mathematics, 至今以出版了 200 多种，覆盖面很广，但多数是基础数学方面研究生教材。

2） UTM, 即 Undergraduate Texts in Mathematics, 该系列比上面系列出现得晚一些，也没有列序号，因此品种也略少一些，似乎只有几十中，大部分是本科生数学教材。

3） Universitext, 这是 Springer Verlag 另一套无序列号的数学丛书，以数学系高年级的教材为主，里面不乏好书。

4） LNM, 即 Lecture Notes in Mathematics, 这是规模最大的丛书，以专著和研究生课程的讲义为主，现已有几千种。

Springer 还有几个系列非常专门，这里就不介绍了。 Springer 的数学书差不多都是醒目的黄色封皮，印刷和装订都很考究。书的数学质量也很高，很受读者欢迎。但是大部分书都适合于有较好数学训练的人阅读，建议我国数学系研究生和高年级本科生使用。

供大学生阅读的数学课外读物历来比较少。美国数学会在几年前推出的简装的系列丛书 Student Mathematical Library 倒是针对大学生的。 大部分书不到 200 页，选材比较有趣，非常有特色。美国数学会仿效 Springer Verlag, 也出版一套黄封面的研究生数学丛书，里面不乏好书。此外，美国数学会有一个翻译书系列，以俄罗斯和日本的数学译著为主，多数是研究性的专著，但也有一些高质量的教材。另一套值得推荐的系列丛书是伦敦数学会的 Student texts, 对象以数学专业高年级大学生和研究生为主，每本的篇幅为 200 页上下，内容覆盖的范围很广，基础数学方面的更多一些。

美国数学协会（Mathematical Association of America）的刊物 American Mathematical Monthly 是大学生（甚至低年级的大学生）可以看懂的数学刊物，和我国的“数学通报”相仿，但它的知名度和文章质量远高于“数学通报”，里面大量的文章可以作为大学数学系学生的课外读物。它每期都有一些竞赛性质的趣题。

下面谈谈供非数学专业使用的外国数学教材，其中最重要的是 Calculus. 美国的微积分教材品种很多，根据对象不同深浅也不一样，正象我国的高等数学课程分理工类、医学类、经济类等等一样。美国的微积分 教材篇幅很大，一本书一般都在 600 页上下，而且是大开本的。由于这是出版量最大的数学教材，印刷非常考究，校对也仔细，所以错误极少。我国的高等数学教材大部分比较简洁，其优点是信息量大，缺点是不利于自学。美国的微积分教材一般浓度不大，非常注意由浅入深，描述和解释性的话比较多，特别注意讲解实例，习题也很丰富，一般的读者只要没有英语方面的问题读起来是很快的。其他大学低年级的公共数学课程的教程也有这样的特点。这些教材之所以篇幅越来越大有商业上的原因，它们有更多的选学章节和附录供使用者选择，这样的书销路会好一些。它带来的负面作用是容易使自学者抓不住重点，所以我国读者使用这些教材需要注意，先在序言中看清作者写书的构思和意图，有的作者还给出导读表。有重点地阅读能更有效地掌握该课程的精髓。

1.4 外国数学教材的来源

自改革开放以来，我国在引进外国教材方面作出了巨大的努力。教委和科学院每年都花费大量外汇购买各种原版科技书籍。然而这些原版书价格非常昂贵，一般的读者很难承受，多数由图书馆采购，按我国现在的条件，一般大专院校的原版书的数量是非常有限的。

近十几年来，我国的一些出版单位如世界图书出版公司、高等教育出版社、机械工业出版社等购买了国外一些大的出版公司的部分书刊的版权后在中国影印出版，其价格只及原版书的五分之一到十分之一，种类也越来越多，这是喜欢外国教材的读者的一个重要书源。

随着信息时代的到来，电子书籍成了一个最诱人的书源。虽然数学电子书不象文科书籍那样容易在互联网上找到，但是它们的数量也是以惊人的速度增加。例如 Springer 在网上提供了它的全部电子出版物的收费网上资源，供集团订购。我国若干高校的图书馆（如清华、复旦）已经订购，那些学校的师生可以在所在校园自由下载。象它的系列丛书 GTM, UTM 自 1997年来的教材几乎全部可以下载。

近年来出现的网上维基百科全书（Wikipedia）已成为炙手可热的工具。从它的数学内容来看，覆盖面相当广，搜索非常容易，在 Google 主页上键入一个学术名词一般第一条就是 Wikipedia 的条目，可见其点击率之高。虽然作者是匿名的，但从绝大部分条目来看可以发现这些作者相当专业，准确性非常高，文章短小精悍，引用的参考资料对读者非常有用，不由得对这些作者的敬业精神表示敬意。

在外国数学书籍的学习过程中，难免会碰到一些在你所读到书中没有解释或没有解释清楚的术语，这时最好的办法就是去请教 Wikipedia。当然， Wikipedia 只是工具，绝不能替代教科书。

1.5 美国的大学数学教学

美国的大学数学课程的内容和世界上其它国家没有太大的区别。由于学生基础差异比较大在课程的设置和学时安排方面有它的特点。由于美国在科技方面仍是超级强国，对于它的大学数学教学作一定的了解是有好处的。

1.5.1 微积分(calculus)

毫无疑问，最重要的数学课程是微积分，这相当于我国的高等数学。大部分微积分课程都包含了空间解析几何甚至简单的线性代数和微分方程。多数学校（包括一些一流的名校）的数学系学生在大一修和理科其他专业同样的微积分课程，而在后续课程中再学数学分析，这一点和我国有很大的区别。看上去有重复，实际上花的学时差不多，学起来也轻松一点，这也符合循序渐进的原则，值得我们借鉴。对于非数学专业，其后续课程按需要而设置，例如理工类的很可能开“advanced calculus”课程，内容包括数理方程、Fourier分析等，生物医学类的专业可能学概率统计。

在近代的美国数学教学中有过两次重大的改革。第一次是 50年代末和六十年代初的“新数学运动”，此前法国的 Bourbaki学派对数学发起强烈的冲击，把公理化数学推向顶峰，产生一系列以前不可想象的重大成果。而历来美国的数学教育落后于欧洲先进国家。部分有声望的美国数学家发起了新数学运动，把一些抽象数学概念灌输给中小学生。当时也产生一些写得很通俗的中小学教材，使学生很早就学集合和映射，培养他们数学的理性思维。然而结果却差强人意，简直可以用“大失败”来形容。这不能怪那些发起改革的数学家，更不能怪 Bourbaki，只能说在美国当时不具有这样的条件。有人怀疑能教这些中小学新教材的中小学教师是否合格就是一个大问题。就像大革命失败后有一个反动的白色恐怖期一样，新数学运动后中小学的数学教学便变本加厉地复旧，使初等数学课本变的超级简单，甚至把平面几何的证明题从中学教材中全部砍掉。美国入大学比中国容易的多，只要肯付学费好赖能进个大学，这些中学生进入大学后大多数人见数学要头疼的。

80 年代的统计显示每年 30万修工科微积分课程的大学生中只有 14 万及格。美国的科技和教育界深刻认识到这个重大问题并痛下决心解决，于是在 80 年代末美国国家自然科学基金会出资一千一百万美元设立一个微积分改革的项目，这是美国近代的第二次数学教学改革。该项目是一个成员众多的集体项目，由哈佛大学牵头，除了一些大学数学教授外也吸收一批各应用领域（如物理、计算机科学、各类工程学、经济学、生物学、各类人文科学）的专家参与，参与此项目的最显赫的人士可数费尔兹奖得主 David Mumford 。第一个引人注目的就是一套教材，俗称哈佛教材，即我们书评中微积分的第一本教材。和新数学运动相反，哈佛教材所教的仍是传统微积分的内容，只是在强调的重点上不同。它突出对微积分在概念方面上的自然理解以及在各学科中的应用，在相当大的程度上放弃数学的严格性。全国有很多大学投入这项改革，包括哈佛、杜克、密歇根等一些名校，据 1999 年代统计在美国有五百多所大学和大专投入微积分改革运动，每年修改革微积分的人数达 30万，占修微积分课程学生总数的百分之三十二。那些院校除了使用哈佛教材外还新建专门的计算实验室。不少大学同时开改革的微积分和传统微积分两门课程供学生选修，经比较后调查的结果显示占一半的院校修改革微积分的学生比传统微积分好，另百分之四十说差不多，百分之十说不好。

在美国数学界对微积分改革和哈佛教材也有各种不同看法，甚至有人把它贬得一钱不值，特别一些理工类名校不屑使用这套教材。有个别原先参与改革的院校如加州大学洛杉矶分校后来退出了改革。

1.5.2 线性代数(linear algebra)

近五十年来，线性代数也成了大学低年级的热门课程。和微积分一样，美国的线性代数也分两步走，先学线性代数第一教程，再学它的后续课程。第一教程是面向各专业的学生的，很多大学数学系的学生也学第一教程。

1990 年十多个美国大学教授在美国国家自然科学基金会资助下开了五天会专门讨论线性代数第一教程的改革，会后向数学教育界提出五条建议（见five）。我们选一些要点概述如下：作为公共基础课程的线性代数的大纲应优先考虑授课对象的需求。需要学线性代数的学科主要有：计算机科学、电子工程、航天工程、系统工程、物理学、经济学、统计学、运筹学等。同时也得考虑少数修此课的数学专业学生的需求。由于相当数量的一部分学生不再修它的后续课程，本课程必须有一定的完整性。线性代数的应用的讲解是必要的，但要简明，使个不能专业的学生都能听懂。课程的深度按学生的数学基础来定。建议此课程以矩阵为主，而不是以抽象的线性空间和线性变换为主，这有利于培养学生的线性代数计算和应用能力，这和培养数学系的学生并无冲突。课程的核心内容如下：

1）（3 个教学日） 矩阵的加法和乘法，转置，各种运算的性质，分块矩阵的运算法则。特别要详细讲解矩阵乘法 AB 的如下解释：

i) Ax 是 A 的列的一个线性组合， AB 中每一列是 A 的列的线性组合。如果 D 是对角阵，则 AD 中的每一列是原来列的放大或缩小。如果 P 是一个置换矩阵，则 AP 的列是 A 的列的一个置换。

ii) AB 的每一行是 B 的行的线性组合，...

2）（4 个教学日）线性方程组，包括高斯消去法、初等矩阵、阶梯形矩阵、解答存在性和唯一性、逆矩阵、 LU-分解。

3）（2-3 个教学日）行列式，余子式，按行或列展开， $|AB|=|A||B|,$ Cramer 法则 。从二阶和三阶引入行列式的计算和性质，尽量避免 冗长的证明。

4）（7-8 个教学日） n 维实空间 ${\mathbb R}^n.$ 线性组合、线性相关、线性无关、基、子空间、生成元、子空间的基、矩阵的行空间、列空间、零空间、矩阵所定义的线性变换、矩阵的秩=行秩=列秩、重新解释线性方程组、秩+零空间维数=列数、内积、向量的长度、正交性、标准正交基、正交阵。不必证明所有定理。

5）（6 个教学日）特征值、特征向量、特征子空间、方阵的对角化、对称阵和它的正交对角化、二次型。

6）（4 个教学日）正交投影、Gram-Schmidt 正交化，QR-分解，最小二乘法。

总共 26-28 个教学日，余下时间可以讲授选学内容。这里的教学日只有 50 分钟的课堂时间，比我国的课时少。

参加讨论会的代表强烈推荐数学系必须设立相应的后续课程，例如抽象线性代数、矩阵分析、数值线性代数，使数学系学生有一个学年的线性代数训练。

美国大学的公共线性代数课程大致上都按上面的精神设计的，这也可以在他们使用的教材中反映出来。对此有所了解有助于我们对外国教材的选用。

目前我国大学的数学教学数学专业和非数学专业的界线过于明显。笔者认为数学分析和线性代数这两门数学系的主课可以借鉴美国的方式，每一门都分两个阶段，第一阶段学一个学期的公共课程，第二阶段学有严格证明的后续课程，不失为一种合理的安排。

1.6 本书的目标

2007年复旦大学数学学院和校图书馆外国教材中心组织一批力量对国外大学的的数学教材进行调查研究。选择一部分优秀的教材向国内同行和学生进行介绍，旨在帮助国内大专院校师生和自学数学的同志选择合适的外国教材，对于最常用的一些教材，我们对它们在国外的使用情况作

了统计和调查。所有的书都由熟悉该书内容的教师书写介绍，其中有不少书在教学中使用过，对于书的特色和难易程度都有较明确的评论。我们相信我们的选书标准是高的，所以数量相对来说不大，所覆盖的范围也并不是太广。对于我们选中的书籍，大部分都作了简评，结合中国高校的情况列出一些使用要点。为了使读者更加全面地了解所选的教材，我们还选载了一些国外读者的比较中肯的评论，不光是讲优点的评论，也有很多讲书中的不足之处的，评论者多数是使用过该书的教师和学生。

在互联网上可以查到不少热心数学人士的网页上的一些读书指导，提供一些数学好书的清单，大部分都比较简略，由于是个人行为，收集的面也有一定限制。我们尝试组织一批精通业务的专家合作也提供一些对国内师生更有用的调查资料，起个抛砖引玉的作用。由于时间和人力物力的关系，这一次选的书的数量和范围有限，我们希望这只是这个工作的一个开头，以后根据条件是可以大大扩充本书的内容的。

本书分两个部分，第一部分是非数学专业（即公共基础课）的数学教材，第二部分是数学系的教材，它们又按不同的数学分支进行编排。本科生和研究生教材就不分了，因为它们间也没有非常明确的界线。对于大部分书除了一些基本资料外都有以下几项参考指标：

适用范围，预备知识，习题数量，习题难度，推荐强度（最高是 10）。

希望这些指标对读者选书提供帮助。

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2 非数学专业的数学教材

在国内外高校中，高等数学是占课时最多的课程之一，因为几乎每个系每个专业都多少要学微积分，或许还要学线性代数、概率统计、微分方程等。这些数学和数学专业所学的数学有很大的不同，它们所强调的是计算和应用，而数学专业的学生需要学系统的理论并且训练证明定理的能力，所以数学专业的数学书籍有一定深度，不适合于工程类、医学和文科各专业的学生使用。理科有些专业（如物理、力学等）对数学的某些分支要求比较高，也可以使用数学系的教材。

我国高校的高等数学按深浅一般分几类，有的学校分 3类，有的分 4类，最低的一般是文科数学，最高的是对物理系和力学系开设的数学，国外大致上也是如此。

我们对美国的微积分教材和线性代数教材分别进行了调查研究，各自精选了十本左右有影响力或使用院校比较多的教材向读者介绍。我们列举的使用院校是根据非完全的统计，仅供读者选书时参考。

2.1 微积分

微积分是大学数学最基本也是最重要的课程，可以毫不夸张地说高中的数学教育的目标就是为微积分打基础。从历史上看，牛顿发明微积分是为了解决当时物理学不能解决的问题，这形成了数学的一个飞跃，随着数学的发展，为微积分建立严格的理论基础成为一个迫切的任务，经过数学家们不懈的努力，在 19 世纪就形成非常严格的微积分理论，被称为数学分析。现在国内大学数学系学的“微积分”大部分就叫“数学分析”。而非数学系大学生学的“微积分”则含在一门叫高等数学的课程中。

在英语国家中是没有 Advanced mathematics 这门课的，他们的 Calculus 课程对应我们的高等数学，他们的 Mathematical analysis 或 Advanced calculus 对应我们数学系学的数学分析。还有些书名含 Real analysis 这词组，这就要看书的内容了，有可能是数学分析，也可能是比数学分析更深的实变函数论。如果一本书名是 Vector analysis, 则它就是讲多变量的微积分，相当与我们高等数学后半部分的内容。

1) Calculus, third edition
作者：Hughes-Hallet,Gleason,McCallum et al.
出版商：John Wiley & Sons, Inc. (2002) ISBN 0-471-40826-3
页数：623
适用范围：理工类大学本科生微积分教材
预备知识：高中数学
习题数量：大
习题难度：低
推荐强度： 9.3
使用学校：
Duke University, University of California at San Diego, Northern Michigan University, University of Cincinnati, University of California at Merced, Virginia Polytechnic Institute and State University, University of Massachusetts at Amherst, Florida State University, Georgia Institute of Technology, Harvard University, Oklahoma State University, Sonoma State University, St. Louis University, Winona State University, University of Rhode Island, Berea College, The

University of Arizona Jacksonville State University, Willamette University, Arizona State University, Western Oregon University, University of South Carolina, Marquette University, Western Washington University

书评：在美国非数学专业的微积分教材中 Thomas 的 Calculus 统治了很多年， 80 年代我在美国任教时这是指定的标准教材。虽然该教材不断修改和再版，但这么多年由一本教材垄断并非正常。Hughes-Hallet,Gleason,McCallum 等一批有志于微积分教材改革的人士合力推出这本全新的微积分教材，简称为哈佛微积分，这是一套受美国国家自然科学基金会重金支持的教材。

本书的内容和传统的微积分没有任何不同，但是更突出重点。象交响乐的一个乐章里有陈述部、展开部和再现部一样，本书对每一个最重要的概念从不同的角度反复讲解，这种一唱三叹的方法很容易让初学者抓住重点。这正是作者提到的“三步法则”(Rule of three): 图像、数值和解析式。另一个特点是降低微积分计算部分的要求而重视对基本概念和方法的正确理解，作者认为用大白话 (plain English)来理解数学比记住一些公式更重要。所以，象极限、导数、积分这些概念的第一次出现都用大量精心设计的文字、生动的图象来解释，然后再用一系列实例来展示其威力，最后在选学内容中写出精确的定义。

本书的另一特色是习题的多样性，应用题的数学很简单，但涉及各科学，特别在生物、医学、经济和人文科学中的应用的习题数量很多，这是以前的微积分教材所没有的，在学习和做题过程中学生可以在早期就建立数学建模的思想。

本人在 2003 年在美国使用此教材教过一学期，学生程度参差不齐，即使基础较差，凡用功的学生都能达到本教材的基本要求。经过实际使用，本人体会到作者在此教材上倾注的心血。错误极少，虽然是多人合作，但章节间的衔接非常自然。本书还配有习题详解 Instructor's solutions manual, 760 页和概念测验 Concept tests 306 页。目前已被包括哈佛大学、杜克大学在内的一批大学定为大一微积分教材。

本书还有后续本：多变量微积分，仍保持原来的风格，主编的次序改变了，编委名单也有一些改动，菲尔兹奖得主 David Mumford 的名字出现在编委名单中。（杨劲根）

国外评论摘选

i) This is not the classic calculations approach to the subject. It is a totally new way of thinking and mastering the subject with out having to do page upon page of number crunching. Use this book along with a graphing calculator and you too can learn to literally see what happens when equations are manipulated. A begining student conceptually gains an understanding of the subject with out getting bogged down in plugging and chugging and derivations. It's written in plain English.

ii) The authors of this text dislike the "plug and chug" methods of other texts, possibly necessitating an instructor more strongly than other books. The book stresses graphs and "real life" applications, making it more realistic and less abstract than other Calc books may seem. Contains useful formulas and rules on inside covers and selected answers section at the back. Overall a great book to use in class.

2)书名:Calculus 系列书

作者:James Stewart

出版商:Thomson/Brooks/Cole

适用范围:非数学专业大学一年级

预备知识:高中数学
习题数量:大
习题难度:从容易到中等都有
推荐强度: 9

书评:

Stewart 的教材以前我不了解，这次调研外国高等数学教材的过程中发现了他的书的使用率是在各同类教材中名列前茅的。仔细查查，他一个人大约写了八本不同的微积分教材，应该是针对不同对象的，或者说分 A,B,C,...类的。我翻阅的一本是 Calculus 第五版，一千一百多页，包含多重积分和二阶常系数线性微分方程。我的印象是：这是一本朴实无华的相当标准的教材，包含了理工科一年级大学生应该学习的所有内容，在很多关键章节的写法是很细致的。应用题很多，但以物理中的应用为主，多少算是还微积分的本来面目，很多章节后还有一些供学生培养独立研究能力的课题，如彩虹的原理，电影院里座位的视角分析等。在单变量微积分和多变量微积分之间插了几章关于空间解析几何，其数量比较恰当。

下面列举这个系列中的五本书的使用院校情况。（杨劲根）

i) Calculus : early transcendentals （2003 第五版）

使用学校(30多所)：

University of California at Berkeley, Columbia University, Saint Joseph's University, Louisiana University, Salisbury University, University of Minnesota, Rensselaer Polytechnic Institute, California State University at Channel Islands, University of Massachusetts at Amherst, San Joze State University, Michigan State University, Tufts University, University of Michigan at Ann Arbor, University of Virginia's College at Wise, University of California at San Diego, Loyola University at Chicago, Tennessee Technological University, College of Charleston, Asheville Buncombe Technical Community College, University of West Georgia, Georgia University at South Bend, Purdue University, University of Washington, Florida State University, California State University, Indiana University, Southeast Grinnell College, Carnegie Mellon University, Vanderbilt University, Dartmouth College, California State University at Dominguze Hills, Idaho State University, Athabasca University in Canada, The University of Texas At Austin, University of Southern California, University of Pennsylvania, California Polytechnic State University

ii) Single variable calculus （2003 第五版）

使用学校(20多所)：

Hunter College of CUNY, Louisiana State University, Florida Atlantic University, University of Illinois at Urbana-Champaign, College of Charleston, Johns Hopkins University, Wake Forest University, Emory University, Florida State University, California State University at Stanislaus, Boise State University, University of Washington, The University of Western Ontario, Stony Brook State University of New York, College of the Holy Cross, San Diego State University, Oberlin College, University at Albany, State University of New York, Loyola College in Maryland, University of Missouri-Columbia, Saginaw Valley State University, Duquesne University, Rivier College

iii) Multivariable calculus （2003 第五版）

使用学校(20多所)：

Harvard University, Hobart and William Smith college, California state University at Dominguez Hills, University of Minnesota, University of Michigan, University of Connecticut, Rustgers the State University of New Jersey, University at Buffalo, Temple University, University of Minnesota at Duluth, Brown University, Kennesaw State University, Klarkson University,

Binghamton University, Boise State University, University of Colorado at Colorado, Springs University of Minnesota, Morris University of Rhode Island, Stony Brook University, Oberlin College, University of California at Irvine

iv) Calculus : concepts and contexts （2003 第三版）

使用学校(近20所)：

Mount Saint Mary College, Whittier College, University of Richmond, The University of Kansas, Kalamazoo College, Howard University, North Carolina State University, Northeastern University, Graceland University, Washington University in St. Louis, Wright State University, Stanford University, University of Minnesota, University of Tennessee, Northwestern University, University of Cincinnati, Utah State University, Oklahoma State University, University of Wyoming

3)书名：Applied Calculus

作者：Deborah, Hughes-Hallet et al.

出版商：John Wiley & Sons, Inc. (2006) ISBN 0-471-68121-0

适用范围：生命科学、管理和文科各类大学本科生微积分教材

预备知识：高中数学

习题数量：大

习题难度：低

推荐强度：9.2

使用学校：

Macalester College, Temple University, Indiana University,Purdue University, University of Rhode Island, Idaho State University, University of Sioux Falls,Loyola University Chicago

国外评论摘选

i) APPLIED CALCULUS, 3/E brings together the best of both new and traditional curricula to meet the needs of today's students. The author team's extensive teaching experience and proven ability to write innovative and relevant problems has made this text a true bestseller. Exciting new real-world applications make this new edition even more meaningful to students in management, life and social sciences. This book will work well for those departments seeking a middle ground for their instructors. APPLIED CALCULUS, 3/E exhibits the same strengths from earlier editions including the "Rule of Four", an emphasis on concepts and modeling, exposition that students can read and understand and a flexible approach to technology. The conceptual and modeling problems, praised for their creativity and variety, continue to motivate and challenge students.

ii) This is a magnificent calculus book. It is aimed at students in business, the social sciences, and the life sciences. This is done by first the examples and problems. But perhaps even more important the wording of the text is such that these students will understand what they are trying to convey and to clearly show them how calculus can be used to solve problems in their particular field.

At the beginning of the book, three pages of the Preface, the applications discussed in the text are listed by: Business and Economics, Life Sciences and Ecology, Social Sciences, Physical Sciences. Under these headings are subjects like: Value of a Car, AIDS, Cancer Rates, Abortion Rate and so on. These are subjects that will have some interest and applicability to students rather than the old traditional problems like water flowing into and out of a bucket that used to be the mainstream of teaching calculus.

4)书名：Advanced Calculus, 2nd Edition
作者： Patrick M. Fitzpatrick
出版商： Brooks/Cole (2005),机械工业出版社影印
页数：590
适用范围：数学系与理工科其他专业的本科生
预备知识：高中数学
习题数量：较大
习题难度： 具有一定难度
推荐强度：9.3

使用学校：
University of Northern Iowa, University of Alberta, University of Colorado at Denver, University of Central Florida, Virginia State University, San Diego State University, University of Rhode Island, University of California, University of Colorado, University of Central Arkansas, Fayeiteville State University, Brigham Young University, University of Calgary, Oregon State University, University of Illinois at Urbana-Champaign, University of Wisconsin at Whitewater

[作者简介] Patrick M. Fitzpatrick拥有格兰特大学博士学位，是纽约大学科朗研究所和芝加哥大学的博士后，1975年进入马里兰大学College Park分校任教，现在是数学系教授和系主任，同时它还是巴黎大学和佛罗伦萨大学的客座教授。他的研究方向是非线性泛函分析，在该方向著有50多篇论文。
书评: 本书以清晰、简洁的方式介绍了数学分析的基本概念：第一部分讲述单变量函数的微积分，包括实数理论、数列的收敛、函数的连续姓和极限、函数的导数和积分、多项式逼近等；第二部分把微积分的概念推广到多维欧几里得空间，讨论多变量函数的偏导数、反函数、隐函数及其应用、曲线积分和曲面积分等。数学分析已经根植于自然科学和社会科学的各个学科分支之中，微积分作为数学分析的基础，不仅要为全部数学方法和算法工具提供方法论，同时还要为人们灌输逻辑思维的方法，本书在实现这一目标中取得了引人注目的成果。本书一方面按传统的和严格的演绎形式介绍微积分的所有主题，另一方面强调主题的相关性和统一性，使读者受到数学科学思维的系统训练。 本书的一大特点是除了包含必不可少的论题，如实数、收敛序列、连续函数与极限、初等函数、微分、积分、多元函数微积分等以外，还包含其他一些重要的论题，如求积分的逼近方法、Weierstrass逼近定理、度量空间等。例如本书专门用一章讨论度量空间，从而把在欧几里得空间讨论微积分时使用的许多概念和导出的结果扩展到更抽象的空间中，引导读者作广泛深入的思考。 另外，与第一版相比，第二版增加了200多道难易不等的习题。全书贯穿了许多具有启发性的例题，并且本版还为教学考虑进行了许多实质性的改动，例如将选学材料与前后内容的关联度降到最低，单独放置，既不影响教学和读者自学的进度，又能让读者集中攻破一些难点，这样使得全书的叙述更简洁、更自然。本书曾于2003-2004年作为马里兰大学教材。（高威）

国外评论摘选
i) A great book. Starts with two very good chapters on linear algebra, adapted to the needs of calculus, and then proceeds to introduce you to the contemporary way to do multivariate calculus, including existence theorems connected to completeness. Very thorough treatment of integration, including integration of forms on manifolds, up to the Stokes theorem, built upon a

fine chapter on differential manifolds, exterior differential forms, riemannian metrics, etc. Good illustrations and beautiful typesetting add to the joy of reading it. Plenty of exercises and chapters on applications to physics and differential geometry.

ii) This is the best book on mathematics I've ever come across. The superbly written text succeeds in guiding the reader in an easy, clear-cut, graceful way through the realm of what he modestly calls "Advanced Calculus". Some minor misprints are to regret, but they don't even come close to blurring the fact that this is - no doubt about that - an unsurpassable masterpiece.

iii) As Spivak's "Calculus on Manifolds", this book is labeled with a very modest title. It should be something as "All you wanted to know about analysis on manifolds but were afraid to ask". This book is a must-reading for the analyst. It covers everything from the most basic vector space concepts up to the fundamental theorems of classical mechanics, running through multivariate calculus, exterior calculus, integration of forms, and many topics more, always keeping a very modern and rigorous style.
The undergraduate may find it a little difficult, but the effort is worth it. For the graduate student and the working mathematician it is an almost-daily reference.

iv) This book is out of print, but is available from Sternberg's website. Search on his full name at Google.

5) 书名：Calculus: early transcendental functions, 4th ed.

作者： Ron Larson, Robert P Hostetler, Bruce H Edwards

出版商： Houghton Mifflin, Boston (2007) ISBN 0-618-60624-6

适用范围：对数学要求不高的专业的本科生微积分教材

预备知识：高中数学

习题数量：大

习题难度：低

推荐强度：9.2

使用学校：

Houghton Mifflin College, Chandler-Gilbert Community College, South Carolina Technical College, Penn State University, The Behrend College, University of Colorado at Denver, Alamo Community Colleges, Johnson County Community College, The Community College of Baltimore County, Emory University, Jackson Community College, Michigan State University, Tri-country Technical College, Rivier College, Rutgers: the State University of New Jersey, Trident Technical College, Mississippi College, Jacksonville State University, Collin County Community College, District Hobart And William Smith Colleges, Oakland Community College

国外评论摘选

i) I have taught calculus for over 20 years, from about half a dozen books: Thomas, Swokowski, Anton, Stewart, and others. Two years ago our university adopted the 6th Edition of Larson. As a pedagodical tool, this text is head and shoulders about all the others. The text uses abundand graphics, a clear design, concise writing, thoughtful examples, and carefully crafted exercises to make calculus accessible to students. I have never had so many students volunteer compliments about a text. This text is simply the "best of the best."

ii) This textbook is much better than the one that is currently a bestseller (Stewart). It explains concepts and examples clearly, showing every step so that we don't have to wonder how did

something happened. It is best suited for someone who doesn't have a lot of time to spend on reading long discussions of theorems... and for someone who doesn't want to go too deep into material and wants to quickly get the concepts. But don't think it is some Dummies or Made Easy guide, it is still a textbook that takes time to read. What I like most about this book is that the authors' style of writing is very clear and friendly: Not many big words or abstract phrases.

6) 书名：Calculus, 9th ed.

作者： Saturnino L Salas, Einar Hille, Garret J Etgen

出版商： John Wiley & Sons (2003) ISBN 0-471-23119-3

适用范围：数学系、物理系或力学系本科生微积分教材

预备知识：高中数学

习题数量：中等

习题难度：中等

推荐强度：9.2

使用学校：

Clark University, University of Houston, James Madison University, Johns Hopkins University, University of South Florida, Georgia Institute of Technology, Athabasca University in Canada, University of Washington, 台湾国立成功大学, New York University, The University of Texas at Austin, Georgia State University, University of Chicago, University of Illinois at Urbana-Champaign, New York University, National University of Ireland at Galway

国外评论摘选

i) This is a superb textbook and it's easy to see why the book is in its ninth edition. What I really enjoyed (yes, I know this may sound a little incongruous in relation to calculus) was the step-by-step build-up of knowledge with good, clear examples. Also, for the problems at the end of each section, all the odd problems have solutions, so one can get some practice (something that is unfortunately rare for many textbooks).
Before going through this book, I had minimal exposure to calculus and what I had seen wasn't very favorable. This book was a key reason why I now really enjoy the subject and feel very comfortable in this area.

ii) I used this book in my first engineering calculus course. The professor was incredibly theoretical and did not teach from the book which made matters somewhat difficult. However, he was showing us the meaning of math which I found refreshing. This book serves its purpose as one which teaches the mechanics of solving problems but very little in developing an intuitive feeling for mathematics. I must admit that the multitude of exercises were very helpful in getting comfortable with difficult mechanical problems. For single variable calculus it is a standard book with good examples, excellent diagrams, and some applications. Getting into multivariables, the ideas are not connected well and seem segragated from the rest of material. I guess as a brief overview, it makes its point but should not be used as a text for multivariable calculus. If you are interested in theory I recommend Apostol's Calculus which covers a great range of material with rigorous foundation. As far as exercises go, Michael Spivak's Calculus is quite challenging and will keep you occupied for months.
All-in-all, a great book for brush up and single variable material but not to be used for higher dimensional analysis.

7) 书名：Calculus, 3rd ed.

作者： Monty J Strauss, Gerald L Bradley, Karl J Smith

出版商： Prentice-Hall (2002) ISBN 0-130-95005-X

适用范围：对数学要求较高的专业的本科生微积分教材
预备知识：高中数学
习题数量：中等
习题难度：中等
推荐强度：9.2

使用学校：
The University of Texas at Arlington, Texas Tech University, Devry University, Northwestern University, Utica College, Rutgers: the State University of New Jersey, Whatcom Community College, University of Wisconsin at Green Bay, King's College, University of London, Dartmouth College

国外评论摘选
i) I learned calculus from this book, and i think that as a text it is excellent. I learned very little from my lecturer, and almost 90 percent of my three good grades in calc 1,2 and 3 can be attributed to the pages of this book. On the other hand, by the end of the year my book had nearly fallen apart.

ii) Many people say that this book is bad. On the other hand, I think is very challenging. The exercises are not as simple as in other calculus textbooks. The book explains everything well and provides you with many examples. I am a math major and this book has been really helpful.

8) 书名：Calculus, 9th ed.
作者： Dale E Varberg, Edwin J Purcell, Steven E Rigdon
出版商：Prentice-Hall (2007) ISBN 0-131-42924-8
适用范围：理工类本科生微积分教材
预备知识：高中数学
习题数量：中等
习题难度：中等
推荐强度：9.2

使用学校：
University of Wisconsin at Madison, The University of Chicago, Iowa State University, University of South Carolina, California State University at Northridge, Syracuse University, Worcester Polytechnic Institute, Oregon State University, Saint Louis University, The Ohio State University, Southern Oklahoma Technology Center, Southern Illinois University at Edwardsville, Saint Louis University, Denison University, York University, The University of North Carolina at Chapel Hill, Virginia State University, 台湾国防管理学院

国外评论摘选
i) When I was 15, this was the book that I taught myself Calculus from. Now that I'm a professor, this is the book that I use to teach Calculus. In this review I will give the pros and cons of using this book from both a student's and teacher's perspective.
A Student's Perspective
When learning Calculus, I read every page of this book and did every problem. Students will complain that examples and discussion in each chapter seem inadequate to do all of the

problems at the end of the section. I feel that this is part of the design of this book. The problems are intended to be instructional. Indeed this book has a corresponding student solutions manual that helps students to check their work and see if they are "getting it". The problems in the book range from extremely elementary up to moderately challenging. If, instead of instructional problems, this book had given enough examples and text to explain all of the ideas, it would have to be over 2000 pages long. Students should think of the problems in each section as being part of the instruction instead of problems to test previously acquired skills.

When teaching myself from this book, I was able to do all but a few of the problems. Granted I had to spend a considerable amount of time struggling with some of them, but for a talented and dedicated student, every problem in the book is accessible and most are extremely instructive. I should also mention that the book is very well written. Having never actually read a math text book from cover to cover back then, I didn't have too much problem tackling this one. It's very rare that a math text be thorough, informative, and easy to read. This one manages to be all three.

The main drawback of the book is that the students solutions manual is absolutely essential and will be an additional cost. Even if money is tight, as it often is for students, make certain that you buy this manual.

A Teacher's Perspective

As I said above, the problems in this book are intended to be instructional. For this reason it is imperative that a teacher not just lecture from the text and examples, but dig into the problems and carefully choose the most instructive ones for in-class presentations or homework assignments. If you only lecture from the text and examples, you'll only be teaching your class a small fraction of what this book has to offer. If you use this for a course, do as many examples as you have time for. I dedicate one lecture per week to doing nothing but working problems. It might be best to work though the even numbered problems for your class, as the odd numbered ones all appear in the student solutions manual.

The layout of the book is a little bit flawed. This book is aimed at three semester Calculus sequences in state universities and liberal arts colleges. It is not a meant to challenge exceptionally bright students. For this reason parts of chapter 2 seem inappropriate-specifically the sections on the rigorous definition of limits and continuity. If you're teaching a calculus course to non-math majors at modest universities, why would you force students to wade through the muck of mathematical proofs of continuity and existence of limits? In my experience the students absolutely hate this part of the course and gain nothing from it. If you have a few bright kids in your class, you can work with them on an independent study of the more theoretical areas such as this. Also, there are few chapters in the book that are out of place. For example, the chapter on integrating to find the volumes and surface areas of solids of revolution comes way too early while the chapters on transcendental functions, inverse functions, and L'Hopital's Rule come way to late.

Overall the presentation of new ideas is very good in this book, with one notable exception. The book introduces the natural logarithm (ln x) through it's definition in terms of the antiderivative of 1/x. From there it uses the inverse function theorem to derive the exponential function and it's properties. I, and my students, find it more natural to define the Euler number, e, in terms of continuously compounded interest, and then derive the natural logarithm and its properties from the exponential function. It's a matter of taste, but the later approach seemed more lucid to my students. You may want to supplement your lectures in this way.

One of my favorite features of this book is that not only does it cover all the material from a traditional three semester Calculus sequence, but it also has chapters on analytical and numerical solutions to ordinary differential equations as well as an appendix containing more theoretical material for brighter students. If you find yourself teaching an unusually talented bunch of kids, the appendix on mathematical induction as well as the aforementioned sections on ODEs and proofs of continuity and existence of limits can make great supplements to challenge those eager to dive into mathematics.

ii) Ok, let me start by stating that because this is "the shortest mainstream calculus" text out there, it does not mean this has less value. It would seem to be so, but this is the exception to the rule where shorter texts means dumber texts. Explaining mathematics is a bit of an art: you have to choose in what sequence things are to be layed out to the reader, so this means you have to choose how you will relate the explanations to one another. The Purcell I read (the 1st edition - it was my dad's) is quite masterfull at that. Often, when my college standard text got the explanations too verbose and confused, I looked for my Purcell copy and there it was, crystal clear: short, mathematically rigorous, to the point.

2.2 线性代数

选的8本广泛使用的线性代数中，Hoffman-Kunze 的教材最深，适合于对线性代数要求高的专业使用，其次是 Strang 的 Linear Algebra and its Applications. 其它教材一般都比较浅。

1) 书名：Linear Algebra and its Applications
作者： Gilbert Strang
出版商： Thompson Learning, Inc. (1988) ISBN 0-15-551005-3
页数：505
适用范围：理工科大学本科基础数学二学年的教材
预备知识：微积分
习题数量：大
习题难度：容易到中等
推荐强度：9

使用此书的部分院校
Massachusetts Institute of Technology , University of California，University of Delaware，Indian Institute of Technology, Bombay，University of Maryland，State University of New Jersey，Tulane University，State University of New York Institute of Technology，SUNY Institute of Technology，Rivier College，New York University，Duke University，University of Colorado at Denver，Yale University，University of Houston，Loyola University，Drexel University，Tufts University，Stanford University，University of Regina，North Carolina State University，Brown University，Dartmouth College，University of Washington，Georgia Institute of Technology，Pennsylvania State University

书评： 本课程是麻省理工学院数学系为全校设置的王牌课程之一，至少已有30年的历史。 作者亲授的全套课程录象已经在 MIT 的官方 网站上免费下载。本书从实用的角度包含了线性代数的全部内容，对基本概念的理解方面作者不惜用较多的文字 作解释，并且几乎手把手地教读者学会使用一些常规的线性代数方法。然而，本书决不是一本“傻瓜书”，它对 读者的预备知识虽然不高，对智商还是有一定的要求，比较适合我国重点大专院校使用。
我观看过该课程的部分录象，视频和音频质量不很高，有一定的英语听力的人可以听请每句话。看录象比看书更有启发性。顺便提一下，Strang 教授在 MIT 开设的另外两门应用数学课程 18.085,18.086 也有录象，可在 http:\\ocw.mit.edu\ 上找到。

在Amazon 网站上此书有67 篇读者评论，五颗星的31篇，一颗星的19篇（如下面所摘录的第5篇），中间的很少， 这在一定程度上说明了这本书的特点， 同时也提醒读者这本书是不是适合于你。（杨劲根）

国外评论摘选

1) 就Linear Algebra 而言,我还没看到比 Gilbert Strang 的书更好的书。他的 Linear Algebra and Its Application 虽然旧，但经典，就像 Rudin 的书一样，难以被替代。他有一本比较新的书，Introduction to Linear Algebra，1993 年的。如果想深入，那么他的另一本巨著 Introduction to Applied Mathematics 则最适合不过了，这本书把 linear algebra 跟其他数学分支结合在一起，配上他启发性很强的描述，感觉好像在看小说，新奇，激动，期待。 现在 Gilbert Strang 的两门课 linear algebra 和 applied mathematics 都被 MIT 放到网上了，有全部的上课现场录像，还有很多相关的学习资料，上课的录像可以在线看或是下载下来看。 Gilbert Strang的讲课风格跟他的写作风格一样，充满睿智和启发性，还带点情节， 比起大部份的数学教育者沉闷的讲课模式和呆板的板书，Gilbert Strang 的课很难让人睡着，当然前提是英语听力水平不能太差。建议去看看，感受一下大师的风采， 同时也感受一下 MIT 的气氛。

2) The Mathematics Department used linear algebra books by Howard Anton, Bernie Kolman, and David Lay for many years. I took a chance two years ago and adopted Gilbert Strang's linear algebra book for a large engineering course. We used the second edition of Introduction to Linear Algebra, Wellesley-Cambridge Press. Several colleagues said it couldn't be done, but students and the instructor survived nicely to see another day. Many students said they enjoyed the book. Gilbert Strang's enthusiasm for the subject matter comes through in the text and students find it a refreshing change. Another strong point is an extensive set of problems. Many problems probe the subject in a way that requires students to think about linear algebra. Routine problems are not forgotten. This is good. Students can work on problems that help them put the subject in their own voice. A third strength is the layout of topics. Matrix multiplication and elementary row operations from a matrix viewpoint are developed first, and this provides an opportunity to discuss row reduction, matrix inverse, and the decomposition with little extra effort. Other standard subjects follow in order and orthogonality arrives early. Computation is not ignored and the text is organized so that computation is optional. LU I worked to adapt my notes and style to the text. After a while, I discarded my old notes and discovered freshness in the subject that I had not known for some time. Enrollment in the course for engineers has increased dramatically in the last two years. More than 250 students studied linear algebra and matrix theory at Drexel University in the spring of 2005. All day students taking linear algebra at Drexel used Gilbert Strang's book. I plan to use it again.
Herman Gollwitzer,Mathematics Department,Drexel University

3) I had the opportunity to learn linear algebra from Prof. Strang's online video lectures at MIT. This book will be a good comapanion to those lectures. All of you who hate Linear Algebra should take it from me : Watch the lectures along with the book, you will do no wrong. Strang's insights as he lectures, will make you fall in love with Linear Algebra.
Rajesh Kumar Venugopal, Syracuse, New York

4) 這是本非常適合自修的書，書中的用字都是很基本的單字，讓英文不是很好的我也能輕鬆地閱讀； 内容由浅而深，观念清析，圖示更是一絕，封底有一個解釋 linear transformation 的圖，完

全表達出 linear transformation 的精髓，令我嘆為觀止，解釋 SVD 的圖也同樣令我印象深刻。另外，這本書在 2003 年出版了第三版 也已經在我必買的書單之中了。

5) Strang tells us in the preface that linear algebra is a beautiful subject, and he is correct. Yet he seems intent on strangling its theoretical beauty with a matrix based approach to vector spaces, and an ugly preoccupation with ${\mathbb R^n.$ It's clear that this book was not written to be either a lucid explanation of how to use linear algebra, nor was it intended to be an aesthetically pleasing exposition of theoretical linear algebra. It was written somewhere in between, and it is an unhappy medium. If you are interested in a theoretical treatment of linear algebra, there are sorrowfully few good texts available. The title of Axler's "Linear Algebra Done Right" is a result of this fact, and if you are seeking a mathematically pure treatment of the subject, that book is a much better choice. If you're not interested in the theory, but only the applications, you should still be able to find a much better text than Strang's.

2) 书名：Introduction to Linear Algebra, 3rd edition
作者：Gilbert Strang
出版商： Wellesley Cambridge Press (2003)
页数：
适用范围：理工科大学本科二学年
预备知识：微积分
习题数量：大
习题难度：容易
推荐强度：8.3

使用学校：
Case Western Reserve University， College of the Redwoods，University of Houston，University of Miami，University of Minnesota，University of Colorado at Denver，Cornell University，Massachusetts Institute of Technology，Loyola University，Drexel University，University of Maryland，Columbia University，Brown University，Rutgers, The State University of New Jersey，Michigan Interdisciplinary and Professional Engineering (InterPro)，University of Nevada, Reno，University of Alabama at Birmingham，College of the Redwoods，Wellesley College，Mount Holyoke College，University of Wyoming

国外评论选摘：
i) People say that mathematical truths never change, and that's true enough. New concepts, applications, and techniques keep emerging, though, so math teaching needs to keep up with the times. Strang has done an outstanding job of keeping this book current and relevant.
It's not a mathematician's math book - this is aimed at people who need results and needs computational techniques more than they need crystalline theorems. That's why it's so helpful to see applications like Markov models, Kirchoff's laws, and Google's analyses of the web. It's also helpful to see examples worked in Mathematica and MATLAB, the tools of choice for desktop exploration of numerical systems. It's startlingly easy to come up with a 100x100 system of equations, and just nuts to try to solve it by hand.
Strang assumes some amount of calculus in this book, something that other books on linear algebra sometimes skip. That raises the bar for the readership, but also opens up topics like

change-of-basis in function space, including Fourier analysis. It also allows differential equations to be addressed as linear systems. Even without calculus, though, a reader is exposed to the singular value decompostion, QR and other matrix decompositions, and considerations in performing the computations. I found a few oddities, such as the description of a matrix's condition number. That has great physical meaning when it's taken as the ratio of the matrix's highest and lowest eigenvalues, but Strang gives a definition that I found less intuitive.
Such oddities are rare, though. Even though this book covers many topics, its emphasis is on clear and applicable presentation. I recommend this to anyone studying linear algebra or who, like me, has to brush up on basics not used in many years.
ii) Gilbert Strang is a very experienced teacher of Linear Algebra, and this book is written as a text based on his MIT linear algebra class. Math majors will not find the 'definition-proposition-lemma-theorem-proof-corollary' treatment here. Instead Strang, aware of the need to teach non-math majors the subject, explains linear algebra in a simple but effective way --examples, diagrams, motivations. This book is one of those with which you can skip class the whole semester and get good grades (but don't do it! get your education in the classroom).

3) 书名：Linear Algebra and its Applications, 3rd ed.
作者：David C. Lay
出版商：Addison-Wesley
页数：445
适用范围：工科经济农医类本科二学级数学教材
预备知识：微积分
习题数量：大
习题难度：容易
推荐强度：9

使用学校：
Ohio Northern University，University of Kentucky，University of North Carolina at Charlotte，University of South Carolina, University of Memphis, Agnes Scott College，Alamo Community Colleges，Bates College，Boston University，Florida State University，Michigan Technological University，Salisbury University，Stony Brook University，University of Maryland，University of Connecticut，University of Massachusetts Amherst，University of Missouri-Rolla，University of Oregon，University of Texas At Austin，Boise State University，Brigham Young University，New Mexico State University，New York University，San Jose State University，Yale University，Westmont College，Rivier College，University of Delaware，University of London, University of Richmond, University of Rochester, Eastern Mennonite University, Princeton University, University of Colorado at Denver, City University, Cornell University， University of Nebraska at Omaha

书评：
作者是在序言中提到过的 1990 年美国线性代数第一教程研讨会的主要与会者之一，所以本书可以代表这个课程的主流。
这是一本标准的非数学专业的中等程度的线性代数教材，虽然有 445 页，但是浓度不大，可以在一个学期学完。

本教材的特点在于其应用性。每一章都已一个实际问题开始，该章结束时给出用本章的数学解决那个实际问题的方法。这些问题及所在的章节罗列如下：
第一章 线性方程组 ----------经济学中的线性模型
第二章 向量和矩阵 ----------营养学中的问题
第三章 矩阵代数 ----------计算机图形和自动设计
第四章 行列式 ----------平行六面体的体积
第五章 向量空间 ----------宇航和控制系统
第六章 特征值和特征向量 -----生态保护中的动力系统问题
第七章 正交性和最小二乘法----北美洲的统计数据之调整
第八章 对称矩阵和二次型 -----多通道图象处理
本书的另一个特色是图例丰富多样，有助于初学者比较直观地理解线性映射等抽象的概念。为了便于使用者学后查阅，书后还附有书中主要术语的小词典。（杨劲根）

国外评论选摘
i) This text is a dream to read compared to many other mathematics texts. Lay's writing style is clear, and he rightly stays away from using wording that distracts the reader from the theory he presents. Mathematical notation is introduced before it is used, and proofs are placed in an appendix. Overall, this is a very good book for undergraduate study. It won't carry you through graduate classes, but it might be useful as a support book if you have a weak background in the topic. Math majors who love concise formalism and extended proofs should stay away from this book. Engineers, business, physical science, and social science majors will find the text very helpful.
ii) Math texts are notoriously poorly written and difficult to follow for the typical undergrad without the guidance of a professor. This book is an exception to the norm. Not everything, but most things, are presented in a way that most students will be able to absorb on their own.

4) 书名： Elementary Linear Algebra, 9th edition
作者：Howard Anton
出版商： John Wiley & Sons
页数：
适用范围：工科经济农医类本科二学级数学教材
预备知识：微积分
习题数量：大
习题难度：容易
推荐强度：8.5

使用学校：
The City College of New York，University of Texas at Dallas，Hartnell College，Rivier College，UC Santa Cruz，University of Colorado at Denver，McGill University，Athabasca University Canada's Open University，Victoria University of Wellington, New Zealand，Brandon University，Louisiana State University，Indiana University-Purdue University，State University of New York College at Brockport SUNY Brockport，University of Manitoba，The Richard Stockton College of New Jersey，Florida Atlantic University，Saint Vincent Colllege，

University of East Anglia，Norwich University，University college Dublin，Cardiff University，University of Essex，University of Calgary，Durham University，Queens College，wellesley College，Lehman College，Cayuga Community College

国外评论选摘
i) I used Anton in my linear algebra class a few years back and I have referred to it often since. Anton's approach is to introduce the notation and basic tools, i.e. vector and matrix arithmetic, within the intuitive geometric settings of the Euclidean plane and space. Once the basic concepts of Euclidean vector spaces have been mastered, Anton moves into abstract vector spaces, linear transformations, and eigenvectors. One chapter is spent on complex matrices, and another chapter deals with numerical issues and least-squares applications. The only topic which is noticably missing is the singular value decomposition, but other than that, Anton is a remarkably complete text. The definitions and theorems are clearly presented, along with the motivating intuitions. The exercises at the end of the chapter sections are a nice balance between computational and theoretical problems. Overall I highly recommend Anton as a first linear algebra text.
ii) The Anton book appears to be the standard in teaching undergrad LA, but I personally didn't like it very much. Part of the problem is due to several misprints in the early chapters. Some of the definitions of basic concepts are confusing at best, wrong at the worst. I found myself relying on the Hubbard-Hubbard "Vector Calculus, Linear Algebra, and Differential Forms" to get through the course. The explanations were more concise and easier to understand. If you'r eteaching yourself, Hubbard-Hubbard is the way to go.

5) 书名：Elementary Linear Algebra (application version), 9th edition
作者：Howard Anton
出版商：John Wiley & Sons
页数：
适用范围：农医人文科学类本科二学级数学教材
预备知识：微积分
习题数量；大
习题难度：容易
推荐强度：9.0

使用学校：
Murray State University，Stetson University，Athabasca University，The University of Tennessee at Martin，University of Toronto，City College of San Francisco，Drexel University，Eastern Michigan University，Towson University，University of Wales，University of Iowa，Stony Brook University，McMaster University，York University，University of Southern Indiana，Binghamton University，University of Melbourne，University of Stirling，College of the Canyons，Middlebury College，Elon University，Kennesaw State University，University of Manitoba，University of Colorado at Colorado Springs，University of Guelph，University of West Georgia，University of Victoria，Chaffey College，Wayne State University，Rowan University

书评：

这是我用过的内容完整的最浅的线性代数教材，以计算和应用为主，但决不是一本“傻瓜书”。虽然绝大部分定理没有证明，但是诸如矩阵的列空间、秩、齐次线性方程组的解空间一类概念还是会让一部分学生头疼的。
不象很多教材在写完向量空间后就马上写线性变换，本书作者把线性变换放到最后，在它之前安排了内积空间和特征值，原因大概是线性变换这部分的内容比较抽象，放在后面较合适。由于深度的限制，通过这本教材不太可能在几何上对线性代数有深入的认识。对于时间宽余的学生，可以把这本书当作入门读物，学完后再念一本更深的教材。（杨劲根）

国外评论选摘
i) 這本書比較簡單，比較適合線性代數基礎比較差的學生，可當成入門的書籍，這本書的另一個重點在於它有三分之一的篇幅在談線性代數在各個領域的應用，可讓你看到線性代數抽象的數學背後廣大的應用。
ii) The book starts by describing matrix manipulations and determinants. These are very tangible things to most maths students. Accordingly, explaining how to take determinants or to invert a matrix lets you build confidence in your knowledge. Also, these topics lends themselves readily to many problems for you to do.
After this, the book heads into more abstract territory. Null and range spaces and the rank nullity theorem, for example. You are exposed to the concept of an abstract vector space. Which invariably some students always trip over. So the grounding in the early chapters can mitigate this awkwardness.
The last chapter touches lightly on the interesting applications, like chaos and fractals. But mostly to pique your interest in proceeding further in the field.
iii) This is the text I used this previous semester for my Linear Algebra class. I had no linear algebra background before taking this class. That being said, this was one of the roughest classes I've ever got through only because the book kept going against the grain in every way possible. I didn't even begin to understand the entire point of linear algebra until about chapter 7 and 8 when the chapters started going into the general cases, and even now, I know how to "solve" all the problems without even knowing their meaning, which seems totally pointless to me. The selected answers to the problems in the book are in no particular pattern. It's not "all odds" or "all evens"; it's just scattered and it made doing homework a nightmare. I felt like I was back in elementary school while reading this book, because back then all I did was learn "methods" of solving problems without understanding "why". The book almost never discussed the purpose or main idea of the subjects it discussed. The "explanations" it gave would be based off of other vague topics. For example "What is the Eigenvector Problem? Well, the eigenvector problem asks if there is a basis for R^n in a $n \times n$ matrix consisting of eigenvectors of said matrix", OK so What's a basis? "A basis a set of vectors for a vector space S is linearly independant and/or set that spans the space S" and the cycle kept hitting me with one definition after another without giving me a big picture or anything. A bit of the book is about "applications" of linear algebra, but doesn't help until you've understood the meat of the book that came beforehand. Also, there were no teachers' solutions manuals available when I took this class, because the distributers have been extremely lax about getting them out (why? who knows). I'm not just saying this book is bad because I was lazy and didn't do well. I worked extremely hard to do "well" in this class. I must have read this book twice through and like I said before, I can solve all the problems but please don't ask me to explain their significance or validate their existence, because I can't. STAY AWAY!

6) 书名：Linear Algebra with Applications, 3rd edition
作者：Otto Bretscher
出版商： Prentice-Hall

使用学校：

San Francisco State University，University of Utah，Pennsylvania State University，Agnes Scott College，Harvard University，Johns Hopkins University，University of Minnesota，McGill University，Colby College，Santa Clara University，University of California，State University College at Buffalo, Queen's University, Georgia Institute of Technology, Northeastern University, Purdue University, Loyola University, Iowa State University

国外评论选摘

i) The explanations and examples are generally very clear, and there isn't a lot of distracting nonsense. In many textbooks they try too hard to teach through "Real World" examples. i find such examples confusing because they obscure the math behind the example. I also felt this book had a nice mix of easy, medium and challenging problems. And it feels like the author really understands and strives to clarify many of the hurdles faced by Linear Algebra students. Make no mistake about it, Linear Algebra is a tough class that requires a lot of dilligence and abstract thinking. This book isn't going to guarantee you an A. But if you work through it, and if you have a helpful teacher, you'll be on the right track.
By the way, I am a Computer Science major, and while I consider myself decent at math, I'm by no means a math genius.
ii) This text was developed by the author during his time on the mathematics faculty at Harvard for specific use in the second semester of a two semester, undergraduate sequence on multivariable calculus and linear algebra. It is intended for physics, chemistry and strongly quantitative economics majors. As such, in terms of complexity it is more par with a collegiate abstract algebra text, with a clear focus however on linear algebra. The "applications" portion of the title is a bit of a misnomer, as examples only occur in the problems and almost never in the examples (which are designed instead to show the theoretical precepts and continuity underlying the field). In general, this text is above the intellectual capabilities of but the most dedicated users of applied mathematics, and those especially is the fields of economics and finance as generally taught at the undergraduate level would best look elsewhere. Most prominently, the text has almost no redundant examples, which makes it a enjoyably lucid read for those who grasp concepts quickly on the first go, but a dead end for those who come up short. I would not as professor think of assigning this book to non-Ivy caliber students outside of pure math; even Harvard students seemed to struggle with it at times.
iii) I was required to purchase this book for a course called Linear Algebra with applications. This book seems to just cut out important theorems, proofs and other pieces of explanation commonly found in other text books I have looked through, and rather than making up for it with a decent explanation or summary for what it omits, it leaves gaping holes in many topics. It gives partial proofs and explanations at times and leaves other pieces "for you to solve as exercises." It's like the [person] who made this book only wrote half a math book, and left the other half for you to figure out in problems at the end of the chapter.

7) 书名：Linear Algebra with Applications, 5th edition

作者：Steven J．Leon

出版商：Prentice-Hall

页数：491

适用范围：理工科本科二学级数学教材

预备知识：微积分

习题数量：大

习题难度：中等

推荐强度：8.5

使用学校：
Rowan University, Arizona State University, Florida International University, Northern State University, University of Illinois at Chicargo, University of Puerto Rico, Colorado State University, State University of New York Institute of Technology, SUNY Institute of Technology, University of Hawaii, Ohio State University, University of Minnesota, Texas A\&M University, University of Massachusetts Dartmouth, University of Texas at Dallas, University of New Mexico, Boise State University, Baruch College, University of Oslo, University of Missouri-Columbia, University of Mississippi, Utah State University, Kansas State University, University of California, Irvine, Brigham Young University, Cornell University

书评：
本书目前已出版到第7版了，我这里只找到第5版。这本书的前六章的编排十分传统，内容也比较规范，从 Gauss 消元法、矩阵和行列式到向量空间和线性变换，再将正交性和特征值，没有讲线性变换的标准型。最后一章（第七章）讲数值线性代数，即线性代数的近似计算方法，一般的线性代数教材不含这方面内容。
本书几乎所有的定理都有证明，证明比较简洁，对读者理解有一定要求。如果用一个学期学本书的前一半还是比较轻松的，后半本显然要难一些。（杨劲根）

国外评论选摘
i) First of all, I would like to say this book is not for beginers. If you have no idea what a matrix is, don't use this book. However if you have taken an introductory course in linear algebra or you already have a reasonably well foundation in this subject, then you should have no problem in understanding following the text. Although the explaination in this book is not particularly outstanding, it does treat some advanced topics like eigenvalues, numerical linear algebra elegantly. I would like to recommend this book to persons who would like to seek a more advanced linear algebra book for reference or self studying.
ii) Leon's text on linear algebra isn't bad, but there is room for improvement. Chapters 1, 2, and 3 do a good job of introducing the basic concepts of linear algebra, including matrix row operations, determinants, and linear independence. The book seems to lose clarity beginning in Chapter 4. The concepts become more abstract and Leon's notation interferes with the ability to clearly understand what he is talking about when it comes to linear transformations and issues regarding $R(A)$ and orthogonality. Very important results are frequently understated as well. In a few cases, there aren't enough examples to go around - especially in Chapters 4 and 5. It is ironic compared to the relative overexplanation found in Chapter 1, for example.

8) 书名：Linear Algebra Done Right, 2nd edition
作者：S. Axler
出版商：Springer (UTM) 1997
页数：491
适用范围：理工科本科二学级线性代数教材
预备知识：高等数学
习题数量：中等
习题难度：一般
推荐强度：9

书评：
本教材比较适合已经学过一些基本的线性代数（如大学一年级的“高等数学”中的线性代数部分）的学生，内容和篇幅适合一个学期。
除了标准线性代数教材所需具备的条件外，本书最大的特点是线性变换的特征值的存在性的证明避开了行列式。按照常规的教程，先把线性变换的特征多项式定义为行列式 $|xI-A|,$其零点是特征值。利用复数域上的代数基本定理，特征值总是存在的。本书中的证明是这样的：对任意线性变换 $A,$ 任取非零向量 $u.$ 则 $u, Au, A^2u , \ldots, A^n u$ 线性相关，于是存在多项式 $f(x)$ 使$f(A)u=0.$ 将 $f(x)$ 分解成 $f(x)=c(a_1-x) \cdots (a_n-x).$ 则$(a_1 I - A) \cdots (a_n I -A) u=0.$ 因此某个 $a_i I -A$是不可逆的，这意味着 a_i 是一个特征值。
作者认为行列式是一个非常不直观的概念，在线性代数教程中过早地使用行列式是违反学习规律的，这也成为本书起名的主要原因之一，虽然多少有点“狂妄”，但是线性代数似乎应该这样来学。
这本教材自出版来很快受到很热烈的欢迎，值得国内大学从事线性代数教学和改革的人员关注。由于本书的严密逻辑和行文的严谨，自学这本书也是比较容易的。（杨劲根）

国外评论选摘：
1) I have used this text for a beginning graduate course in linear algebra, mostly because I prefer its treatment of eigenvalues and eigenvectors over Hoffman and Kunze, and it sticks to the basics: complex scalars. It also has a good treatment of inner product spaces. The basic concepts and theorems are indeed presented cleanly and elegantly. Its use of linearly independent sequences (rather than sets) is a little nonstandard (what if the set of vectors is infinite?) but the adjustment is minor. Two things though I found treated in a less than desirable fashion: He pretends that we don't know about matrices, doesn't want to develop the machinery, and the treatment of coordinate vectors and matrix representations suffers. Students also get no sense of how to compute the solution of concrete vector space problems, which is easily done once the theory is established, and which is an essential skill to have after a second course in linear algebra. I have to give them supplementary notes. Second, the treatment of determinants suffers, apparently for ideological/political reasons. I think students deserve a straightforward development of determinants simply because that theory is widely used in applications, in engineering, and in discrete mathematics, and it has its own beauty. It is not hard to do, and I do it myself from notes, adapted from the treatment of Hoffman and Kunze. Now that undergraduate linear algebra courses have in many places dropped any substantial theorem-proving component, students need a serious course in linear algebra which can take them, e.g. all the way into Jordan form. There are not many good books for this, and this text does a good job with the basics without overkill on the abstraction, so I use it despite the drawbacks mentioned above.
2) I have no doubt that this is one of the most thought provoking math books that I have come across. I used this book for a linear algebra course last fall '08 and I learned a ton. Specifically about the structure of vector spaces and linear operators. However, the most important function that this book serves is to move students towards the methodology of mathematics, which means proof construction and counter examples. It also trains students to let go of their intuitions. But you can not self-study this book, there are no answers and more importantly the structure of the course begs for instruction. I would recommend before taking this course doing what i didn't do and have had to do since, make sure you have your first course of linear algebra solidly under your belt, and that doesn't mean having gotten an A in the prior class is sufficient. Go through the most difficult proof driven exercises in your first text, that should serve as practice for easiest homework problems in this book.

All that said, there are serious limitations to this book. It would be nice if the author worked out 1 comprehensive semi-difficult exercise in each chapter of the text. While struggling to solve the problems can be enlightening, there is only so many times I can read the same sections over and over again, looking for some insight from the kiddie exercises provided by the author. It would also help if some of the kiddie exercises were accompanied with graphs, especially when describing the sums of vector spaces. Sometimes a picture is worth a thousand words - sometimes!
Last but not least, the author has a copyright on the solutions to the book. Where he does not allow professors to post homework solutions to exercises. This had a devastating effect on the class I was in, because there were many students who were lost in the first couple of homework sets and basically were never given a chance to figure out what was going on. Pedagogically, this is unacceptable. Furthermore it sets a dangerous trend, math problems simply stated should not be copyrighted. For this reason I suggest that people not purchase this book, but I still strongly recommend that they get a hold of it.

2.3 其它

书名：Differential equations, 2nd ed.
作者：P. Blanchard; R. Devaney; G. Hall
出版商： Brooks/Cole Thomson Learning (2002)
页数：697
适用范围：理工科大学非数学专业数学教材
预备知识：初等微积分和线性代数
习题数量：大
习题难度：容易
推荐强度：9.4

使用院校：
Harvard University, Saint Joseph's University, Florida Atlantic University, Agnes Scott College, HaverFord College, Indiana University, University of Connecitut, University of Georgia, Boston University, Portland State University

书评： 这是一本应用性的常微分方程教材，在复旦大学外国教材中心将它列为哈佛教材的一种。它对学生的数学预备知识的要求不高，只要最基本的微积分和线性代数就够了。本书的内容覆盖了常微分方程方面的所有重要内容。
这本教材最大的特点是在定性理论方面比同样深度的教材多而详细，作者并不把它集中在一章讲述，而是把它贯穿在全书中，用这样的方法由浅入深地把一些比较难说清楚的概念讲的非常清晰。
本书的英文非常通俗易懂，虽然理论性的证明很少，但是对主要定理的解说十分具有说服力。我曾经使用此教材教过一个学期，对象是相当于我国非重点工科大学的学生，学生反映此教材难度比较合适，他们基本上能掌握其中的重要内容。
第一章是一阶常微分方程，在介绍了一些必要的基本概念和应用模型后作者立刻非常明确地用典型实例详细解释了三种方法：1. 解析方法-分离变量法 2. 定性理论 - 向量场方法3. 数值方法 - 欧拉折线法。每种方法都占十几页。接下来叙述并详细解释了解的存在性和唯一性定理。接下来讲

述相直线和平衡解，又化整整一节讨论分歧理论。 这样的处理方式的一个显著的优点是用最短的时间是学生了解了常微分方程的概貌。
第二章是一阶常微分方程方程组，但以两个函数的方程组为主。其主题不变，继续讲述处理同一类问题的三种不同方法。由于维数的增高，向量场更有意思，书中举了各种形形色色的例子使读者知道各种变化。
第三章是一阶线性常微分方程方程组，这是任何一本常微分方程教科书必须包含的内容，本书也不例外，写的非常清楚，没有可挑剔之处。
第四章的标题是受迫振动和共鸣，其实就是二阶常系数线性方程。这也是标准内容，和第三章一起，这两章是以解析方法为主的。
第五章是非线性方程组，重点自然放在定性分析上，先详细讲述二维情形，在化较少的笔墨写三维情形。
第六、七、八章分别是 Laplace 变换，数值方法（特别是 Runge-Kutta 方法），离散动力系统。
前四章的内容是必学的，后四章的内容相对独立，可以根据不同的专业选将若干章。一般说来，一个学期讲完五章是绰绰有余的。（杨劲根）

国外评论摘选

i) As a differential equations instructor I used Boyce and DiPrima for many years. Its a good, solid presentation of differential equations and a great reference. However, I was always disappointed that my students ended up with no "feel" for differential equations. Also I became convinced that more methods were needed for nonlinear differential equations. After using a couple of other books which seemed to be slanted toward more qualitative approaches I came across Blanchard's book. I used it as a textbook for my class for several years now and I have found it to be a near perfect match to my goals. Some consider it wordy but I appreciate the motivation and insight the authors try to bring to the concepts. As a result it is not a good reference but as a textbook it is great. There are plenty of graphical tools. Quite surprising to me is how much the book illuminates DE's by simply analyzing the components of the DE, even before any solution is attempted. These features, along with some integrated applications, gives students much more of the "feel" for differential equations I have been looking for.

ii) This book is unique. Most differential equations textbooks simply provide formulae for different types of problems, but you don't really see the big picture. This book lets you see the big picture, but omits many of the most useful formulae that you may need in your career. This for that. It would be nice to see a book with the best of both worlds, but if you simply want to learn and understand the topic, this book is the way to go. Also, there is a good emphasis on qualitative and numerical techniques. Students often feel like they get less out of a mathematics class when qualitative and numerical techniques are emphasized over more analytic approaches. However, those of us who have worked in the "real world" know that the qualitative and numerical techniques are probably even more important. I have worked as a research statistician and my research areas emphasize computing. When I'm presented with real problems and real data (which, in my career, usually comes in large, unmanageable quantities), do I usually pull out my notebook and tackle the problem in a very precise manner, working out an exact solution? No, quite often I cannot realistically do that. Now I'll admit that I don't use much from this particular field on the job, but it still applies. Moving on, I must also mention that the book does a very good job at explaining these qualitative and numerical techniques in addition to things that are more analytic, although it sometimes a little too verbose. Regarding applications, the book covers a lot of fields and does put a big emphasis

on applications. Physics, biology (especially population growth models), and electrical/computer engineering receive the most treatment. Overall, I would say that the book does an excellent job at including plenty of applications and choosing meaningful ones.
I don't have much to say about the exercises. Most aren't too contrived and they mixed up the difficulty fairly well. However, I would have liked to see more "hard" problems.
In summary, I'd recommend that you pick up a different book if you need a reference for work or research, but pick this one up if you actually want to learn and UNDERSTAND the basics of differential equations.

iii) I used this book in a 2003 summer course in DE, and found it to be a wonderful introduction to the subject. I am not sure what some of the other people meant by saying it wasn't for math majors- I am one and found it wonderful. Not everything needs to be concise, (I gave Rudin's book five stars too BTW, so I AM a fan of some concise books). It gave diverse examples of applications from all over--physics, EECS, ecology, biology, etc. The CD-Rom is a great learning tool. Ultimately analytic techniques are NOT what DE is about, and this book tries to show the student how to use qualitative and numerical methods early on. Anyone who wants to know DE must become familiar with numerics and the qualitative way of analyzing the equations. This book will show you how to THINK about DE, and not how to mindlessly attack an equation based on its form. This is the intro ODE book to which all others ought be compared.

书名：Concrete Mathematics, 2nd ed.
作者：R.Graham, D.Knuth, O.Patashnik
出版商：Addison Wesley (1994)
页数：624
适用范围：大专院校计算机专业数学教材
预备知识：基本微积分
习题数量：大
习题难度：从容易的习题到研究性的题都有
推荐强度：9.2

书评： 这是非常特别的一本教材。首先书名就与众不同，一不小心会误读为“离散数学”，事实上从内容上看，它包含离散数学的很多内容，特别是组合数学和数论，但作者在序言中声明本书是“离散数学” 和“连续数学”的混合物。
三个作者排序是按姓氏的，本书的第一位作者 Ronald Graham 是组合数学的权威之一，曾任过美国数学会主席。第二作者 Donald Knuth 是计算机科学界的传奇式人物，现任斯坦福大学教授，他的巨著《The Art of Computer Programming》是计算机程序设计的圣经，本书包含了学习上述巨著的几乎全部数学预备知识。
上世纪末美国数学会曾在它的官方出版物上举行公开的辩论，探讨数学发展的方向，最后没有明确的结论。现代数学是向抽象化的方向发展的，数学家更加注重数学问题定性的研究，其重要性是不容质疑的。但有不少有识之士担心这样下去会有脱离实际的危险，所以他们提倡看得见的数学。这是这本书的初衷。对此书有兴趣的读者不妨先看一下序言，以便更清楚地了解这本书的特点。
全书分 9章，依次为：递归、求和、整值函数、数论、二项式系数、一些特殊的数、母函数、离散概率、渐近。每章中包含丰富的内容，有很多问题和例子在其它同类书中很难找到，一些比较

难的问题的出处都一一写明。本书的重点是讲述解决问题的方法，牵涉到很多数学的常用技巧，看上去比较初等，但对读者的要求还是比较高的。另外本书的趣味性很强。习题很全面，几乎所有习题有答案，这对自学非常便利。
数学系和计算机系的本科生阅读本书一定有不小收获。（杨劲根）

国外评论摘选
1) i) Unless you're very used to this type of mathematics, this book will, as other reviewers comment, prove hard work. However, even someone with little formal maths background like myself can get a lot out of it. It's beautifully written and well-presented, and on the whole the pacing is OK, although sometimes it goes much too fast for casual reading. Once I've made my way through it, I suspect it will make a very useful reference book too; it's full of useful techniques for solving real-world problems, at least if you work in a field that sometimes requires you to solve recurrences and work with tricky integer functions.
Although often corny, the marginalia do give you something of the feeling of being on a course, rather than just reading a textbook. As well as daft jokes, there are hints as to the relative importance of some sections (including ``skip this bit on first reading'' as well as ``this is the critical part'' -- both kinds very helpful).

ii) This book is not light reading, but it's worth it. It has most value as a reference tool, and covers well some areas of maths which are important to CS. Moreover, the information is presented in a light-hearted way, with lots of inline jokes (mainly very corny) and margin notes from students who took the lecture course behind the book. The examples tend to help, and there are plenty of exercises with worked solutions. Also lots of references to the primary literature.

书名：Discrete Mathematics
作者：Dossey,Otto,Spence,Vanden Eynden 原著，俞正光等改编
出版商：Addison Wesley (2002) 高等教育社（2005），ISBN 7-04-016632-1
页数：562
适用范围：大专院校计算机专业离散数学教材
预备知识：基本微积分
习题数量：大
习题难度：容易
推荐强度：8

书评：
离散数学并不是数学的一个分支，它是计算机和信息学专业的一门数学基础课，内容一般包括集合论、数理逻辑、初等数论、抽象代数、组合数学等，但每部分内容都不是非常系统和完整。从某种意义上讲，这是一门大杂烩课程。由于内容的繁多，要学完全部离散数学一个学期是不够的。对于一个学期的离散数学课，一般适合于选讲其中一部分。
本书从实用角度出发，以组合数学为主线安排了一个单学期的教程，最难的部分抽象代数完全没有，初等数论和数理逻辑也很少，有一章讲述逻辑线路和有限自动机，涉及了最基本的布尔代数。叙述方面也以概念的直观解释和算法为主，不强调定理的证明，所以比较适合于数学程度比较低的大学生使用。如果授课对象是层次高的计算机专业学生，这本书就显的太浅，内容也不够丰富。

本书英文浅显易懂，例子非常多，作者们似乎花了工夫认真编写这本教材，错误非常少，习题虽然数量大，但很有意思。
下面登载两篇国外的评论，代表两种观点。（杨劲根）

国外评论摘选
1) As a student at Illinois state, I'm skeptical about all of the professors abilities... After all, these are the guys that consistently screw up addition in front of class. After having a chance to complete half of this book in my Discrete Math course (mind you, I'm not a math major) I have definitely gained respect for ISU's math department.
I'm not sure if most authors really teach classes, or if they write books to fulfill their publishing requirements. I can tell you that the authors of Discrete math had the students in mind.
I've found this book to have exceptional examples, and well-explained, READABLE prose. If you wanted to pick up a copy for self study, this would be a good book.... Yes a professor would be nice, but these guys did a good enough job that the book stands alone.
2) If you are looking for a book for a course in discrete mathematics where the emphasis is on graph theory, then this book will probably satisfy your needs. However, for any other type of course, it will most certainly prove to be inadequate. Nearly half the book is devoted to graph theory, and while many theorems are listed, very few are proven. The working computer scientist may find that acceptable, but most mathematicians will find it inadequate. Logic and the basics of proof are relegated to an appendix. The first chapter covers some combinatorics and the basics of algorithmic analysis, which is meant to be a primer. However, it requires the use of set terminology, set notation and basic counting techniques. Since set theory is covered in chapter 2 and counting techniques in chapter 7, I consider the order to be inappropriate. Recurrence relations, circuits and finite state machines are also covered in other chapters. There are a large number of exercises and the solutions to the odd numbered ones are included. Sets of problems to be solved by programming a computer are given at the end of each chapter, some of which are easy, but many of which are hard. Only students who have had a programming course could be expected to be able to do any of them without significant help. This is a book that does not satisfy my requirements for a discrete mathematics textbook. I consider logic to be a critical topic that must be covered, so I will not consider using any book where predicate and propositional logic are not covered in depth. While I do not expect my students to construct rigorous proofs, I do expect them to be able to construct simple proofs and follow some of the relevant more complicated ones.

3 数学分析和泛函分析¶

数学分析是数学专业的最重要的基础课，是理论性较强的微积分，它以严格的极限理论 作为基础。我国的综合性大学数学系学生一年级的课程一般都将数学分析列为重点课程，复旦大学安排三个学期的数学分析课。美国大学数学系一般分两步：先让学生修和非数学专业一样的微积分，即 Calculus（属于 lower division 的课程），再修数学分析（属于 upper division 的课程），所化的时间大致上差不多。到底哪种方法好也没有定论，大概对中学数学基础较差的学生按照美国的办法容易接受一些，事实上我国数学系一年级的数学分析有不少学生是吃不消的。

数学分析的英文书名大致有三种： mathematical analysis, real analysis, advanced calculus. 可能有些差别，一般说来，advanced calculus 和我国的数学分析比较接近，适合没有微积分基础的数学专业学生学习， mathematical analysis 多半是针对学过初等微积分的人， real analysis 更深一些，接近我国的实变函数论。

泛函分析是数学分析的自然延伸，它的基础除了数学分析以外还需要线性代数和实变函数论 和少量的复变函数论。外国的有些数学分析教材中也包含测度和勒贝格积分等泛函分析中需要用的内容，所以很多学校在本科生阶段就不设实变函数课了。

书名：Introducton to Analysis
作者： Arthur Mattuck
出版商： Prentice Hall (1999) ISBN 0-13-081132-7
页数：460
适用范围：大学数学系本科基础数学学生教材
预备知识：微积分初步知识
习题数量：大
习题难度：较大
推荐强度：9.8

书评: 本书是麻省理工学院的 Arthur Mattuck 教授教授这门课程多年经验的基础编写而成的，是一本实分析的优秀入门教程，深受读者欢迎。 本书主要讲述单变量函数的分析理论，侧重于讲述实数理论的基本思想，特别是用分析的方法对函数进行估计。 本书从基本的实数理论讲起，内容主要包括数列与函数的极限和连续性，级数理论，微分理论，Taylor展开， Riemann积分理论，Lebesgue积分理论等等。本书的一个鲜明的特点是，对书中的定理不只是叙述，而是从来源讲起， 对读者以启发为主，侧重于揭示数学思想。 例如，对微积分的两个基本定理，其证明较一般书中繁琐，但是其证明给出了微积分的重要思想， 即积分是微分的无穷积累， 微分是积分的局部化，并且，还分析了两个基本定理之间的关系。另外，书中还给出许多重要的应用。
本书比较适合作为我国综合性大学数学系实分析课程一学年的外文教材，也可以作为 程度较好的数学系本科生进一步深化实分析概念的课外读物。 （王泽军）

国外评论摘选

1) This is an unusual and beautifully written introduction to real analysis. The presentation is carefully crafted and extremely lucid, with wonderfully creative examples and proofs, and a generous sprinkle of subtle humor. The layout of the pages is exceptionally attractive. The author has clearly put a great deal of thought and effort into producing an analysis text of the highest quality. Most of the book concentrates on real-valued functions of a single (real) variable. There is a gradual and careful development of the ideas, with helpful explanations of elementary matters that are often skipped in other books. For instance, prior to the chapter on limits of sequences, the book has a chapter on estimation and approximation, discussing algebraic laws governing inequalities, giving examples of how to use these laws, and developing techniques for bounding sequences and for approximating numbers. Proofs involving "epsilons" and "arbitrarily large n" make their first appearance here.
The overall presentation of the book is carefully thought out. Each chapter is broken up into small sections, and each section emphasizes one principle idea or theorem. The proofs of the main theorems are lovely, and give both intuitive explanations and rigorous details. Genuinely interesting examples and problems illuminate the key ideas. Each chapter contains a mix of problems: "questions" that help students test their grasp of the main points of each section, "exercises" that are intermediate in scope, and more difficult "problems". (A solutions manual is available for instructors from the publisher.)
The careful explanations, even of "elementary" matters, and two appendices on sets, numbers, logic, and methods of argumentation, make the book suitable for a first analysis course in which students have had no prior exposure to proofs. There is ample material for a one-semester, or in some cases a one-year, course.
In summary, I believe that this is the best introductory real analysis book on the market. Students and instructors alike will find it a joy to read.

2) The book is slow to begin but it does a great job in explaining all the concepts. The author explains the proofs and theorems and it introduces some intermediate ideas to understand the theorems and definitions. The book contains a lot of exercise of different nature and difficulty. It covers a great range of subjects but not enough on the Rn. The book is basic in it contain, it is not difficult to read and follow. It can serve as an introduction to analysis. I would recommend it if you want an introduction to analysis.

书名：Mathematical Analysis, Second Edition
作者： Tom M. Apostol
出版商： Addison Wesley (1974), 机械工业出版社影印
页数：492
适用范围：大学数学系本科生
预备知识：高中数学
习题数量：中
习题难度：中等
推荐强度：9.8

[作者简介] Tom M. Apostol, 美国数学家，生于犹他州。他于1946年在华盛顿大学西雅图分校获得数学硕士学位，于1948年在加州大学伯克利分校获得数学博士学位，1962年起任加州理工学院教授，美国数学会、美国科学发展协会（A.A.A.S）会员。对初等数论和解析数论有研究，他的著作很多，除本书外，还著有《Calculus, One-Variable Calculus with an Introduction to Linear Algebra》、《Calculus, Multi-Variable Calculus and Linear Algebra with Applications》等。

书评: 本书第一章以公理化的方式引入了实数系和复数系，接下来介绍了集合论和点集拓扑的一些基本概念和内容，为后面微积分理论的展开打好基础。从第四章开始，作者开始介绍极限、连续和导数等微积分的基本概念。在第六章作者引入了有界变差函数与可求长曲线的概念，接着就对Riemann-Stieltjes积分进行了介绍，而Riemann积分则是它的特例。第八第九章是对级数和函数序列知识的讲解。第十章介绍Lebesgue积分，第十一章介绍Fourier级数以及Fourier积分，第十二章介绍多元微分学，第十三章介绍隐函数与极值问题，接下来的两章是关于多重Riemann积分与Lebesgue积分的介绍，最后一章介绍了复变函数的Cauchy定理以及留数的计算。 本书是一部现代数学名著：自20世纪70年代面世以来，一直受到西方学术界、教育界的广泛推崇，被许多知名大学指定为教材。作为一本大学数学系的本科教材，本书仔细而又不累赘地向读者介绍了微积分的思想，涵盖了数学分析绝大部分的基本知识点，并配有覆盖各级难度的练习题，适用于初次接触数学分析的读者。无论对于教学还是自学，都不失为一本理想的教材。另一方面，本书对于实分析和复分析中的部分内容也有所介绍，这其实也是很多美国大学数学教材（Mathematical Analysis或者Advanced Calculus）内容设置的共同点。例如作者在第十章有对Lebesgue积分的介绍。不过与一般实分析教材里的思路不同，作者采用了Riesz-Nagy的方法引入了Lebesgue积分，此方法直接着眼于函数及其积分，从而避免了对于测度论知识的要求；同时作者还进行了简化、延伸和调整，以适应大学本科水平的教学。 （徐晓津）

国外评论摘选

1) If you're the type of person who likes crisp and clear proofs but don't want to have the proofs be as skinny as Rudin's then this is the perfect book. Apostol's writing style is not only accessible and clear but the organization of the text is excellent too. There are plenty of problems with a good mix of difficulty levels. He also throws in an example here and there to give you firm footing on some difficult topics. If I had to recommend one analysis text this would be it.

2) I own analysis texts by Apostol, Rudin, Bear, Fulks, Protter, and Kosmala. This one by Apostol gets my vote as the best all-around text on the subject. It's rigorous, elegant, readable, and has just the right amount of explanatory text. This would be my first choice as an undergraduate textbook, a self-study text, or as a supplemental reference to another text. I also recommend Bear for his elegance and witty style, and Kosmala for his thorough explanations. But if you are going to buy only one, make it this one.

3) I've never been a big fan of Apostol. He tends to make things more difficult than they really are. Some of the reviewers commented that they are impressed with the elegance of the proofs, which makes me wonder if they are as confused as Apostol. As an example let's consider his proof of the FTC. There is an easy and elegant proof which you find in most books, but Apostol tries to be cute and gives an obscure and ugly proof. Mathematics is an art, and Mr. Apostol is no Picasso.

书名：Principle of Mathematical Analysis, 3rd edition
作者： Walter Rudin
出版商： McGraw-Hill (1976), 机械工业出版社影印
页数：334
适用范围：数学系一、二年级学生与理工科高年级学生

预备知识：高中数学，最好具备微积分的初步知识
习题数量：287 道习题，较大
习题难度： 较难，但是很多有难度的题目有提示
推荐强度：9.5

[作者简介] Walter Rudin 1953年于杜克大学获得教学博士学位。曾先后执教于麻省学院、罗切斯特大学、威斯康星大学麦迪逊分校、耶鲁大学等。他的主要研究领域集中在调和分析和复变函数。除本书外，他还著有另外两本名著：《Functional Analysis》和《Real and Complex Analysis》，这些教材已被翻译成13种语言，在世界各地广泛使用，以本书作为教材的名校有加利福尼亚大学伯克利分校、哈佛大学、麻省理工学院等。

书评: 本书前二章介绍了从高中数学到大学数学过渡中的基本知识：实数与复数理论，基础拓扑理论。第三章介绍数列与级数。第四章介绍函数的连续性。第五章介绍微分的概念。第六章介绍Riemann-Stieltjes积分的概念。第七章介绍了数学分析中很重要的一个概念：函数序列与函数项级数的一致收敛性。在第八章作者列举了几个特殊的函数项级数，如幂级数、Fourier级数等作专门讨论。第九章介绍多变量函数。第十章介绍了微分形式的积分。在最后第十一章对勒贝格积分作了初步的介绍。
本书内容相当精练，结构简单明了，这是Rudin著作的一大特色。例如在第六章积分部分，作者直接介绍了Riemann-Stieltjes积分，而一般数学分析课程中的Riemann积分就是它的特例。书中的习题经过了精心挑选，有助于学生掌握数学分析的基本概念及提高逻辑推理的技巧。本书第3版经过了增删与修订，更加符合学生的阅读习惯与思考方式。 本书适合作数学系学生学习数学分析课程的参考书，也适合作为具有一定微积分知识的理工科高年级学生提高分析水平与能力的教材。 本书是一部现代数学名著，一直受到数学界的推崇。作为Rudin的分析学经典著作之一，本书在西方各国乃至我国均有着广泛而深远的影响，被许多高校用做数学分析课的必选教材。本书涵盖了高等微积分学的丰富内容，最精彩的部分集中在基础拓扑结构、函数序列与函数项级数、多变量函数以及微分形式的积分等章节。
[零星感悟] 作者从学生的角度出发来考察问题的接受难易程度，并在整本书的结构上做了精心的安排和调整。 比如说，从理论上讲，从有理数的概念出发引入实数的概念是非常正常和符合逻辑的，但是Rudin通过以往的教学经历发现学生对这样的做法不容易接受，因此Rudin从有序集与具有上（下）确界的性质入手来介绍实数，显得简洁而具有新意。 在第九章多变量函数中，一个关键的问题就是反函数存在定理的证明。记得以前看过的书上证明都比较复杂。在此书中，Rudin利用压缩映射的不动点理论，大大简化了证明过程。 (刘东弟)

国外评论摘选

1) OK... Deep breaths everybody...
It is not possible to overstate how good this book is. I tried to give it uncountably many stars but they only have five. Five is an insult. I'm sorry Dr. Rudin...
This book is a good reference but let me tell you what its really good for. You have taken all the lower division courses. You have taken that "transition to proof writing" class in number theory, or linear algebra, or logic, or discrete math, or whatever they do at your institution of higher learning. You can tell a contrapositive from a proof by contradiction. You can explain to your grandma why there are more real numbers than rationals. Now its time to get serious.

Get this book. Start at page one. Read until you come to the word Theorem. Do not read the proof. Prove it yourself. Or at least try. If you get stuck read a line or two until you see what to do.
Thrust, repeat.
If you make it through the first six or seven chaptors like this then there shall be no power in the verse that can stop you. Enjoy graduate school. You half way there.
Now some people complain about this book being too hard. Don't listen to them. They are just trying to pull you down and keep you from your true destiny. They are the same people who try to sell you TV's and lobodemies.
"The material is not motivated." Not motivated? Judas just stick a dagger in my heart. This material needs no motivation. Just do it. Faith will come. He's teaching you analysis. Not selling you a used car. By the time you are ready to read this book you should not need motivation from the author as to why you need to know analysis. You should just feel a burning in you chest that can only be quenched by arguments involving an arbitrary sequence ${x_n$ that converges to $x \in X.$
Finally, some people complain about the level of abstraction, which let me just say is not that high. If you want to see abstraction grab a copy of Spanier's 'Algebraic Topology' and stare at it for about an hour. Then open 'Baby Rudin' up again. I promise you the feeling you get when you sit in a hottub for like twenty minutes and then jump back in the pool. Invigorating.
No but really. Anyone who passes you an analysis book that does not say the words metric space, and have the chaptor on topology before the chaptor on limits is doing you no favors. You need to know what compactness is when you get out of an analysis course. And it's lunacy to start talking about differentiation without it. It's possible, sure, but it's a waste of time and energy. To say a continuous function is one where the inverse image of open sets is open is way cooler than that epsilon delta stuff. Then you prove the epsilon delta thing as a theorem. Hows that for motivation?
Anyway, if this review comes off a combative that's because it is. It's unethical to use another text for an undergraduate real analysis class. It insults and short changes the students. Sure it was OK before Rudin wrote the thing, but now? Why spit on your luck? And if you'r a student and find the book too hard? Try harder. That's the point. If you did not crave intellectual work why are you sitting in an analysis course? Dig in. It will make you a better person. Trust me. Or you could just change your major back to engineering. It's more money and the books always have lots of nice pictures.
In conclusion: Thank you Dr. Rudin for your wonderfull book on analysis. You made a man of me.

2) What has been said below is all true. Rudin really does have some excellent moments in this book, except perhaps the chapter on Lebesque integration, which is one of the crappiest expositions of the topic I have found so far. Get the texts by E. Stein (Real analysis), or Bartle's small book on lebesgue integration. There is even a probability text by A.Shiryaev ("Probability, 2nd edition") which has a truly amazing treatment of the Lebesgue integral. Anyhow, the rest of the book is excellent, concepts of single-variable analysis are very well explained, the proofs are short and enlightening. The multivariable calculus is also very well explained. I gave it 3 stars because it's not a good book for self-study. There are hardly any explanations, the beginning student will likely get really frustrated. In order to enjoy this book you either have to know analysis already, so this would be a second text, or you have to take a course that uses Rudin as textbook. Then you won't have any problems, since the teacher will probably end up explaining the stuff in class. I have heard complaints even by some of the world's best mathematicians at Princeton against the fact that Rudin's text is so terse. To me that's not even impressive. It's just arrogance or laziness on the behalf of the author. So don't feel too bad if you read it and find that things aren't explained very well; that's because they aren't! The false sense of reward comes from banging your head against the wall before finding the answer, and being thankful you finally got somewhere rather than committing suicide (only thing is, if anything, you may have just reinvented the wheel...) But by then, you'll have wasted

a lot of time already. If you have nothing else to do, and are incredibly patient, this is no problem at all, otherwise, it's a real waste of your time. You could also be a genius, in which case, none of these points are even an issue; then you can prove all theorems in the world, so congratulations, I look forward to meeting you! Oh yeah, and there are no solutions to the problems provided to check if your answers are right or not, so good luck.

书名：Advanced Calculus, 2nd Edition
作者： Patrick M. Fitzpatrick
出版商： Brooks/Cole (2005), 机械工业出版社影印
页数：590
适用范围：数学系与理工科其他专业的本科生
预备知识：高中数学
习题数量：较大
习题难度： 具有一定难度
推荐强度：9.3

[作者简介] Patrick M. Fitzpatrick拥有格兰特大学博士学位，是纽约大学科朗研究所和芝加哥大学的博士后，1975年进入马里兰大学College Park分校任教，现在是数学系教授和系主任，同时它还是巴黎大学和佛罗伦萨大学的客座教授。他的研究方向是非线性泛函分析，在该方向著有50多篇论文。
书评: 本书以清晰、简洁的方式介绍了数学分析的基本概念：第一部分讲述单变量函数的微积分，包括实数理论、数列的收敛、函数的连续姓和极限、函数的导数和积分、多项式逼近等；第二部分把微积分的概念推广到多维欧几里得空间，讨论多变量函数的偏导数、反函数、隐函数及其应用、曲线积分和曲面积分等。 数学分析已经根植于自然科学和社会科学的各个学科分支之中，微积分作为数学分析的基础，不仅要为全部数学方法和算法工具提供方法论，同时还要为人们灌输逻辑思维的方法，本书在实现这一目标中取得了引人注目的成果。本书一方面按传统的和严格的演绎形式介绍微积分的所有主题，另一方面强调主题的相关性和统一性，使读者受到数学科学思维的系统训练。 本书的一大特点是除了包含必不可少的论题，如实数、收敛序列、连续函数与极限、初等函数、微分、积分、多元函数微积分等以外，还包含其他一些重要的论题，如求积分的逼近方法、Weierstrass逼近定理、度量空间等。例如本书专门用一章讨论度量空间，从而把在欧几里得空间讨论微积分时使用的许多概念和导出的结果扩展到更抽象的空间中，引导读者作广泛深入的思考。 另外，与第一版相比，第二版增加了200多道难易不等的习题。全书贯穿了许多具有启发性的例题，并且本版还为教学考虑进行了许多实质性的改动，例如将选学材料与前后内容的关联度降到最低，单独放置，既不影响教学和读者自学的进度，又能让读者集中攻破一些难点，这样使得全书的叙述更简洁、更自然。本书曾于2003-2004年作为马里兰大学教材。（高威）

国外评论摘选

1) A great book. Starts with two very good chapters on linear algebra, adapted to the needs of calculus, and then proceeds to introduce you to the contemporary way to do multivariate calculus, including existence theorems connected to completeness. Very thorough treatment of integration, including integration of forms on manifolds, up to the Stokes theorem, built upon a

fine chapter on differential manifolds, exterior differential forms, riemannian metrics, etc. Good illustrations and beautiful typesetting add to the joy of reading it. Plenty of exercises and chapters on applications to physics and differential geometry.

2) This is the best book on mathematics I've ever come across. The superbly written text succeeds in guiding the reader in an easy, clear-cut, graceful way through the realm of what he modestly calls "Advanced Calculus". Some minor misprints are to regret, but they don't even come close to blurring the fact that this is - no doubt about that - an unsurpassable masterpiece.

3) As Spivak's "Calculus on Manifolds", this book is labeled with a very modest title. It should be something as "All you wanted to know about analysis on manifolds but were afraid to ask". This book is a must-reading for the analyst. It covers everything from the most basic vector space concepts up to the fundamental theorems of classical mechanics, running through multivariate calculus, exterior calculus, integration of forms, and many topics more, always keeping a very modern and rigorous style.
The undergraduate may find it a little difficult, but the effort is worth it. For the graduate student and the working mathematician it is an almost-daily reference.

4) This book is out of print, but is available from Sternberg's website. Search on his full name at Google.

书名：Advanced Calculus, 5th Edition
作者： Wilfred Kaplan
出版商： Addison Wesley (1991), 电子工业出版社影印
页数：741
适用范围：理工类本科高年级学生与研究生
预备知识：高中数学和初步的微积分学基础
习题数量：大
习题难度： 有难度
推荐强度：9.5

[作者简介] Wilfred Kaplan于1939年在Harward大学师从Hassler Whitney 获得博士学位，后任教于Michigan大学。
书评:
本书除了全面地介绍微积分的知识，还介绍了线性代数、矢量分析、复变函数、以及常微分方程、偏微分方程等方面的知识。全书共分为10章：前两章介绍了线性代数和偏微分；第三章介绍了散度、旋度和一些基本的恒等式，还介绍了n维空间中的张量；第四、五章介绍了积分理论，包括定积分、重积分、曲线积分、曲面积分、Stokes公式等；第六章介绍级数理论；第七章介绍Fourier级数理论；第八章介绍复变函数的解析理论；第九章介绍了常微分方程理论；第十章介绍了偏微分方程。本书内容丰富，编写深入简出，在每一章都有相当篇幅的内容打了"*"号，这些内容属于基础理论的深化与拓广，可供教师教学时选用，或供基础好的学生选读。 本书的前身是作者应他的一位工程学同事的建议所著，目的是让工科学生在掌握初等微积分的基础上进一步扩充数学知识，提高数学水平与能力。初稿写成后，曾用于工科大三学生的教学。付诸印刷后，被Michigan大学指定为理工科高年级学生的教材。 因为本书的写作初衷是提供给工科学生，并且作

者认识到数值方法具有实用价值和帮助读者更深入的了解微积分理论，所以本书不仅介绍了理论知识，还涉及到相关数值方法，这也是本书的一个特点。 本书另一个特点是十分方便读者自学自测。比如说，本书中的定义都有明确标示，所有的重要结果都作为定理以公式的形式给出；书中不仅提供了大量难易不同的习题，更给出了习题答案；此外，还提供了大量的参考文献，并在每章的末尾给出了推荐阅读的书目。 本书为方便教师安排教学进度，注意在各章有机联系的同时，尽量减少每章节对前面章节知识的依赖程度。作者还在序言中为一学期每周四小时的课时提供了具体的教学安排建议。 (高威)

国外评论摘选

1) This book is simply the best that I have found for math texts. Kaplan does not expect much from the reader; he explains basically everything besides Calculus I material. Kaplan's writing is lively and is (relatively) easy to read. He gets to the point and keeps everything easy to follow. I am still in awe about how much material (look below) he was able to fit into this relatively small book and still keep it so clear. The examples are clear and concise. The problems in the book compliment the understanding of the material; they start out easy and guide the reader to do more difficult problems. This book is MORE THAN SUFFICIENT FOR SELF-STUDY.

2) Any student who is taking analysis/advanced calculus course should read chapter 2 of this book, especially if he is confused or is struggling on the excellent but relatively abstract/ concise texts of Rudin, Apostal, Bartle, Marsden et al. I've never seen a book which can explain the concept of Jacobian and Inplicit function theory in such a clear way!!

3) It is good and clear book. Excellent for undergrad students who want to dig into calculus a bit deeper. But it is too easy for an advanced undergrad or a grad student in any technical field. I recommend the books published by Springer.

书名：Advanced Calculus
作者： Lynn H. Loomis, Shlomo Sternberg
出版商： Jones & Bartlett Pub (1989)
页数：592
适用范围：大学数学系本科生教材
预备知识：高中数学
习题数量：大
习题难度： 中等
推荐强度：9.6

[作者简介]Shlomo Sternberg，美国Harvard大学教授，他于1957年在约翰霍普金斯大学获得博士学位。Shlomo Sternberg是一位杰出的数学家，尤其因他在微分几何上的贡献而闻名。

书评:
本书第零章是关于集合、映射等基础知识，接下来对向量空间作了介绍；第三章引入了微分的概念，接下来作者又对紧性、完备性和点积空间进行了介绍。第六章是有关微分方程的简单讲解，第七章介绍了多重线性函数，第八章引入了积分，第九第十章介绍了可微流形以及流形上的微积分问题，第十一章介绍了外微分，第十二章介绍了位势理论，而最后一个章节对微积分在经典力

学上的应用作了介绍，向读者展现了数学的威力。 本书是一部优秀的分析教材。与一般的微积分教材不同，它大体上可以分为两个部分：第一部分介绍了赋范向量空间上的微分知识，第二部分主要介绍了可微流形上的微积分知识。本书既有基础的章节，例如第一第二章对于向量空间的介绍，也有对于读者而言要求比较高的内容，比如第九章中关于切空间和李导数的概念。作者在用朴实的语言向读者介绍微积分的概念和思想的同时，也尽可能地展现了不同的观点：例如对于隐函数存在定理的证明，作者就给出了三种证明方法，揭示了数学的魅力。本书的另一大特色在于丰富的习题，练习题的题量大，并且难度不一，作者还把一些重要定理的证明放在了习题中，因此对于读者而言，尽可能多地完成书后习题可以更好的把握和巩固知识，提高分析能力。 本书可以根据教学的需要选取部分章节，程度较好的数学系本科生也可选用此书作为微积分的课外读物。 （徐晓津）

书名：Problems and Theorems in Analysis
作者： George Polya and Gabor Szego
出版商： Springer Verlag (1978)
页数：第1卷389页；第2卷391页；共780页
适用范围：数学专业高年级学生与研究生，数学教师与数学工作者
预备知识：数学分析，高等代数，复变函数
习题数量：第1卷776道；第2卷884道。这是一套习题书
习题难度： 难，有的习题甚至为研究者的最新成果，难度很大
推荐强度：9.8

[作者简介] George Polya（1887-1985）匈牙利数学家，早年在苏黎世瑞士联邦理工学院任教，后入美国籍，1942年起在美国Stanford大学任教。Polya 在数学的广阔领域里都有深入的研究，特别在泛函分析、数理统计和组合分析等方面尤为突出。Polya不仅是数学家，也是一为优秀的教育家，他始终把高深的数学研究和数学的普及与教育结合起来。
Gabor Szego（1895-1985）匈牙利数学家，早年在柯尼斯堡大学任教，后入美国籍，也在美国Stanford大学任教。他主要的贡献是在数学分析与数理方程方面。《分析中的问题与定理》一书是George Polya 与 Gabor Szego 最著名的著作。Polya曾经这样评论他与Szego的合作：这是一段美妙的时光；我们专心致志、充满热情地工作。我们有着同样的背景。我们象同时代其他匈牙利数学家一样，受到Leopold Fejér的影响。我们都是那个为中学生创办的强调解题的刊物 Hungarian Mathematical Journal 的读者。我们又对同样的课题、同样的问题感兴趣，但往往是一个人对某一个课题知道得多，而另一个人对其他的课题知道得多。这是一次绝妙的合作。我们的合作成果-《分析中的问题与定理》，是我最好的工作，也是Szego最好的工作。

书评:
本书两卷，共分九个部分。第一部分主要收录无限序列与无限级数方面的问题。第二部分是有关积分的各种问题。第三、第四部分是关于单复变量函数的问题，内容包含了数学系本科生与研究生的复分析课程中的主要问题。第五部分主要涉及代数的零点确定问题。第六部分讲多项式与三角多项式。第七部分为行列式与二次型的问题。第八部分为数论方面的题目。第九部分为数学中与几何有关的一些问题。 本书与其说这是一部教科书，不如说这是一部字典，因为它收录了分析

学中的各种问题和定理。这是一本有着突破传统意义的书。它对问题巧妙的系统性安排与归纳，给学生创造了自主性思考的可能，最大程度上启发学生的研究能力和创新能力，这也是它不同于其他一些平庸的习题参考书的地方。作者甚至试图用很多哲学的观点来阐释它所选出的题目的代表性，比如有关特殊和一般的问题，要知道早期著名的数学家迪卡尔曾经说过："我学数学是为了追求最终的哲学。"正是这种理念的融入，使得这本书在学术界的地位尤为突出，不只是学生，很多教授和数学工作者都以此书为参考书，并对此书给予了高度的好评。
[零星感悟] 什么是好的教育？给学生一套完善的体系然后让学生在这样的体系下寻找机会自己去发现和解决问题，这样的完善的体系才是好的教育的关键。此习题书不同于其他习题参考书的特点也就在此。它给我们数学系高年级学生与研究生提供了在不同主题下精心安排的问题，启发我们独立思考和研究问题的能力，是一本不可多得的分析习题书籍。 第一部分的习题139让我们明白了很多问题就像两个点决定一条直线一样，是有两个极端的线性组合而得出的结论。 第六部分的习题92让我们明白了掌握一个领域的知识就像了解一个城市的所有交通路线。真正的掌握就是从任何一个出发点，你都可以找到最短的路线达到你想要达到的目的。 (刘东弟)

书名：Functional Analysis
作者： Walter. Rudin
出版商： McGraw-Hill Book Company ISBN: 0-07-054225-2
页数：397
适用范围：数学类专业本科高年级学生和研究生
预备知识：数学分析 复分析 实分析 线性代数
习题数量：中等
习题难度： 中等偏难
推荐强度：9.5

书评： W.Rudin的《泛函分析》是一本分析数学方面的经典名著，多年来一直被国外一些高校用于研究生教学。 全书由三部分组成，第一部分是线性泛函分析基础，作者在线性拓扑空间的框架下建立了开映射定理、 闭图像定理、逆算子定理、共鸣定理和线性泛函延拓定理等基本定理，介绍了赋范线性空间的对偶性、 紧算子的概念与性质。作为这些理论的应用，作者还专辟一章介绍了Stone-Weierstrass定理、插值定理、 不动点定理、紧群上的Harr测度等知识。第二部分介绍了Fourier变换和广义函数理论， 并给出了这些理论在微分方程方面的应用。第三部分在Banach代数的基础上， 介绍Hilbert空间上有界正规算子的谱理论，并进一步建立了无界正规算子的谱定理，最后还介绍了 类算子半群。第一部分是全书的基础，第二部分和第三部分则是可供平行阅读的两个独立部分， 读者可根据需要选择使用。全书叙述严谨，条理清晰，理论的展开较为详尽。 该书既可用作泛函分析课程的教材，也可供数学工作者查阅参考。（童裕孙）

国外评论摘选
1) Hardly can I find words to highlight the goodness of this book. As mentioned by other readers ,it provides elegant, direct and powerfool proofs of the three theorems which constitute the cornserstones of functional analysis (Hanh-Banach, Banach-Steinhaus and Open mapping). These theorems are, in addition, studied in their most general context, namely topological vector spaces.

Specially appealing is its treatment of distributions' theory. It is, as far as I know, the only text which start by defining the rigurous topology on the set of test functions and then obtains the convergence and continuity of functionals (distributions) in terms of this topolgy, which is, indeed, the only way to present and gain insight into these concepts and to reach some results such as completness. In doing otherwise one risk definitions can emerge as artificial and rather arbitrary.
It is, without any doubt, a must have book for those with interest in pure mathematics as well as for those who, eventually, realize that the only way to dominate their area is saling through mathematics.

2) No other book covers the elements of distributions and the fourier transform quite like Rudin's Functional Analysis. This is a must for every budding PDE-er!

3) I enjoy perusing Rudin's "Functional Analysis" at this stage in my life. It is fairly nice tome for functional analysis, and its general treatment of topological vector spaces (as opposed to the standard Banach space examples studied in a typical functional analysis class) is now well-received.
However, as a student, I was put off by this book. At times, I found it difficult to tie the theory present to the basic examples which were relevant at the time (such as L^{p spaces). For a first time learner, I would suggest the book of Kolmogorov and Fomin (which is a Dover book, by the way), and would wait until later for this book.

4 单复变函数¶

复变函数论是数学系本科生的必修科，也是很多数学分支的研究生课程，其重要性是公认的。
粗看起来，数学分析中的导数和积分推广到复数上就是复分析了。但是复的可导函数，即解析函数，具有实的可导函数不具备的优美性质。它中的一些主要定理如柯西定理、柯西积分公式、刘维尔定理、留数公式等都有强大的威力。
由于单复变函数论的基本理论非常成熟，国外的优秀教材也比较定型，我们下面介绍的基本教材都可以作为首选。
学习复变函数前学生应有非常扎实的数学分析基础，特别是无穷级数和广义积分的收敛性。

书名：Complex Analysis, 3rd edition
作者：Lars V. Ahlfors
出版商：McGraw-Hill (1979)
页数：331
适用范围：大学数学系本科生,数学专业研究生
预备知识：数学分析和线性代数
习题数量：小
习题难度： 中等
推荐强度：10

书评： 出自数学大家之手的这本书已经公认为单复变函数论的经典著作， 既可以选取部分内容作为我国综合性大学数学系本科生的复变函数论教材， 又可以用来作为大学高年级学生和研究生的选修课内容，同时它又是从事复分析研究的标准参考书。 有关单复变函数论的教材、参考书不下几十种，但是除了干巴巴的概念、 定理的正确叙述与严格证明之外提供大量解释性文字的书本并不多见，而在这些叙述中既没有多余的话， 又能使读者开阔视野并感受到作者深厚功力的更为少见，本书恰恰为其中的佼佼者。这本书已经出了三版。 在这第三版中大部分内容未作更动，叙述依然简洁而流畅，但是作者彻底改写了第八章， 以层论的观点描述Riemann面上整体解析函数的存在性，使经典的内容现代化。（张锦豪）

国外评论摘选
1) This book has been, since its first edition in 1953, the standard textbook for rigorously learning complex analysis, and not without a reason. The wonderful theory of this branch of mathematics is appropriately emphasized and thoroughly constructed, leading to more general and precise results than most textbooks. While the constant appearance of new texts on the field can only help appreciate the subject from a different perspective, few give you such a deep and serious treatment like this gem. Postscript: An earlier reviewer claims that Ahlfors never defines the set of complex numbers, while this is indeed done in the fourth through sixth pages in a much more analytical way than generally found elsewhere. It is quite possible to dislike this author's style or approach (or anybody's for that matter), but it would be difficult to charge Ahlfors with being sloppy with his writing.

2) How can anyone fail to read this book? The exposition is rigorous, coherent, precise without being either pedantic or overwhelming. A certain level of mathematical maturity is requisite, such as one might acquire in the course of digesting Rudin's "Principles of Mathematical Analysis" or Apostol's book. This is not a compendium of results and exercises for engineers or physicists, it is a concise introductory text in pure mathematics. In that sense it is too abstract and proof oriented for that aforementioned audience which would be better served by a text in mathematical methods. Even pure mathematics students would benefit from supplementing this book with more detailed, computationally oriented books such as Conway or Boas. It's unrealistic to expect to find everything in one text and to further expect it to remain cogent and approachable. Ahlfor's beautiful little book has justifiably remained a classic for four decades.

3) I'm not sure why the other reviews are so positive. The book is very thorough and rigorous I'm sure, but the explanations are terrible. Everyone I've talked to in my class agrees that it's extremely difficult to learn from if you don't already know complex analysis, because the definitions and order of treatment are very un-intuitive. Example: residue at a is defined as the number R that makes $f-R/(z-a)$ the derivative of a single-valued analytic function in $0<|z-a|<\delta;$ why didn't he even mention that it's the coefficient of $1/(z-a)$ in the Taylor expansion? And he didn't even give any examples of specific residues. I ended up using a mathematical methods for physics book; it was the only way I could develop any kind of intuition for the subject.

书名：Introduction to complex analysis
作者： Kunihiko Kodaira
出版商： Cambridge University Press (1984)
页数：256
适用范围：大学数学系本科
预备知识：数学分析和线性代数
习题数量：无
推荐强度：9.8

书评： 日本岩波讲座的基础数学中由小平邦彦撰写过三本关于复分析的小册子，其中的I、II 分册被译成英文出版为本书。其内容与我国综合性大学的复变函数论课程基本相符。 本书体现了数学大家小平邦彦一贯的写书风格，起点低，过程详尽，深入浅出，流畅而易读。 本书以复可微（有连续导数）的条件引入全纯函数的概念，为后续的处理带来很大方便。 同时以Cauchy积分定理为主线，从简单到复杂，循序渐进地揭示了这个定理的成立与拓扑的关系。 以远较一般教科书为多的篇幅介绍了Riemann球，引入"局部坐标""齐次坐标"等概念， 并顺理成章地接着用来导出分式线性变换的群伦性质。 本书处处体现了小平邦彦深厚的研究功力与广阔的视野。 对于希望将来在Riemann面、Teichmuller空间、多复变函数、复几何、 代数几何等方面进一步深造的有志者来说是一本不可多得的基础好书。（张锦豪）

书名：Functions of One Complex Variable
作者： John B. Conway
出版商： Springer-Verlag (1973)
页数：313
适用范围：大学数学系本科或数学专业研究生一年级

预备知识：数学分析和线性代数
习题数量：大
习题难度： 从易到难都有，大部分中等
推荐强度：9.6

书评： 本书虽然为大学生学习单复变函数论而写，但是内容十分丰富。 作者用整个一章介绍最大模原理，除了我国教材中通常出现的内容外， 还证明了Hadamard三圆定理与Phragmen-Lindelof定理。 对多复变函数的近代理论有深远影响的Runge定理、Mittag-Leffler定理、Weierstrass定理等也给予详尽的介绍。本书还包含了大Picard定理等值分布理论的基础。 同时以解析函数芽层的现代观点描述了解析延拓这一重要现象，并引入Riemann面， 进一步再用复流形的现代概念进行提升，非常精彩。 本书内容远远超过我国综合大学复变函数论课程的需要， 所以同时可以用来作为大学高年级与研究生一年级的选课教材。 本书观点颇高，论述严谨，排版紧凑。 虽然作者声称只需基本微积分以及关于偏导数等少量预备知识即可阅读本书， 但由于介绍预备知识的叙述水平超过了一般的数学分析，因此初学者若没有一定的数学天赋则很难自学。 但毫无疑问，这是每一位学习或应用复变函数论者的极好参考书。（张锦豪）

国外评论摘选
1) We're using this book for my graduate level complex analysis course, and over all, I'm pleased with it. Aside from some goofy notation (i.e., an empty box to represent the empty set?), it's pretty well written. The pace of the text isn't too fast or too slow, and there are plenty of exercises of a varying degree of difficulty to help you learn the material.

2) An ideal text for a first-year graduate students in mathematics studying Compex Analysis. And this depend how the professor present the material. The exposition is complete and very clear, including a lot of optional material for the curious. which could be very useful to those preparing for a qualifying exam in analysis at the PhD level.

3) This book was the recommended textbook for a course in Complex Analysis I took at college. I had already done a 1st course on analysis, but that didn't help me too much. This book, littered with loads of proofs and lemmas, is a little too terse, and the author expects students to understand a lot on their own. Concepts in Complex Analysis need to be demonstrated using examples, and diagrams, if possible. Like for eg. the concept of branches in complex functions. The book starts of defining the complex logrithmic function. The author never says what a branch exactly is. He writes down a hell lot of proofs and expects the student to figure out that the complex logarithm is infact a multi-valued function, and that a branch is essentially a "slice" of this multivalued function. Similiar problems crop up when the author discusses fractional linear transforms. Instead of showing whats happening with simple diagrams, the author makes things look extremely complicated with his equations and theorems. This book makes learning complex analysis a very mechanical exercise, devoid of all fun.

书名：Complex Analysis, 3rd edition
作者： Serge Lang
出版商： Addison-Wesley (1993)
页数：321
适用范围：大学数学系本科或数学专业研究生一年级

预备知识：数学分析和线性代数
习题数量：中
习题难度： 中
推荐强度：9.7

书评： 本书第三版较之第一版增加了许多超出本科生学习的内容。全书分为三部分， 其第一部分与我国综合性大学的复变函数论教材大致相当，第二、第三部分为进一步学习的内容， 可供大学生高年级或研究生低年级的选修课之用。本书将Cauchy定理分为两部分介绍，从局部到整体， 从简单到复杂，使读者很容易接受。特别是在一般Cauchy定理的证明中借用了分析味更浓的Dixon证明， 避开了初学者理解拓扑内涵的困难。将对数函数的介绍与解析延拓的放在一起，使读者从更一般的角度理解如何选取多值函数的单值支。 本书的另一亮点是介绍了Zeta函数并用来证明素数分布定理。作者是位著名的数学家， 学识广博，擅长撰写数学基础类教材。一些深刻的定理在他的处理下通俗易懂， 所以本书虽然述及到许多深入的复分析内容，读来也是毫无困难，值得向初学者推荐。（张锦豪）

国外评论摘选
1) A person with absolutely no knowledge of complex numbers could begin with page one of this book. However, I think that some exposure to analysis is helpful before finishing the first chapter, but not necessary. I found this book easier to read and understand than some real analysis books, yet it helped me further understand real analysis in the process. I'm sure this is due to mere repetition of some of those concepts over a different field. As the author mentions in his foreword, the first half of the book can be used as an undergraduate text (Jr/Sn years) and the second half can also, but I would NOT have enjoyed it in undergraduate studies. I found it worthy of a first course in complex numbers at the graduate level. I especially liked it after studying real numbers. The placement of the chapter subject matter can be altered (to some degree) to ones liking. I think Lang has provided good examples and problems. There's a solutions manual (by Rami Shakarchi) for this text somewhere.

2) if you want an introduction to complex analysis, I advise you to pass on this book, and read Churchill and Brown's introductory book. Having said this, part I of Lang's book will seem mostly review if you follow my advice. Part II, on Geometric Function Theory, is more advance material that is presented reasonably well.

5 多复变函数¶

多复变函数论是现代基础数学的重要分支，除了单复变函数论外，它还要使用代数、拓扑、泛函分析等很多深刻的知识。所以一般只设为研究生课程。

书名：An Introduction to Complex Analysis in Several Variables, 2nd edition
作者：Lars H?rmander
出版商： North-Holland (1973)
页数：254
适用范围：数学专业研究生
预备知识：实分析与泛函分析初步,复变函数,微分形式略知一二
习题数量：无
推荐强度：10

书评： 作者于1964年在Stanford大学介绍了多复变函数论，对其讲义稍作修改后即成本书第一版。 除了最后一章最后一节外，第二版基本保持原样。本书前后观点统一，读来似有一根红线贯穿始终。 作者处理单复变的预备知识的第一章会给习惯复变函数论方法的人以耳目一新的感觉。 反映作者将超定偏微分方程理论应用于多复变函数论所做巨大贡献的第四章，是本书的最精彩部分。 这一章系统地介绍了拟凸域上$\bar{\nabla-$算子的 理论及其应用，值得反复阅读，细细品味，否则会入宝山而空返，到此一游而已。本书叙述简洁而流畅。 阅读本书无需很多单复变函数论的知识，但若没有扎实的实分析功底，读时有如陷入泥泞之地。 目前本书已被公认为多复变函数论的经典著作，是准备从事多复变函数论、复几何、 超越代数几何等方向研究的必读课本，也可以用来作为大学高年级学生的选修课内容。（张锦豪）

书名：Introduction to Complex Analysis Part II, Functions in Several Variables
作者： B. V. Shabat
出版商： American Mathematical Society
页数：371
适用范围：数学专业研究生
预备知识：复变函数
习题数量：大
习题难度：中等
推荐强度：9.5

书评： 本书为作者两卷书的第二部，第一部讲单复变函数。本书起点较低，通俗易懂。 对于深受前苏联教育体制影响的我国大学培养出来的学生来说， 它的内容与叙述方式以及对预备知识的要求都非常贴切我国学生的知识结构。 本书对概念的介绍非常清楚而到位，例子较其它书籍为

多，还附有大量难度适中的习题，因此是值得推荐的多复变函数论入门书。但是本书没有提供系统的方法供研究者参考。 本书可作为大学生高年级与研究生的选修课教材。（张锦豪）

书名：Topics in Complex Function Theory I, II, III
作者： Carl L. Siegel
出版商： John Wiley & Sons, Inc. (1969,1971,1973)
页数：186,193,244
适用范围：大学数学系本科或数学专业研究生
预备知识：复变函数论，抽象代数初步，代数拓扑初步
习题数量：无
推荐强度：9.8

书评： 这是一本述及复分析高端论题的经典著作。本书根据作者于1964年在德国哥廷根大学 为时两学期的演讲基础上写成的。作者以椭圆积分与椭圆函数及单值化理论作为第一卷的内容开始， 继之以自守函数和阿贝尔积分成就第二卷，最后在第三卷将读者引入多变量阿贝尔积分与模函数。 本书写作风格独特：不追求概念的天衣无缝的表达，而侧重其内涵与相互联系的阐发，同时包含了有关领域的几乎所有重要结果，以及在其它地方很难找到的一些熟知结果的证明。 因此，不仅初学者能沿着作者独辟的蹊径很快到达最前沿的研究领域， 许多专家也会从其叙述中感受到这位数学大家的深厚功力，得益良多。（张锦豪）

6 代数¶

数学系本科生课程中，线性代数是最重要的，一般要学一年，其次是抽象代数。一般这些代数知识还不够，在研究生阶段又要学一些更加专门的代数，如群表示、李代数、交换代数、同调代数、环论等。

数学系使用的线性代数教材和非数学专业的教材有很大的不同，前者注重定理的证明，习题中证明题的比重很大，而后者注重计算，习题也以计算题为主，因此非数学专业的学生如果使用数学专业的线性代数教材有可能会不适应。

在此，我们介绍了几本比较基本的优秀代数教材，大多是本科生或低年级研究生使用的。

书名：Linear Algebra, 2nd ed.
作者：K. Hoffmann and R. Kunze
出版商：Prentice Hall, Inc. (1971)
页数：407
适用范围：大学数学系本科低年级教材
预备知识：微积分
习题数量：600 多道习题
习题难度：各种难度都有
推荐强度：9

使用学校：
Central Michigan University，University of North Dakota，Indian Institute of Technology, Bombay，University of Pittsburgh，University of Texas，Johns Hopkins University，West

Virginia University，University of Houston，Simon Fraser University，Washington University in St. Louis，University of Notre Dame，University of Wisconsin-Madison，Cornell University，University of South Carolina，University of Rhode Island，University of Missouri，University of Maryland，Stony Brook University，University of Michigan，Purdue University，University of Kansas，United Arab Emirates University，Rice University，Kenyon College，Temple University，Louisiana State University，Sonoma State University，North Carolina State University，University of Iowa

书评： 这是一本线性代数方面久负盛名的教材，在麻省理工学院作为大三课程的教材使用多年，第一版于 1961年出版，对此后世界各国编写的各种线性代数教材有很大影响。比方说，北大和复旦的线性代数教材的内容和编排次序与此书非常接近。
虽然作者在序言中声称为了照顾数学基础不是特别扎实的工科学生把很多地方写的浅一些，但是据我看这本教材只适合数学系或者对线性代数基础要求很高的专业用。
本书内容完整，线性代数的所有内容都有了，甚至多重线性代数的若干内容如外积也包含了。从第一章开始就定下一个基调：本书讲解的线性代数是在任意域上的。在叙述了域的定义后，先给读者一颗定心丸：但是几乎所有例子和习题是在数域上的。接下来马上又警告大家：奇怪的域是有的，有限多个 1 相加可以等于 0. 这样的安排虽然对学生的要求比较高，我觉得还是利大于弊，可以一步到位了。另一个类似的情形出现在第五章讲行列式时一上来就定义任意交换环上的行列式。我国的教材大多把行列式定义为一个数，然而在例题和习题中很多行列式中含变元，使人觉得不很严格。提起行列式， Strang 教授在 MIT 的视频课程给我深刻印象，他并不先讲行列式的定义，而是先将它需要满足的几个关键性的性质，最后推出只存在唯一的方式来定义行列式满足这些关键性质。本书基本上也是按照这样的方式处理的，虽然费力一些，但把它的本质讲的比较透彻，这比急功近利地急急忙忙讲行列式的计算技巧深刻。更进一步，接下去作者马上讲交换环上模的概念以及交错型和外积，虽然显得有些过火，但这样的安排还是合情合理的，因为这些内容和行列式的关系太密切了。也许作为课堂教学这些内容只能跳过，否则一个学年讲不完这本书。
后半部分的内容是线性变换的各种标准形、内积空间以及它上的线性算子，比较常规。大量习题和这些习题的覆盖度大也是本书的一个特色。（杨劲根）

国外评论摘选
i) I got this book for my Linear Algebra class about four years ago. This is a great book if you are getting a degree in mathematics. It won't help if you are just trying to get by the class and don't like math. It is not very practical but if you are looking for a real math book on Linear Algebra this is it. It contains a wealth of theorems that only a math lover would appreciate. If you really want to learn about Linear Algebra from a rigorous mathematical point of view this is it. This book taught me so much.
ii) This was the textbook they used to use at MIT in the past few decades. Virtually, however, nobody uses this book in a regular undergraduate course anymore. Instead of developing the ideas in the familiar context of the real numbers, Hoffman and Kunze give a more abstract (and general) discussion. For example, the theorems about determinants work in all commutative rings. The rigorousness and the wealth of information are overwhelming for most undergraduates to handle. You will not learn anything if you just glance through the pages. Every line requires deep thought. Down-to-earth applications are not included. So I do not recommend this book for engineers.

书名：Lectures on Linear Algebra
作者：Gelfand
出版商：INTERSCIENCE
PUBLISHERS. INC. （60 年代）
页数：185
适用范围：大学数学系本科低年级教材
预备知识：微积分
习题数量：少
习题难度：比较大
推荐强度：8.5

书评：前苏联有一些数学大师写过一些给大学本科用的数学教材，就象我国华罗庚先生写过《高等数学引论》一样。作为泛函分析祖师爷级别的人物 Gelfand 写的线性代数讲义也不失为一本极好的教材。
本书是学院式的，结构和叙述十分严格和简洁，具有 Bourbaki 的风格，但又不象 Bourbaki 那样地追求一般性，所以不要求读者有很高的起点。不象一般的线性代数教材从解线性方程组或行列式开始，本书从 n 维内积空间作为第一章，甚至包括复内积空间，可谓开门见山。第二章讨论线性变换，特别是正交变换和酉变换。第三章是若当标准型，第四章是多重线性代数。
本书原文是俄文的，在 60 年代有多种文字的译本，包括中文本，由于是繁体字，现在已打入冷宫了。我在大学刚毕业不久（1971年）阅读此书得益非浅，感觉对线性代数乃至泛函分析有更深的认识，这是我把这本老书向读者介绍的主要原因。本人认为此书层次较高，适合于已经有一些初等线性代数知识的人读，它不一定对提高解题能力有利， 但对提高数学观点是绝对有益的。（杨劲根）

国外评论摘选
1) The professor who recommended this book made the comment that every time you re-read it, you notice something else that you missed the last time you read it. This is absolutely true. I must say, the first time I picked up this book, I did not like it. The notation was not what I was used to, and the book dives right in, assuming a lot of background (matrices, determinants, etc.) but covering material which many people find boring (bases, etc.). However, when you read deeper, there's a lot here. Once you get past the ugly notation, the proofs are extraordinarily clear. And in spite of the books small size, there is a remarkable amount of motivation and discussion.
Like the other reviewer said, this is not a book to learn linear algebra from for the first time: this is an advanced book that is useful for graduate students who have already had a linear algebra course and who want to learn more topics, or understand topics on a deeper level.
This is an excellent book; the bottom line is that it's so cheap that there's no excuse NOT to buy it.
2) This is the best treatment of linear algebra that has been published. It starts with n-dimensional linear spaces and ends with an introduction to tensors. An excellent description of dual spaces is concisely presented. NO INDEX!

3) Lucid and clear notation , complete explanations . This books was first published in 1937 but until now it remains best text book in the field .

4) This is a good book if all you need is a condensed reference on theorems and proofs and it assumes that you go for practice (and instruction) elsewhere. If you are trying to actually learn linear algebra (especially on your own and especially if you want to learn how to solve practical problems) get one of Gilbert Strang's books and watch his video lectures at MIT web site. Another thing that I dislike about the Gelfand's book is that it puts too much emphasis on index notation - instead of matrix notation which is natural for linear algebra, almost all formulas and theorems are presented at very low level using expressions consisting of variables with multiple indices. Naturally it gets very messy and hard to follow at times. This doesn't present any more information than equivalent matrix notation but introduces unnecessary complexity and makes things that are really easy to understand very confusing.

书名：Linear Algebra Gems
作者： David Carlson, Charles R. Johnson,David C. Lay, A. Duane Porter
出版商： The Mathematical Association of America (2000左右)
页数：328
适用范围：大学数学系本科低年级参考读物
预备知识：微积分、线性代数
习题数量：123 题
习题难度：大
推荐强度：8.5

书评： 这本书不是线性代数的教材，而是兴趣浓厚的学生或教线性代数的老师的参考读物，同类 的书并不多见。美国最大的数学组织是美国数学会（AMS），其次就是美国数学协会（MAA），它 的主要目标是推动数学教学，尤其是大学本科的数学教学，和 AMS 一样，它也有不少出版物， 其中最主要的是美国数学月刊，简称 Monthly, 是一份历史悠久并且享有盛名的数学教育刊物， 上面的文章质量高于我国的数学通报，另有一个刊物 College Mathematics Journal，我国数学界不太熟悉。 本书是从多年的 Monthly 和 College Mathematics Journal 中选出几十篇与线性代数有关 的短文，又约稿请人写了若干文章，总共74篇按内容进行分类而构成的。大部分文章是教学心得和 若干有名的定理（比如若当标准型）和习题的进一步探讨。各篇文章互相独立，每篇文章一般在一个或半个小时内读完， 非常适合于充当大学生课外读物，特别是对大学生数学竞赛很有帮助。
内容共分十部分如下
PART 1 - PARTITIONED MATRIX MULTIPLICATION
PART 2 - DETERMINANTS
PART 3 - EIGENANALYSIS
PART 4 - GEOMETRY
PART 5 - MATRIX FORMS
PART 6 - POLYNOMIALS AND MATRICES
PART 7 - LINEAR SYSTEMS INVERSES AND RANK
PART 8 - APPLICATIONS
PART 9 - OTHER TOPICS
PART 10- PROBLEMS
象第 1，2，5，6，7 部分一看就知道有不少有技巧性的内容。第十部分是习题，大部分是竞赛级别的题。 （杨劲根）

书名：Algebra
作者： Michael Artin
出版商： Prentice Hall (1991), 机械工业出版社影印
页数：618
适用范围：大学数学系本科基础数学一学年的教材
预备知识：微积分和线性代数
习题数量：大
习题难度： 各种难度都有
推荐强度：9.8

书评： 本书是美国大数学家美国科学院院士 Michael Artin 的力作，从70年代早期开始就作为麻省理工学院数学系高年级 本科生教材，是一本极具特色的优秀教材，深受使用者欢迎。 与传统的抽象代数教材不同，本书以数学中的重要实例为主线索，引导出抽象的概念，对读者以启发为主，又不缺乏数学的严格性。虽然教材的主要内容是基本的代数结构，但字里行间不乏现代数学的烙印。代数数论、代数几何、表示论中的一些基本思想 也时时涌现，如整二次型的原理和应用、二次域的理想类、不定方程、紧群表示等。 全书分14章，从矩阵运算引入群概念直到最后一章伽罗华理论一气呵成，不使人感觉600多页篇幅的冗长。 本书的习题是作者20多年积累而得，很多是作者独创的习题，例如有一道2x2魔方的问题是70年代3x3魔方游戏刚问世时作者 编制的群论习题。大约有四分之一的习题有一定难度。 本人80年代在 MIT 攻读研究生期间为此课程作过多次助教，主讲人为作者本人或其他资深教授，每次大约有三十人修课，主要 学生是基础数学各专业的学生，也有一些计算机专业的本科生及研究生选修的。学生反映此课程质量很高，但比较难。
本书比较适合我国综合性大学数学系抽象代数课程的外文教材，尤其适合一学年。对于半年的抽象代数课程，则可选用部分章节。 程度较好的数学系本科生可选用此书作为抽象代数的课外读物。 （杨劲根）

国外评论摘选

1) Pretty much any introductory abstract algebra book on the market does a perfectly competent job of introducing the basic definitions and proving the basic theorems that any math student has to know. Artin's book is no exception, and I find his writing style to be very appropriate for this purpose. What sets this book apart is its treatment of topics beyond the basics--things like matrix groups and group representations. I suppose many introductory books shy away from much of the material on matrix groups in Artin's book because it involves a little analysis (and likewise for the section on Riemann surfaces in the chapter on field theory). However, Artin correctly realizes that a reasonably mathematically mature student--even one who doesn't know much analysis--will be able to profit from and enjoy the relatively informal treatments he gives these slightly more advanced topics. Of course these topics can also be found in graduate-level texts, but I for one would much rather be introduced to them via an example-based approach such as that in Artin than through the diagram-chasing obscurantism in more advanced books. I happened upon this book a little late--in fact, only after I'd taken a semester of graduate-level algebra and already felt like analysis was the path I wanted to take--but I'm beginning to think I would have been more keen on going into algebra if I'd first learned it from a book like this one.

2) I bought this book for a class that I ended up dropping. In the beginning, I hated this book. I found Herstein's "topics in algebra" much better, and more to the point. It was only when I was getting bored with Herstein that I bothered to pick this up again. I was pleasantly surprised. A lot of the material flowed very smoothly - exactly as if Artin was teaching the material to you. It must however be noted that people tend to love or hate this book. This is predominantly due to the author's writing style. Given how expensive this book is, you might perhaps want to peruse it somewhere before deciding to buy it. But if you do, you'll get a solid exposition on most of the introductory topics in algebra as well as some insight on groups and symmetry, lie groups, representation theory, galois theory and quadratic number fields. And a whole lot of intuition as well, for the more regular topics. Give this book a chance - it's worth the effort and money.
3) As an undergraduate I learned, or tried to learn, algebra from this book. Artin's pedagogical methods just didn't work for me. Although his idea of teaching through concrete, geometric examples sounds great in principle, in practice it's not so successful. It is very hard to see the forest for the trees, since Artin is so chatty and discursive. When he is discussing examples, he sometimes puts specialized results on par with more general theorems, which may be misleading. Many proofs are only sketched, and occasionally theorems are stated after their proofs, necessitating a rereading of the preceding paragraphs in order to grasp the points of the proof. The chapters on representation theory (Ch. 9) and arithmetic of quadratic number fields (Ch. 11) are nonstandard topics and interesting in themselves, but again, the level of detail tends to obscure, rather than enlighten.
The one saving grace of the book is the excellent problem sets at the end of each chapter. In doing them you will learn the algebra that the main body of the text attempts to impart.

书名：Codes and Curves
作者： Judy Walker
出版商： American Mathematical Society (2000)
页数：66
适用范围：大学数学系本科高年级参考书
预备知识：抽象代数
习题数量：小
习题难度： 容易
推荐强度：8.5

书评： 这本小册子是1999年美国数学会在 Princeton 组织的暑期学校的一门课程的讲稿，是代数几何码的入门读物。代数几何码是新发现的 一种纠错码，目前仍有大量问题在研究。本书前一半对纠错码的基本知识和若干经典的纠错码作了扼要的介绍，重点是 Reed-Solomon 码，因为代数几何码是它的推广。然后，作者不加证明地清楚地叙述了有限域上平面代数曲线的基本知识， 最后介绍了代数几何码以及好的代数几何码的构造方法。
本书的一个显著特点是提供了六个供本科生研究的课题。本人曾指导复旦大学数学系毕业班的六名学生报告这本书，并围绕 六个课题查阅文献资料，写作毕业论文，取得很好的效果。 （杨劲根）

国外评论摘选

1) The book gives an overview of algebraic coding theory. The first chapter introduces error correcting codes, the Hamming distance, Reed-Solomon codes, and concludes with a brief

exposition of cyclic codes. The second chapter discusses some upper bounds on the minimum distance of a code such as the Singleton and Plotkin bounds.
The second theme of this book are algebraic curves. Chapter 3 contains the basic definitions and some examples of algebraic curves. The concept of a nonsingular curve is explained in Chapter 4. This chapter also contains a half page explanation of the genus of a curve. The Riemann-Roch theorem is finally covered in Chapter 5.
The two themes come together in Chapters 6 and 7. These chapters discuss the basic principles of algebraic geometry codes.
This little book gives the reader a first taste of an intriguing field. The most surprising part is how much is covered in so few pages [the main text without appendices has 44 pages]. The explanations are always accessible for undergraduate students of mathematics, computer science, or electrical engineering. The prerequisites are some knowledge of abstract algebra, but most material is reviewed in the appendices.
It is a lovely little book that is written in a lively style. The book nicely complements the typical college courses on coding theory. If you want to get an idea what algebraic geometric codes are and you want a quick answer, then this is the book for you.
2) There is a free version of the book available on the website of the University of Nebraska-Lincoln.

目录：

书名：Introduction to Commutative Algebra

作者： Michael Atiyah & I.G.MacDonald
出版商： Addison-Wesley Publishing Company (1991)
页数：126
适用范围：大学数学系本科基础数学高年级或研究生低年级教材
预备知识：抽象代数和点集拓扑
习题数量：大
习题难度： 较大
推荐强度：9

书评： 英国皇家科学院院士 Michael Atiyah 是当代大数学家，曾或菲尔滋奖。本书是交换代数的入门书籍，是一本优秀教材， 特别适合于代数几何、代数数论和其他代数专业的研究生使用。本书的篇幅虽小，内容却很丰富，包含了交换代数的核心内容。 学过一学期抽象代数的人可以顺利学习本书前九章，学习第十和第十一章需要点集拓扑的基本知识。正文中的定理的证明简明易懂，有很多重要的定理安排在习题中，所以要掌握此书内容必须化工夫做每一章后的大部分习题。
本书以诺特交换环和有限生成模作为重点，这正是代数几何和代数数论中出现最多的代数结构。作者在序言中说到域论没有 涉及，这可以从别的优秀教材（如 Nagata 的“域论”）中得到补充。国外很多名校的数学教授将此书作为交换代数教材的首选。我国引进此书也很早，它很受师生的欢迎。 （杨劲根）

国外评论摘选

1) Some people believe that, for getting into algebraic geometry (by this I mean Grothendieck-like AG, with schemes and all that), one needs a monolithic training in commutative algebra (something like both volumes of Zariski-Samuel, for example). I disagree. This little book seems to be specially suited to those who want to learn AG. It's a bit too brisk, specially at the beginning - if you don't already have an acquaintance with the basics of groups, rings and ideals, you may run into trouble - but very illuminating. Masterful choice of topics, great exercises (as a matter of fact, about half the topics of the book, and more specifically the ones that are directly related to AG, are treated in the exercises, some of them quite challenging) - like one said before, it looks like a "chapter 0" of Hartshorne's book on AG. The authors consciously estabilish relations between the commutative algebra and the modern foundations of AG over and over along the way, illuminating both topics. For the algebra itself, it also gets on well with Rotman's "Galois Theory" and MacDonald's out-of-print introduction to AG, "Algebraic Geometry - Introduction to Schemes", besides being the perfect preamble in commutative algebra to the books of Mumford and Hartshorne. A gem.
2)The strongest aspects of Atiyah & MacDonald's book are its brevity, accessibility to undergraduates, and subtle introduction of more advanced material.
Audience: I think an undergraduate with a solid understanding of material from a first course in abstract algebra (i.e., the chapter on rings--the modules chapter would help, but isn't necessary--from M. Artin's book 'Algebra' is more than sufficient) and some basic point-set topology from an intro real analysis course (or ch1-4 of Munkres) would be sufficient for fully appreciating the material. I think having experience in PS Topology is important for understanding parts of this book well; doing the exercises is possible if you learn it "on the fly," but I hadn't seen Urysohn's Lemma before, and even that caused me some "intuition" hangups; to fully appreciate the material, I would recommend doing a healthy number of problems in topology first.

Material: The material uses concepts from homological algebra, though in a disguised form; students with experience in category theory will find offhanded comments that recast some of the material in that language, but CT is absolutely not essential to understand the material well. It also provides exercises that lead naturally into topics from Algebraic Geometry and Algebraic Number Theory quite readily; a nice set of problems in CH1 walk a student through construction of the Zariski topology, prime spectrum, etc., and some functional properties of morphisms between spectra. Algebraic Number Theory starts showing up after chapter 4 in greater detail, and would lead comfortably into Lang's GTM on ALNT by CH9 (though I only read a bit of Lang, the first chapter felt natural).
The "details left to the reader" are usually reasonably tackled with the tools made available so far, and the book is short enough that one can cover a lot of ideas in a reasonable amount of time; the commentary made by the authors is brief, to the point, and never redundant as far as I can recall, so I consider this a highly efficient book (but not too efficient, it's self contained enough and not uncompromisingly terse).
Exercises: They are quite good, I think. Very few of them follow from "symbol-pushing" or "robotic theorem proving," and usually require some constructive argument. The exercises are mostly chosen to introduce more advanced material, and do a good job in that regard. The longer chapters have 25-30 exercises, and shorter chapters (a few pages) have maybe 10, so there are plenty of problems to do.
Hazards: The material on modules is brisk, the propositions in the first three sections on modules are mostly left without proof; however, the proofs follow from their analogues for rings, and aren't that hard, just be sure to actually do them because they are mentioned only briefly. Also, the book is not typo-free, but this only caused me one major hangup during the semester. After Chapter 3, the proofs are mostly complete, with a spattering of "left to the reader" exercises, which I usually found helpful.
Companion Material: I think Lang's 'Algebra' GTM would make a nice reference for the material on Homological Algebra and other miscellaneous things that come up in the proofs; I remember once a proof in the book required the notion of the adjoint of a matrix over a ring, and so I had to look it up in Lang, and also the basic category theory covered in CH1 of Lang would at least introduce (though in a very rapid way) the "abstract nonsense" mentioned offhandedly here and there. If you have a lot of money, or access to a good library, 'Categories for the Working Mathematician' is a slower and more thorough introduction to that language, and I would recommend at least having a look, though this isn't really central to the material from Commutative Algebra.
3) This is how mathematics texts SHOULD be written. As in technical writing, the smaller text is the better written text. Everything is clean and direct, with clairity obviously a prime consideration. One never gets mired down. The proofs are always as close to a "THE BOOK" proof as possible, with illuminating examples, and plenty of excercises, many with outlines for solution, which makes the book ideal for self study. This book is a revelation. If I had to take only one math text with me to a desert island, this would be the one.
4) This is a difficult book for undergraduates, even ones who have already had some abstract algebra. Many refer to the book's style as "terse", meaning that there is little explanation, few examples, and proofs are very condensed.

书名：HOPF ALGEBRAS

作者： MOSS E. SWEEDLER

出版商： W. A. BENJAMIN, INC. (1969)

页数：336

适用范围：大学数学系本科、数学专业研究生

预备知识：代数、环模基础理论

习题数量：小

习题难度： 容易
推荐强度：9

书评： 1941年，德国数学家H. Hopf在研究代数拓扑时引入了Hopf代数的概念。真正引 起人们对这类代数结构普遍关注的是1965年J.W. Milnor 和J.C. Moore的有关分 次Hopf代数的文章；到上世纪80年代末，量子群概念的出现及其在Knot不变量理 论中的应用将Hopf代数的研究推向一个新的高潮。如今，Hopf代数理论正在诸如代 数群、李代数、表示论、组合论以及量子力学等学科的研究中发挥着重要的作用。M.E. Sweedler所著的\textquotedblleft Hopf Algebra\textquotedblright是历史上第 一本系统介绍这方面Hopf代数知识的书籍。
这本书是从Sweedler给研究生的系列讲座内容中整理出来的，介绍的对象主要是非分 次的Hopf代数。域k上一个增广（augmented）代数H，若带有一个余结合的 （coassociative）和余单位的（counitary）代数映射$\Delta \colon H\rightarrow H\otimes H$，则称$H$是一个双代数（bialgebra），Hopf代数是指带 有antipode的双代数。本书开始先引入sigma-符号；而后一步步将上世纪70年代 以来有关Hopf代数的最新结果，其中大部分是作者与其合作者当时取得的进展，呈现 给读者；最后一章以证明域上由所有有限维交换、余交换的Hopf代数组成的范畴是交换的（abelian）范畴作为结束。整本书的内容简洁，易懂，且自我包含，是一部很好 的关于Hopf代数知识的入门教材。它的不尽完美之处是没有列出任何参考文献 。
本书共有16章，有些章节非常短。前4章，给出了余代数、余模、及Hopf代数的 初步介绍，其中包括从模中构建余模的有理模构造方法及余代数的基本定理---该定 理阐明了在一个余代数中，任意有限个元素均包含于一个有限维子余代数中，故而任一 余代数都是有限维余代数的直接极限。第5章讨论了积分（integral）， \textquotedblleft积分\textquotedblright这一名称是因它很像紧群上关于 Haar测度的积分运算而得名。这一章的重要结果是证明任一有限维Hopf代数必存在 一维的积分空间；作为一个推论，本章中导出了群环的著名Maschke定理。
第6章对一个代数引入并讨论了它的对偶余代数。第7章到第9章主要介绍了度量 、smash积和外积等概念，为第13章余交换点（pointed）Hopf代数的结构定理的 证明作了前期的准备工作。
第10章的主要内容是Hopf代数作用的Galois理论；第11章对本原元进行了重点 讨论并引入了分次双代数的概念，在这一章以及后面的第14章中余代数的基本定理起 到了很大的作用。第12章考虑了shuffle代数和相关既约的点余代数万有映射性质 ，此外还讨论了divided power。
第13章中证明了著名的结论：任一既约的点余交换Hopf代数一定同构于Lie代数 或限制Lie代数的包络代数；据此导出了余交换Hopf代数的结构定理。剩下的两章 讨论了仿射群和由交换、余交换Hopf代数构成的Abelian范畴。 （朱胜林）

7 数论¶

数论是数学中历史悠久但又有生命力的分支，也很有趣味。数论有分初等数论、代数数论、解析数论等。初等数论主要用初等的方法讨论整数的性质，如同余方程、不定方程、二次剩余等。
代数数论是讨论代数数的分支，要使用很深的代数工具。近年来，代数数论和代数几何合起来形成了一门称为算术几何的新分支，是非常艰深的。解析数论则是用数学分析和复变函数论来研究数论的问题，当今数学中第一号未解决问题黎曼猜想就属于解析数论的范围。
大学本科阶段学习一些数论是有用的，特别对学习抽象代数有很大帮助。研究生阶段一般只有数论专业的学生才学数论。
我们在这里介绍的数论教材大多是供数学专业本科生使用的。

书名：Elementary Methods in Number Theory
作者： Melvyn B.Nathanson
出版商： Springer-Verlag
页数：509
适用范围：大学数学系本科基础数学学生、数学专业研究生
预备知识：微积分
习题数量：大
习题难度： 中等，多数习题很容易
推荐强度：9.5

书评： 本书是Springer-Verlag出版的研究生系列教材中的一本，编号第195，2000年出版。全书分 为三个部分，第一部分介绍了初等数论的基本内容，整除性，同余，原根，Gauss二次互反律，有限 交换群上的Fourier分析，以及abc猜想的一个简单介绍。第二部分讨论了一些算术函数的性质， 给出了素数定理的初等证明。第三部分介绍了加法数论中的三个问题，即Waring问题， 正整数表为整数的平方和的问题，以及分拆函数的渐进估计的问题。 本书的一个特点是给出了许多深刻的数论定理的初等证明， 比如，Selberg的素数定理的初等证明，Linnik关于Waring问题初等证明， 一个整数表示为偶数个整数的平方和的个数的Liouville方法，以及Erdos关于分拆函数的渐进估计的结果。 事实上，本书的所有的证明都只使用了初等的方法，不涉及解析方法以及其他的高等方法， 因此本书也是一本很好的大学生数论教材。本书第一部分和第二部分作为大学生一个学期的课程是合适的。 （王巨平）

国外评论摘选

1) Every serious student of number theory should have this classic book on their shelf. Even though only "elementary" calculus and abstract algebra are used, a certain mathematical maturity is required. I feel the book is strongest in the area of elementary --not necessarily easy though -- analytic number theory (Hardy was a world class expert in analytic number theory). An elementary, but difficult proof of the Prime number Theorem using Selberg's Theorem is thoroughly covered in chapter 22.

While modern results in the area of algorithmic number theory are not presented nor is a systematic presentation of number theory given (it is not a textbook), it contains a flavor, inspiration and feel that is completely unique. It covers more disparate topics in number theory than any other n.t. book I know of. The fundamental results in classical, algebraic, additive, geometric, and analytic number theory are all covered. A beautifully written book.
Other recommended books on number theory in increasing order of difficulty:
1) Elementary Number Theory, By David Burton, Third Edition. Covers classical number theory. Suitable for an upper level undergraduate course. Primarily intended as a textbook for a one semester number theory course. No abstract algebra required for this book. Not a gem of a book like Davenport's The Higher Arithmetic, but a great book to seriously start learning number theory.
2) The Queen of Mathematics, by Jay Goldman. A historically motivated guide to number theory. A very clearly written book that covers number theory at a graduate or advanced undergraduate level. Covers much of the material in Gauss's Disquisitiones, but without all the detail. The book covers elementary number theory, binary quadratic forms, cyclotomy, Gaussian integers, quadratic fields, ideals, algebraic curves, rational points on elliptic curves, geometry of numbers, and introduces p-adic numbers. Only a slight bit of analytic number theory is covered. The best book in my opinion to start learning algebraic number theory. Wonderfully fills the otherwise troublesome gap between undergraduate and graduate level number theory.
Full of historical information hard to find elsewhere, very well researched. To cover all the material in this book would likely take two semesters, though most of the important material could be covered in one semester. Requires a background in abstract algebra (undergraduate level), and a little advanced calculus. Some complex analysis for sections 19.7 and 19.8 would be helpful, but not at all a requirement. The author recommends Harold Davenport's The Higher Arithmetic, as a companion volume for the first 12 chapters; according to Goldman it is a gem of a book.
3) Additive Number Theory, by Melvyn Nathanson. Graduate level text in additive number theory, covers the classical bases. This book is the first comprehensive treatment of the subject in 40 years. Some highlights: 1) Chen's theorem that every sufficiently large even integer is the sum of a prime and a number that is either prime or the product of two primes. 2) Brun's sieve for upper bound on the number of twin primes. 3) Vinogradov's simplification of the Hardy, Littlewood, and Ramanujan's circle method.

2) My initial reaction through the first chapters was one of embarrassment at my lack of understanding. I could not believe a book, hailed by so many as a standard and essential resource, could be so much out of my reach. Then, amid the last page or so of chapter 1 I had an epiphany. The book, from that point on, was completely clear and logical while retaining an extraordinary amount of breadth in coverage. Add my staunch support and recommendation to the long list of kudos that this book has accrued. There are, to my knowledge, no better books for the beginning student of number theory. If you have any interest whatsoever in the theory of numbers, this book is essential.

书名：A course in arithmetic

作者： J.-P. Serre

出版商： Springer Verlag (1973) ISBN 0-387-90041-1

页数：113

适用范围：大学数学系本科基础数学高年级或研究生低年级教材

预备知识：抽象代数,复分析

习题数量：很少

习题难度： 较大

推荐强度：10

书评： 法国大数学家，菲尔滋奖和阿贝尔奖获得者 Serre 写过不少短小精悍的小册子，大部分从他亲自所讲授的课程的讲稿整理而成。 本书是他非常有代表性的本科生高年级的数论教材，曾在西方评为某年度世界最佳数学教材。
本书并不是数论的系统教程，作者选择数论中三个重要专题扼要叙述了它们的内容和方法，这三个专题是：二次型、素数的 Dirichlet 定理 和模形式。读者可以化较少的时间学到一些近代数论的知识。最令读者欣赏的是定理的证明将大数学家的技巧展现得淋漓尽致，阅读中不禁 拍案叫绝。非定型幺模偶整格的分类定理非常漂亮，但其完整的证明在很多代数教科书中很难找到，本人所知道的就是本书以及 Milnor 和 Husemoeller 写的 Symmetric bilinear form 一书中的证明。这两本书都是70年代出版的，经过这两位菲尔滋奖得主之手的证明已经很难再 作改进，因此后人写的书大多只是引用而不再重写了。
具备抽象代数的知识就可以读懂前半本书，后一半需要复分析的准备知识。由于叙述简洁，习题数量少，作为教材使用会有一定困难。 作为自学的参考书对读者的数学素养也有较高的要求。（杨劲根）

国外评论摘选

1) The book is divided into two parts -- algebraic and analytic. I've only worked through the analytic part. Anything by Serre is worth its weight in gold and this book is no exception; everything Serre covers is of the utmost importance. But Serre's style is extremely condensed and spare, and he makes no concessions to the reader in terms of motivation or examples. I can't digest more than half a page of Serre a day; however if one wants to understand the structure of a theory, Serre is ideal. I worked through "A Course in Arithmetic" over a decade back. As I recall I covered Riemann's zeta function and the Prime Number Theorem, the proof of Dirichlet's theorem on primes in arithmetical progressions using group characters in the context of arithmetical functions, and some of the basic theory of modular functions. All of this material is also covered in Apostol's two books on analytic number theory ("Introduction to Analytic Number Theory", and "Dirichlet Series and Modular Functions in Number Theory"); Apostol goes further than Serre in the analytic part -- which is only to be expected since he is devoting two whole texts to the subject.
2) Serre's work could best be summarized in one word - Elegance. The book comprises of two distinct parts. The first one is the 'algebraic' part. Serre's goal in this section is to give a complete classification of the quadratic forms over the rationals. As preliminaries to reaching this goal, he introduces the reader to quadratic reciprocity, $p-$adic fields and the Hilbert Symbol. After these three, he spends the next chapter detailing the properties of quadratic forms over ${\mathbb Q$ and ${\mathbb Q_p$ (the $p-$adic field). The reason to work over ${\mathbb Q_p$ is the Hasse-Minkowski Theorem (which says that if you have a quadratic form, it has solutions in Q if and only if it has solutions in ${\mathbb Q_p$). Using Hensels Lemma, checking for solutions in ${\mathbb Q_p$ is (almost) as easy as checking for solutions in Z/pZ. After doing that, he spends yet another chapter talking about the quadratic forms over the integers. (Note: the classification goal is already achieved in previous chapter). The second half of the book is the 'analytic' one. The first chapter in this section gives a complete proof of Dirichlet's theorem while the second one studies the properties of modular forms (these are good!) Due to the extreme elegance, the book is sometimes hard to read. This might sound like a paradox, but it's not and I'll explain why. The book takes some effort to read because it's terse and it often takes a while to figure out why something is 'obvious'. However, once you see it all, you'll realize that a great mind was guiding you through the pursuit. The choice of

topics is just right to achieve the goals that the author sets out for himself. Also, I'd rather think for myself and read a smaller book than be given a huge fat tome where the author details his own thought process. This book was my first foray into number theory and I absolutely enjoyed it. If you're considering reading it, I wish you joy in your pursuits.

书名：Introduction to Analytic Number Theory
作者： Tom Apostol
出版商： Springer Verlag (1976) ISBN 0-387-90163-9
页数：328
适用范围：大学数学系本科数论教材
预备知识：微积分,复分析
习题数量：大
习题难度：一般
推荐强度：9

书评： 这是一本非常受欢迎的数论入门教材，写得极其清楚仔细而又不烦琐。虽然书名是解析数论，事实上也包括了初等数论。 由于书的自封性能好，习题又经过精心挑选，适合于大学低年级的数论教材。
本书由于其良好声誉而多次再版，被选入 Springer 的 UTM 系列。同一作者的微积分教材（见本书的另一篇书评）也有好口碑。 （杨劲根）

国外评论摘选

1) I think that there will be little harm if the title of the book is changed to 'Introduction to elementary number theory' instead. The author presumes that the reader has not any knowledge of number theory. As a result, materials like congruence equation, primitive roots, and quadratic reciprocity are included. Of course as the title indicates, the book focusses more on the analytic aspect. The first 2 chapters are on arithmetic functions, asymptotic formulas for averaging sums, using elementary methods like Euler-Maclaurin formula .This lay down the foundation for further discussion in later chapters, where complex analysis is involved in the investigation. Then the author explain congruence in chapter 4 and 5. Chapter 6 introduce the important concept of character. Since the purpose of this chapter is to prepare for the proof of Dirichlet's theorem and introduction of Gauss sums, the character theory is developed just to the point which is all that's needed. (i.e. the orthogonal relation). Chapter 7 culminates on the elementary proof on Dirichlet's theorem on primes in arithmetic progression. The proof still uses $L-$function of course, but the estimates, like the non-vanishing of $L(1)$, are completely elementary and is based only on the first 2 chapters. The author then introduce primitve roots to further the theory of Dirichlet characters. Gauss sums can then be introduced. 2 proofs of quadratic reciprocity using Gauss sums are offered. The complete analytic proof, using contour integration to evaluate explicitly the quadratic Gauss sums, is a marvellous illustration of how truth about integers can be obtained by crossing into the complex domains. The book then turns in to the analyic aspect. General Dirichlet series, followed by the Riemann zeta function, L function ,are introduced. It's shown that the $L-$functions have meremorphic continuation to the whole complex plane by establishing the functional equation $L(s)=$ elementary factor $* L(1-s).$ The reader should be familiar with residue calculus to read this part. Chapter 13 may be a high point of this book, where the Prime Number Theorem is proved. Arguably, it's the Prime Number Theorem which stimulate much of the theory of complex analysis and analyic number theory. As Riemann first pointed out, the Prime Number

Theorem can be proved by expressing the prime counting function as a contour integral of the Riemann zeta function, then estimate the various contours. The proof given in this book , although not exactly that envisaged by Riemann , is a variant that run quite smoothly. As is well known , a key point is that one can move the contour to the line $Re(s)=1,$ and to do this one have to verify that $\zeta(s)$ does not vanish on $Re(s)=1.$ The proof , due to de la vale-Poussin, is a clever application of a trigonometric identity. Unfortunately, the method does not allow one penetrate into the region $0<1,$ where the distribution of zeroes in this region contain the information about the flunctuation of $\Pi(x)$ around $x/\log x.$ The famous Riemann Hypothesis states that the only zeroes in this region lis on the line $Re(s)=1/2.$ After more than 100 years, although the Riemann Hpothesis has natural generalisation to number fields, neither of these RH is proven, which indicates the difficulties of this problem. Recently some new directions, related to quantum statistical mechanics, has been connected with this old problem. If the RH is proven, then the set of prime numbers , although looks completely random locally (like the occurences of twin primes), is governed by clear-cut laws on the large after all. The last Chapter is of quite differnt flavour, the so-called additive number theory. Here the author only focusses on the simplest partition function ---the unrestricted partition. However interesting phenomeon occur already at this level. The first result is Euler's pentagonal number theorem, which leads to a simple recursion formula for the partition function p(n). 3 proofs are given. The most beautiful one is no doubt a combinatorial proof due to Franklin. The third proof is through establishing the Jacobi triple product identity, which leads to lots of identites besides Euler's pentagonal number theorem. Jacobi's original proof uses his theory of theta functions, but it turns out that power series manipulaion is all that's needed. The book ends with an indication of deeper aspect of partition theory--- Ramanujan's remarkable congrence and identities (the simplest one being $p(5m+4)= 0 \pmod{5$). To prove these mysterious identites, the "natural"way is to plow through the theory of modular functions, which Ramanujan had left lots more theorem (unfortunately most without proof). However an elementary proof of one these identites is outlined in the exercises. This book is well written, with enough exercises to balance the main text. Not bad for just an 'introduction'. 2) This book has excellent exercises at the end of each chapter. The exercises are interesting and challenging and supplement the main text by showing additional consequences and alternate approaches. The book covers a mixture of elementary and analytic number theory, and assumes no prior knowledge of number theory. Analytic ideas are introduced early, wherever they are appropriate. The exposition is very clear and complete. Some novel features include: three chapters on arithmetic functions and their averages (including a simple Tauberian theorem due to Shapiro); Polya's inequality for character sums; and an evaluation of Gaussian sums (by contour integration), used in one proof of quadratic reciprocity.

8 代数几何¶

代数几何是核心数学的重要分支，内容比较高深，不太容易入门。由于它所用的知识比较多，学习的周期相对比较长。一般本科生阶段不设代数几何课程。
代数几何对抽象代数、复分析、拓扑等都有较高的要求，特别交换代数和同调代数是它的不可缺少的工具。我们在这里介绍一些目前在国际上使用最多的一批基础性的教材供有兴趣和有志向的读者参考。

书名：An Invitation to Algebraic Geometry
作者：K.Smith etc.
出版商：Springer-Verlag, New York, 2000. ISBN 0-387-98980-3
页数：155
适用范围：基础数学本科高年级或非代数几何专业研究生低年级
预备知识：线性代数，群，环，域扩张，Galois
理论的基础知识，最基本的点集拓扑
习题数量：大
习题难度：容易
推荐强度：8.5

书评： 本书由作者 1996 年的为非代数几何专业的数学研究生开设的 20 小时代数几何课的讲义整理修改而成，是一本非常好的代数几何入门书。
本书从最基本的交换代数代数几何概念开始，用简明的方式引入仿射簇和射影簇。重点的内容是一些经典的例子： Veronese 映射、计数几何、Segr\'{e} 嵌入、 Grassman 簇等。最后介绍代数几何中的一些重大问题如奇点解消、射影簇分类、典范映射等。
本书把代数几何讲解的具体易懂，不拘泥于细节，一些关键定理给出清晰的解释而不是详细的证明，如 Hilbert 零点定理、 B\'{e}zout 定理、 Bertini 定理等，有一部分内容的简单证明放入习题。有些其他教材不提及的问题也作了简短介绍，如 Gauss 映射用来 2 页和 5 道习题。书中不时插入对一些重要问题研究的历史和现状，颇有 Wikipedia 的风格。在三个合适的地方叙述三个未解决的难题： Jabobian 猜想、空间曲线的完全交问题、 Iitaka 猜想。
本书篇幅不大，适合初学者在较短的时间内对代数几何的特点有初步的了解，为进一步的深入学习作准备。（杨劲根）

国外评论摘选

1）This book has a great deal to recommend it:
a. It is a genuinely entry-level book that begins with the definition of a prime ideal and the Nullstellensatz.
b. The style of explanation is clearly geared to noninsiders. In addition to giving examples of algebraic varieties, some "nonexamples" are given that might have occurred to, say, an analyst as reasonable objects to study but that do not qualify as varieties.

c. The illustrations are frequent, relevant, and well executed.
d. The authors go out of their way to help the reader develop geometric intuition and to relate it to the accompanying algebraic description. For example, in the careful treatment of the geometry of a family of hyperbolas in chapter 6, the geometry of the general hyperplane section is beautifully illustrated, and the reader's geometric intuition is stimulated into action.
e. Many of the constructions covered are classical-the Grassmannian, the Veronese, the Gauss mapping, the secant variety of a variety-yet the book almost seamlessly connects this with more modern material, such as resolution of singularities and vector bundles.
f. There is a consistent policy throughout the book of tying in elementary algebraic geometry to recent developments by current leaders such as Kollar, Kontsevich,Mori, Lazarsfeld, and de Jong, so that readers come away with a clear conception of where this is all going and what the next steps might be if a particular topic sparks their interest. Overall, readers will find this book easy to get into and enjoyable to read. Outsiders to the subject will feel that they are hiking up a gently sloping trail, at the end of which they reach a number of pleasant viewing spots from which they. can see rather far in a number of different directions. Students contemplating algebraic geometry as a field of specialization will also find this an attractive and instructive place to start. (by Mark Green, {\em The American Mathematical Monthly,} Vol. {\bf 109}, 675-678(2002))

2）This could be your only book on algebraic geometry if you just want a sound idea of what algebraic geometry can do. If you actually want to know the field, and you do not already have a lot of expert friends telling you about it, then the advanced books will go much more easily with this expert around. It is a terrific guide to the key ideas--what they mean, how they work, how they look. The only book like this one in brevity and scope is Reid UNDERGRADUATE ALGEBRAIC GEOMETRY--with its highly informed, highly polemical, final chapter on the state of the art. Both are very good. This one is more advanced. Beyond what Reid covers, Smith sketches Hilbert polynomials, Hironaka's (and very briefly even De Jong's) approach to removing singularities, and ample line bundles. You do need a bit of topology and analysis to follow it. Smith has very many fewer concrete examples than Reid. They are beautifully chosen classics, like Veronese maps and Segre maps, so they teach a lot. And the more you know to start with, the more you will see in each.
The book does geometry over the complex numbers. It is good old conservative material, with terrific graphics of curves and surfaces. The proofs and partial proofs are very clear, intuitive and to the point. But, in fact, just because the proofs are so clear and to the point they usually work in a much broader setting. Long stretches of the book apply just as well over any field or any algebraically complete field. This generality is only mentioned a few times, in passing, but is there if you want it. Smith describes schemes very briefly, and mentions them at each point where they naturally arise. You will not know what schemes "are" at the end of this book. You will know some things they DO. She has no time for fights between "concretely complex" and "abstractly scheming" approaches--for her it is all geometry.

3）For people just starting on Algebraic Geometry, Robin Hartshorne's book, is very daunting--but it is the ULTIMATE book for professional and advanced readers. But for starters, Karen Smith's "An Invitation to Algebraic Geometry" is simply a SPLENDID way to start working on the basic ideas. The author has some stunning graphs and pictures to help understand material. I loved the book the minute I opened it.

书名：Introduction to Commutative Algebra and Algebraic Geometry
作者： Ernst Kunz
出版商： Birkhauser Boston, (1985) ISBN 3-7643-3065-1
页数：238

适用范围：基础数学本科高年级或研究生低年级
预备知识：线性代数，群，环，域扩张，Galois 理论的基础知识，最基本的点集拓扑
习题数量：大
习题难度： 大部分中等，少量难题
推荐强度：9

书评： 作为交换代数的入门书，它不如 Atiyah-McDonald 的有名，也不如 Eisenbud 的大， 但是我认为对于有志学习代数几何的大学生来讲，这是最好的入门书。此书与大学基础课程 的衔接非常紧密，不管是自学还是用此书当教材都比较轻松，如果再认真做习题则效果更好。 交换代数的内容甚广，作者完全按经典代数几何的需要选择交换代数的内容，重点是多变量 多项式环的商环及它上的有限生成模。除了基础性的材料外，也有少量研究性的题材。现在 代数几何最流行的研究生教材是 Hartshorne 的 Algebraic Geometry, 本书可以认为是 Hartshorne 的教程的前续课程。本书作者是著名的交换代数专家，原书用德文写作，后翻译成英文，美国 代数几何大师 David Mumford 写了序言，称此书是美国学代数几何学生久等的一本书，它填补 了一个空白。（杨劲根）

书名：Basic Algebraic Geometry (Second, Revised and Expanded Edition)
作者： Shafarevich
出版商： Springer-Verlag (1988) ISBN 3-540-54812-2
页数：上册 303 下册 269
适用范围：基础数学研究生
预备知识：近世代数、复分析、点集拓扑
习题数量：大
习题难度：较难
推荐强度：9

书评： 本书是俄罗斯的数学大师 Shafarevich 的力作，由英国著名代数几何学家 Miles Reid 翻译成英文。本书内容非常丰富且不枯燥，叙述和证明清晰，比较容易读，是非常收欢迎的一本 代数几何书，国内外不少院校开设代数几何课曾将此书选为研究生教材。全书分三大部分， 第一部分是射影簇，内容包含经典代数几何，一直讲到代数曲面的分类和奇点，其中不乏其它 教科书中不多见的内容。这一部分占了整个上册，作为一个学期的课程内容够多的。 第二部分是概形理论，用现代的语言来刻画代数簇，最后讲到 Hilbert 概形。本部分内容比较简要，基本上讲清概形和层论的威力。第三部分是复代数流形的拓扑和几何，很多内容如代数簇的拓扑 分类和 uniformization 在其它代数几何教科书很难找到。总之，本书基本上讲述了代数几何的所有方法。 习题非常丰富，大部分的习题很有意思，可以看出是作者和他的助手们多年积累而编成的。 还有一个显著的特点是本书不需要交换代数的预备知识，当然学过交换代数在看此书更加轻松。 （杨劲根）

国外评论摘选

1) I have been a student of AG for the past six years and I have come to the conclusion that Shafarevich is a great place to start. Having said this, one must have the necessary background in algebra and topology. I disagree with the other reviewer about doing this after Hartshorne--start here then do Hartshorne!!!

书名：Algebraic Geometry
作者: Robin Hartshorne
出版商：Springer-Verlag
页数：495
适用范围：基础数学研究生
预备知识：近世代数、交换代数、同调代数、复分析、基础拓扑
习题数量：多
习题难度： 又难又繁
推荐强度：8.8

书评： 本书是现代代数几何的标准教科书，适合代数几何专业的研究生使用。从代数几何的发展历史来看， 60年代由 Grothendieck 提出的以概形为基础的新理论完成了代数几何的一次新的革命，至今代数几何仍以 Grothendieck 的理论为基础，他和 Dieudonne 合写的庞大的 EGA (Elements de Geometrie Algebrique) 可堪为代数几何的圣经。但是由于规模太大，EGA 无法当作教科书使用。事实上在 EGA 之前，Grothendieck 在日本的东北数学杂志上一篇同调代数的长文也是代数几何的奠基性的文献之一。此后，出现了两本有 很大影响的书，其一是 Matsumura 的 Commutative Algebra, 这本看上去象是研究笔记的专著把 Grothendieck 的 EGA 中的交换代数部分和部分同调代数整理出来加以详细证明。另一本就是 Hartshorne 的 Algebraic Geometry, 此书用两章约230页的篇幅介绍 Grothendieck 的概形理论。作者能完成此举得益于两点：第一，所有和交换代数 有关的内容都引用 Matsumura 的有关章节。第二，作者牺牲一般性而大大简化了很多大定理的证明，具体来讲， 在大部分章节作者把概形限制为诺特概形，把态射限制为有限型的态射。这样的简化对代数几何的主流方向的研究 来说影响不大。从某种意义来看，Hartshorne 的书是 Grothendieck 的 EGA 的浓缩简化版。这本书中的很多英文 名词现在已经获得代数几何界的普遍认可。
本书中定理的证明比较简洁，认真的读者需要补充不少细节，从这点来看，此书的浓度比较大，要读懂此书大部分得化一年以上的 时间，对于没有学过交换代数或学的不多的读者，最好先读 Kunz 的 Introduction to Commutative Algebra and Algebraic Geometry 一书。会法文的读者可以参考 Grothendieck 的 EGA 学习，因为有些定理的证明在 EGA 中写的更为详细。 习题也是本书的一大难点，不少研究生抱怨这本书中的习题太难做。网上有些地方甚至有 Hartshorne 习题部分解答下载。 （杨劲根）

国外评论摘选

1) This book is one of the most used in graduate courses in algebraic geometry and one that causes most beginning students the most trouble. But it is a subject that is now a "must-learn" for those interested in its many applications, such as cryptography, coding theory, physics, computer graphics, and engineering. That algebraic geometry has so many applications is quite amazing, since it was not too long ago that it was thought of as a highly abstract, esoteric

topic. That being said, most of the books on the subject, including this one, are written from a very formal point of view. Those interested in applications will have to face up to this when attempting to learn the subject. To read this book productively one should gain a thorough knowledge of commutative algebra, a good start being Eisenbud's book on this subject. Also, it is important to dig into the original literature on algebraic geometry, with the goal of gaining insight into the constructions and problems involved. The author of this book does not make an attempt to motivate the subject with historical examples, and so such a perusal of the literature is mandatory for a deeper appreciation of algebraic geometry. The study of algebraic geometry is well worth the time however, since it is one that is marked by brilliant developments, and one that will no doubt find even more applications in this century. Varieties, both affine and projective, are introduced in chapter 1. The discussion is purely formal, with the examples given unfortunately in the exercises. The Zariski topology is introduced by first defining algebraic sets, which are zero sets of collections of polynomials. The algebraic sets are closed under intersection and under finite unions. Therefore their complements form a topology which is the Zariski topology. The properties of varieties are discussed, along with morphisms between them. "Functionals" on varieties, called regular functions in algebraic geometry, are introduced to define these morphisms. Rational and birational maps, so important in "classical" algebraic geometry are introduced here also. Blowing up is discussed as an example of a birational map. A very interesting way, due to Zariski, of defining a nonsingular variety intrinsically in terms of local rings is given. The more specialized case of nonsingular curves is treated, and the reader gets a small taste of elliptic curves in the exercises. A very condensed treatment of intersection theory in projective space is given. The discussion is primarily from an algebraic point of view. It would have been nice if the author would have given more motivation of why graded modules are necessary in the definition of intersection multiplicity.

The theory of schemes follows in chapter 2, and to that end sheaf theory is developed very quickly and with no motivation (such as could be obtained from a discussion of analytic continuation in complex analysis). Needless to say scheme theory is very abstract and requires much dedication on the reader's part to gain an in-depth understanding. I have found the best way to learn this material is via many examples: try to experiment and invent some of your own. The author's discussion on divisors in this chapter is fairly concrete however.

The reader is introduced to the cohomology of sheaves in chapter 3, and the reader should review a book on homological algebra before taking on this chapter. Derived functors are used to construct sheaf cohomology which is then applied to a Noetherian affine scheme, and shown to be the same as the Cech cohomology for Noetherian separated schemes. A very detailed discussion is given of the Serre duality theorem.

Things get much more concrete in the next chapter on curves. After a short proof o the Riemann-Roch theorem, the author studies morphisms of curves via Hurwitz's theorem. The author then treats embeddings in projective space, and shows that any curve can be embedded in P(3), and that any curve can be mapped birationally into P(2) if one allows nodes as singularities in the image. And then the author treats the most fascinating objects in all of mathematics: elliptic curves. Although short, the author does a fine job of introducing most important results.

This is followed in the next chapter by a discussion of algebraic surfaces in the last chapter of the book. The treatment is again much more concrete than the earlier chapters of the book, and the author details modern formulations of classical constructions in algebraic geometry. Ruled surfaces, and nonsingular cubic surfaces in P(3) are discussed, as well as intersection theory. A short overview of the classification of surfaces is given. The reader interested in more of the details of algebraic surfaces should consult some of the early works on the subject, particularly ones dealing with Riemann surfaces. It was the study of algebraic functions of one variable that led to the introduction of Riemann surfaces, and the later to a consideration of algebraic functions of two variables. A perusal of the works of some of the Italian geometers could also be of benefit as it will give a greater appreciation of the methods of modern algebraic geometry to put their results on a rigorous foundation.

2) This is THE book to use if you're interested in learning algebraic geometry via the language of schemes. Certainly, this is a difficult book; even more so because many important results are left as exercises. But reading through this book and completing all the exercises will give you most of the background you need to get into the cutting edge of AG. This is exactly how my advisor prepares his students, and how his advisor prepared him, and it seems to work. Some helpful suggestions from my experience with this book: 1) if you want more concrete examples of schemes, take a look at Eisenbud and Harris, The Geometry of Schemes; 2) if you prefer a more analytic approach (via Riemann surfaces), Griffiths and Harris is worth checking out, though it lacks exercises.

书名：Principles of Algebraic Geometry
作者： Phillip Griffiths, Joseph Harris
出版商： John Wiley & Sons, Inc., 1978; ISBN 0-471-32792-1
页数：813
适用范围：基础数学研究生
预备知识：近世代数、复分析、点集拓扑
习题数量：无
推荐强度：8

书评： 代数几何原理的作者之一 P. Griffiths 是美国科学院院士，国际著名的数学家，他是 普林斯顿高等研究院教授。本书从比较解析的方面介绍代数几何，它给人的印象是代数几何也很具体， 这是一本代数几何的入门引导课本。
本书前两章处理复流形理论的一些结果和技巧，同时强调它们在射影代数簇上的应用； 第2章开始介绍黎曼面和代数曲线的理论；第3章中介绍了流、陈类、Riemann-Roch 公式等基本工具，然后在第4章中介绍代数曲面理论；本书最后介绍 Quadric Line Complex.
本书的选材十分合适，内容基本自我包容，给读者以直观和易懂的感觉， 但笔者认为该书在排版过程中过于仓促，书中（尤其第0章）累出打印错误和符号冲突。 如果有谁能从现在的版本重新修订并浓缩成一本稍薄的代数几何入门书，那将是代数几何爱好者的福音。（陈猛）

国外评论摘选

1) Once thought to be highly esoteric and useless by those interested in applications, algebraic geometry has literally taken the world by storm. Indeed, coding theory, cryptography, steganography, computer graphics, control theory, and artificial intelligence are just a few of the areas that are now making heavy use of algebraic geometry. This book would probably be one the most useful one for those interested in applications, for it is an overview of algebraic geometry from the complex analytic point of view, and complex analysis is a subject that most engineers and scientists have had to learn at some point in their careers. But one must not think that this book is entirely concrete in its content. There are many places where the authors discuss concepts that are very abstract, particularly the discussion of sheaf theory, and this might make its reading difficult. The complex analytic point of view however is the best way of learning the material from a practical point of view, and mastery of this book will pave the way for indulging oneself in its many applications.
Algebraic geometry is an exciting subject, but one must master some background material before beginning a study of it. This is done in the initial part of the book (Part 0), wherein the reader will find an overview of harmonic analysis (potential theory) and Kahler geometry in the context of compact complex manifolds. Readers first encountering Kahler geometry should

just view it as a generalization of Euclidean geometry in a complex setting. Indeed, the so-called Kahler condition is nothing other than an approximation of the Euclidean metric to order 2 at each point.
The authors choose to introduce algebraic varieties in a projective space setting in chapter 1, i.e. they are the set of complex zeros of homogeneous polynomials in projective space. The absence of a global holomorphic function for a compact complex manifold motivates a study of meromorphic functions and divisors. Divisors are introduced as formal sums of irreducible analytic hypersurfaces, but they are related to the defining functions for these hypersurfaces also, via the poles and zeros of meromorphic functions. For the mathematical purist, a "sheafified" version of divisors is also outlined. Divisors and line bundles are basically "linear" tools used to investigate complex varieties through their representation as complex submanifolds of projective space. In addition, various approaches are used to study codimension-one subvarieties, such as the results of Kodaira and Spencer. Although the famous Kodaira vanishing theorem is clothed in the language of Cech cohomology, this cohomology is represented by harmonic forms, thus making its understanding more accessible. The authors also show explicitly to what extent an algebraic variety can be thought of as a compact complex manifold via the Kodaira embedding theorem. Projective space of course is not the most complicated of constructions, as readers familiar with the theory of vector bundles will know. Grassmannians are an example of this, and they are introduced and discussed in the book as generalizations of projective space. And, just as in the ordinary theory of vector bundles, the authors show how to use Grassmannians to act as universal bundles for holomorphic vector bundles.
The presence of meromorphic functions will alert the astute reader as to the role of Riemann surfaces in the study of complex algebraic varieties. Indeed, in chapter 2, the authors cast many classical complex analytic results to modern ones, and they prove the famous Riemann-Roch theorem, which essentially counts the number of meromorphic functions on a Riemann surface of genus g. The theory of Abelian varieties is outlined, and the reader gets a taste of "Italian" algebraic geometry but done in the rigorous setting of Plucker formulas and coordinates.
Chapter 3 is a summary of some of the other methodologies and techniques used to study general analytic varieties, the first of these being the theory of currents, i.e differential forms with distribution coefficients. It is perhaps not surprising to see this applied here, given that it can handle both the smooth and piecewise smooth chains simultaneously. The currents are associated to analytic varieties and allow a definition of their intersection numbers and a proof that they are positive. The all-important Chern classes are introduced here, and it is shown that the Chern classes of a holomorphic vector bundle over an algebraic variety are fundamental classes of algebraic cycles. Most importantly the authors introduce spectral sequences, a topic that is usually formidable for newcomers to algebraic geometry.
The study of surfaces is studied in chapter 4, with the differences between its study and the theory of curves (Riemann surfaces) emphasized. The reader gets a first crack at the notion of a rational map, and the birational classification of surfaces is shown. Intuitively, one expects that the classification of surfaces would be easy if it were not for "singular points", and this is born out in the use of blowing up singularities in this chapter. Rational surfaces are characterized using Noether's lemma, and a rather detailed discussion is given of surfaces that are not rational, giving the reader more examples of rigorous "Italian" geometry.
2) If you are a graduate student in mathematics or related fields and you are interested in learning algebraic geometry in the Griffiths-Harris way, then I suggest before buying this book to have a good background in the following: 1. Complex Analysis 2. Differential Geometry and calculus on manifolds 3. Homology-Cohomology Theory 4. Undergraduate Algebraic Geometry Do not expect chapter 0, "Foundational Material", to be the place where you are supposed to build your "foundation". You can try the books of Michael Spivak, David A. Cox, Fangyang Zheng, among other books for foundational material but not chapter 0.

However, if you have most of the above-mentioned foundational material, then this book is good in presenting complex manifolds for example in chapter 0 section 2 and also in presenting (complex) holomorphic vector bundles, as well as many other things.
So, in summary, I would say a good book but not for students trying to learn the basics in algebraic geometry.

书名：The Red Book of Varieties and Schemes
作者： David Mumford
出版商： Springer-Verlag (1994) ISBN 3-540-50497-4
页数：309
适用范围：基础数学研究生
预备知识：近世代数、复分析、点集拓扑
习题数量：很少
习题难度： 较难
推荐强度：8.6

书评： 代数几何学家 Mumford 是美国科学院院士，菲尔滋奖获得者，在哈佛大学任教多年。 本书实际上是 60 年代他在哈佛的代数几何课程的讲义。即使到80年代有了 Hartshorne, Shafarevich 写的优秀教科书，很多初学者仍然喜欢 Mumford 的老讲义,油印本在研究生中广为流传。Springer Verlag 经专家的推荐便将这些讲义原封不动地出版了，由于原来的油印的封面是红色的，故此书就被亲切地取名为 red book. 比较可惜的是这本书只有三章，由于种种原因作者未能把原来写讲义的庞大计划执行到底。
代数几何是抽象概念非常多的一门数学分支，初学者需要化很大的精力来理解、消化和记住一大堆基本概念。 Mumford 的讲义用朴实无华的方式解释代数几何这些概念的来龙去脉，一点不落俗套。从目录上看，这些内容和 其它同类的书差不多，事实上具体的论述还是很不一样的，讲义行文的非正式的风格也使枯燥的数学变的生动。 作者自嘲地称这本书里一个定理也没有，这多少有些夸张， 很多应该叫做定理的结论在这位大师面前大概不能称为定理。 （杨劲根）

国外评论摘选

1）In a nutshell, reading this book is like reading the mind of a great mathematician as he thinks about a great new idea. Anyone interested in schemes should read it. But a review needs more detail: The RED BOOK is a concise, brilliant survey of schemes, by one of the first mathematicians to learn of them from Grothendieck. He gives wonderfully intuitive pictures of schemes, especially of "arithmetic schemes" where number theory appears as geometry. The geometry shines through it all: as in differentials, and etale maps, and how unique factorization relates to non-singularity. There is a bravura discussion of Zariski's Main Theorem (the algebraic property of being "normal" implies that a variety has only one branch at each point) comparing forms of it from older algebraic geometry, topology, power series, and schemes. Mumford cites proofs of these but does not give them. In fact, this theorem was one of the first things Mumford could use, to get Zariski to respect schemes.
Many accomplished algebraic geometers say this book got them started. But you probably cannot learn to work in the subject from this book alone--you either have to work with people who work with it, or use some other books besides (maybe both). The other book would probably be Hartshorne ALGEBRAIC GEOMETRY, which is far more detailed, has far more

examples, goes very much farther into cohomology--and is very much longer and denser (though also clearly written).

2）There is a problem in getting going with alg. geo. To learn the geometry you need commutative algebra and to contextualize commutative algebra you need algebraic geometry. Mumford is an excellent book to get going without the need for the heavy prereqs of the more classic books like Hartshorne or Griffiths-Harris. A really good read. This is not however a terrific reference text, you'll need something else as a reference. Its much to expository and their is no index.

书名：Compact Complex Surfaces, 2nd edition

作者：W.P. Barth, K. Hulek, C.A.M. Peters, A. Van de Ven

出版商：Springer-Verlag (2003) ISBN 3-540-00832-2

页数：436

适用范围：代数几何、复几何、微分几何方向研究生

预备知识：多复变函数论、代数几何、复微分几何

习题数量：无

推荐强度：9

书评： "紧复曲面"专著第一版于1984年出版，自其出版以来因其选材精致，重点突出而广受青睐， 内行称其为"BPV"（第一版的三位作者的姓氏的开头字母）。第二版增加了不少新的研究成果， 但笔者认为第二版的组织过于仓促，反而给人画蛇添足的感觉，尽管如此， 这仍不失为一本优秀的专业工具书。

本书第一章列出了必需的预备知识，虽然没有证明，但笔者认为该内容十分恰当。 第二章中，作者分别介绍了曲面上的曲线、Riemann-Roch定理、相交理论；第三章中介绍了曲面的奇点、 纤维化方法和稳定纤维化的周期映射。然后从第四章开始，本书着重讲述曲面的一般性质、 特殊曲面的分类和一般型曲面的典范分类。本书的最后一章中主要介绍曲面的拓扑和微分结构。 笔者认为，本书对于一般型曲面的分类内容略显陈旧。

总的来说，这是一本介绍代数曲面理论的极好工具书。（陈猛）

9 拓扑与微分几何¶

拓扑学是现代数学的基础之一，国内外很多大学数学系都把它列为本科生高年级基础课，甚至是必修棵。按大数学家嘉当的观点，数学中最基本的结构就是代数结构和拓扑结构。
拓扑又分点集拓扑（也称一般拓扑）、代数拓扑和微分拓扑等。点集拓扑讨论拓扑空间的基本概念和拓扑空间的连通性、分离性、紧致性等重要性质。代数拓扑则是用抽象代数中的工具来研究拓扑空间进一步的性质，其内容包括基本群、覆盖空间、同调、上同调、示性类等。一般在本科生的拓扑课程中学习点集拓扑、基本群和覆盖空间。其余是研究生课程的内容。微分拓扑的对象是微分流形的拓扑性质，也属于研究生课程的范围。
初等微分几何主要讨论欧氏空间中曲线和曲面的几何学，需要的数学工具比较少，基本上有数学分析、线性代数和常微分方程的预备知识就够了，它也可称为 古典微分几何，大学本科的微分几何课程一般学初等微分几何。近代微分几何的对象是微分流形、向量丛、李群等，它和拓扑学的关系异常密切。微分几何里又有各种分支如黎曼几何、复几何、辛几何等，这里不一一介绍了。

书名： Elements of Algebraic Topology
作者： James R. Munkres
出版商： Addison-Wesley Publishing Company
页数： 454
适用范围：基础数学研究生一学年的教材和数学系高年级本科生
预备知识：一般拓扑学和线性代数
习题数量：适中
习题难度： 适中
推荐强度： 9.6
书评：本书是美国 MIT 的 James R. Munkres 教授的力作，从 80 年代开始就作为麻省理工学院数学系一年级研究生教材， 是一本极具特色的优秀教材。本书以拓扑中的重要实例为线索，引导出抽象的概念， 对同调论的基本思想做到了深入浅出。本书以现代数学的语言来阐述代数拓扑中同调理论的， 内容安排上由浅入深，即从单纯同调开始，逐渐地过渡到奇异同调及上同调。 虽然教材的主要内容是基本的代数结构，但几何动机、背景和应用贯穿始终。 全书分 8 章，从单纯复形引入同调群概念，到如何定义一般拓扑空间的同调群； 到最后一章又转到流形的同调理论。遵循了从特殊到一般，再回到特殊的哲学规律。 本书的习题和教材的衔接处理的非常好。
本书比较适合我国综合性大学数学系研究生的外文教材，尤其适合一学年。 本教材自封性非常好，故此还特别适合程度较好的数学系本科生进行自学。 （吕志）

国外评论选摘
1) This well written text is one of the standard references in algebraic topology courses because of its conciseness, and I find it very useful as a reference text. However I think it is a little incomplete because of several reasons.
(1)It pays no attention to one basic concept of algebraic topology: the fundamental group.
(2) It doesn't cover Cech homology, important in other areas, like dimension theory for example.

(3) It doesn't stress the most important feature of algebraic topology: its connection to other areas of mathematics (analysis, differential geometry, etc.).
(4) Its list of references is too short, and lacks almost completely HISTORICAL references which are always important to become an expert in any field.
Conclusion: a good reference on homology and cohomology essentials, but not "the" reference on algebraic topology as a whole

2) Algebraic topology is a tough subject to teach, and this book does a very good job. Some prerequisites, however, are essential:
- point set topology (e.g. in Munkres' Topology)
- Abstract algebra
- Mathematical maturity to be willing to follow a definition and argument even when it seems like a weird side-track

In addition, this would not be the first book I would recommend to those interested in algebraic topology. First might be Massey's "Algebraic Topology: and Introduction" that introduces the fundamental group (conceptually easier than homology and cohomology).
At some point, however, a prospective student in topology will have to learn homological algebra and this provides the most concrete approach I know to the subject.
Algebraic topology is a lot of fun, but many of the previous textbooks had not given that impression. This one does.

书名： Lecture Notes on Elementary Topology and Geometry

作者： Singer & Thorpe

出版商：Springer Verlag (1967)

页数：232

适用范围：大学数学系本科高年级教材或参考读物

预备知识：微积分、线性代数、抽象代数

习题数量：少

习题难度：中等

推荐强度：9.2

书评： 本书是大数学家为本科生写教材的又一典范，在 60-70 年代曾用作麻省理工学院数学系本科高年级一学年的课程的教材。 时隔几十年，行家们仍然认为这是一本不可多得的拓扑和几何的优秀入门读物。

本书篇幅不大，包含的内容不少，深入浅出，引人入深。作者不追求完整性，比如前两章的点集拓扑的基本知识不拘泥于一些 公理的仔细探讨，而是简明实用地把几何中最常用的拓扑空间讲清楚。第三章只化 20 几页就把基本群和复叠空间及其关系写清楚的， 值得指出的是前三章看似枯燥的内容中时而出现非常有趣的例子。第四、五、六章是全书的第一重点，讲述拓扑空间的同调群及微分流形 的概念，高潮是 de Rham 定理的证明。这里充分体现了数学中各不同分支间的渗透。最后两章是黎曼几何的导引，讲述了曲面上的 Gauss-Bonnet 公式这一深刻的定理。即使从现在的角度去看，这本书的选材仍然是反映现代数学主流的。

从内容来看，本书是点集拓扑、微分拓扑、微分几何三合一，这正是作者开这门课的宗旨。我国综合性大学的数学系一般对本科生也设有 拓扑和微分几何的课程，一般各占一个学期，这些学校的学生在学完这些课程后常常不知道 de Rham 定理和 Gauss-Bonnet 公式。 Singer-Thorpe 的这本教材也许可以启发我们在拓扑和几何的本科生教学方面作些改革。 （杨劲根）

书名： Topology from the differentiable viewpoint
作者： John Milnor
出版商：The University Press of Virginia
页数：61
适用范围：大学数学系本科高年级参考读物
预备知识：微积分、线性代数、基础拓扑
习题数量：17 题
习题难度：从易到难
推荐强度：9.5
书评： 华罗庚前辈写过好几本题为《从 ... 谈起》的数学小册子，其中一本是 《从单位圆谈起》。我们可以把 Milnor 的这本小书起名为《从单位球面谈起》，事实上本书自始至终不离单位球面。
拓扑学家 John Milnor 是费尔滋奖得主，在 Princeton 执教多年，他在拓扑方面的很多系列讲座笔记被整理出版，成为脍炙人口的数学读物， 这本书就是其中一本。
一本可以让大学三年级学生能看懂的 60 页的小册子，却包含如此多的深刻的定理（从 Sard 定理直到 Hopf 定理）以及完整的证明，这是何等的不可思议！ 这是学拓扑或几何的学生的必读书。（杨劲根）

国外评论选摘

i) Perfect for a first-year graduate or advanced undergraduate course, Milnor takes us on a brief stroll through elementary differential topology. Elegant and self-contained, this book serves as an excellent first taste of the subject. Milnor is a master expositor, and is at his best in this book.
ii) One of the best points of this little book is its brevity and clear exposition of the basic ideas. It makes a great reference guide because it's so short and well-organized. Written by a distinguished mathematician, it's no wonder that other graduate-level texts such as Guillemin & Pollacks "Differential Topology" highly recommend reading it alongside their book. Milnor's booklet is a classic, whose style and ideas surely pervade other texts.

书名： Algebraic topology
作者： Hatcher
出版商： Cambridge University Press (2002)
页数： 542
适用范围：大学数学系基础数学研究生教材
预备知识：点集拓扑、抽象代数
习题数量：中等
习题难度：中等
推荐强度： 8.5

国外评论选摘

i) No serious introductory text on basic algebraic topology has ever achieved this level of clarity, readability and depth. Its richness in examples (in both the main text and the problems) exposes a beginner to the underlying mechanisms of geometry in algebraic topology; its choice and arrangement of topics strike a perfect balance between accesibility and substantiveness; its lively and motivating exposition makes a student reluctant to attend the often boring topology classes. For a novice, this should be the first reading on the subject before (s)he is ruined by the many existing daunting texts; for a veteran, this can be very nourishing, especially if (s)he is already ruined by those either unreadable or shallow 'introduction's.
ii) Allen Hatcher has gone to great length's in order to create a text which, albeit overly verbose, can be used as a gentle introduction to modern Algebraic Topology. Why 'modern'? Compare this text with the tried and tested texts of Spanier, Munkres as well as May and, almost immediately, you will see what I mean. The obvious example is Hatcher's use of CW-complexes as opposed to the more traditional build up beginning with simplices. For the die-hard mathematician who enjoys less fluff, this book is not for you and, in particular, if this is your first venture in Algebriac Topology, you enjoy the theorem-proof-theorem style with a light sprinkling of explaination, then I would recommend J.J. Rotman's text. Whereas, if you enjoy filler, background information, and lots of side-notes or examples, then Hatcher's text would be a perfect fit. Myself, I fall into the category of those who enjoy the more terse texts but, I purchased Hatcher's (the hardcover) because of the clarity and percision found in the proofs. The majority of other texts have a tendancy to obfuscate the underlying meaning that should be unerstood by the up-and-coming mathematician. Of course this approach has it's merits since, in particular, it forces the reader to fill in the blanks but, as a matter of insight, Hatcher's approach is also beneficial. Another positive strength of Hatcher's text lies in the fact that he effectively breaks the subject into it's prime sub-categories in such a way that the reader can begin with either of the four parts of the text without having to rely too much on previous sections. This novel feature allows someone interested in, say, Cohomology to pick up an begin learning about Cohomology without having to waste time making their way through material they are not interested in. Finally, yes you can get the book for free via Hatcher's website but I highly recommend purchasing the hardback text. It is well made, it will last for years, and it becomes truely mobile as compared to burning your eyes out while reading the text on your computer. Moreover, why waste the time printing it out.

书名： Differential forms in algebraic topology

作者： Bott & Tu

出版商： Springer GTM 82 (1982)

页数： 331

适用范围：大学数学系基础数学研究生教材

预备知识：基础数学本科生的大部分知识

习题数量：少

习题难度：中等

推荐强度： 9.3

国外评论摘选
1) The authors of this book, through clever examples and in-depth discussion, give the reader a rare accounting of some of the important concepts of algebraic topology. The introduction motivates the subject nicely, and the authors succeed in giving the reader an appreciation of where the concepts of algebraic topology come from, how they do their jobs, and their limitations. The de Rham cohomology, which is the main subject of the book, is explained in here in a way that gives the reader an intuitive and geometric understanding, which is sorely

needed, especially for physicists who are interested in applications. As an example, they give a neat argument as to why de Rham cohomology cannot detect torsion. In chapter 1, the authors get down to the task of constructing de Rham cohomology, starting with the de Rham complex on R(n). The de Rham complex is then specialized to the case where only C-infinity functions with compact support are used, giving the de Rham complex with compact supports on R(n). The de Rham complex is then generalized to any differentiable manifold and the de Rham cohomology computed using the Mayer-Vietoris sequence.
The discussion gets a little more involved when the authors characterize the cohomology of a fiber bundle. The all-important Thom isomorphism for vector bundles, is treated in detail. The authors give several good examples of the Poincare duals of submanifolds. The connection to ideas in differential topology is readily apparent in this chapter, namely transversality and the degree of a map. In addition, the first construction of a characteristic class, the Euler class, is done in this chapter.
The Mayer-Vietoris sequence is generalized to the case of countably many open sets in chapter 2, and shown to be isomorphic to the Cech cohomology for a "good" cover of a manifold. Good examples are given for computing the de Rham cohomology from the combinatorics of a good cover. The authors then characterize Cech cohomology groups in more detail, introducing the important concept of a presheaf. Presheaves are usually introduced abstractly in most books, so it is a real treat to see them described here in such an understandable way. Computations of the case of a sphere bundle are given, and the role of orientability and the Euler class in giving the existence of a global form on the total space is detailed. The Thom isomorphism theorem and Poincare duality are generalized to the cases where the manifold does not have a finite good cover and the vector bundle is not orientable. A very concrete introduction to monodromy is given and nice examples of presheaves that are not constant are given.
The authors treat spectral sequences in chapter 4, and as usual with this topic, the level of abstraction can be a stumbling block for the newcomer. The authors though explain that the spectral sequence is nothing other than a generalization of the double complex of differential forms that was considered in chapter 2. The crucial step in the chapter is the transition to cohomology with integer coefficients, which is necessary if one is to study torsion phenomena. The De Rham theory is then extended to singular cohomology and the Mayer-Vietoris sequence studied for singular cochains. The authors show that the singular cohomology of a triangularizable space is isomorphic to its Cech cohomology with the constant presheaf the integers. After a fairly detailed review of homotopy theory (including a discussion of Morse theory) the authors compute the fourth and fifth homotopy groups of S(3). The last section of the chapter discusses the rational homotopy theory of Sullivan as applied to differentiable manifolds. The authors discussion is illuminating, and shows how eliminating any torsion information allows one to prove some interesting results on the homotopy groups of spheres. One such result is Serre's theorem, the other being the computation of some low-dimensional homotopy groups of the wedge product of S(2) with itself.
The last chapter of the book considers the theory of characteristic classes, with Chern classes of complex vector bundles being treated first. The theory of characteristic classes is usually treated formally, and this book is no exception, wherein the authors formulate it using ideas of Grothendieck. They do however give one nice example of the computation of the first Chern class of a tautological bundle over a projective space. The Pontryagin class is defined in terms of a complexification of a real vector bundle and computed for spheres and complex manifolds. A superb discussion is given of the construction of the universal bundle and the representation of any bundle as the pullback map over this bundle.
2) This book is almost unique among mathematics books in that it strives to ensure that you have the clearest picture possible of the topics under discussion. For example almost every text that discusses spectral sequences introduces them as a completely abstract machine that pumps out theorems in a mysterious way. But it turns out that all those maps actually have a clear meaning and Bott and Tu get right in there with clear diagrams showing exactly what those maps mean and where the generators of the various groups get mapped. It's clear

enough that you can almost reach out and touch the things :-) And the same is true of all of the other constructions in the book - you always have a concrete example in mind with which to test out your understanding. That makes this one of my all time favourite mathematics texts.

书名：Knot thoery
作者：Livingston
出版商：Mathematical Association of America (1996)
页数：258
适用范围：大学数学系本科生自学读物
预备知识：线性代数，群论
习题数量：多
习题难度：中等
推荐强度：8.8

书评：本书是美国数学协会出版的大学生系列丛书“Carus Mathematical Monographs" 中的一册，是拓扑学中纽结理论的优秀入门书。
本书的预备知识非常少，只要少量的线性代数知识。如果知道一些群论更好，但作者在用到群和二次型时都从头讲起。本书从纽结的历史和直观形象开始讲述纽结的分类，分别从组合、几何和代数三个方面引入各种重要不变量，如 Seifert 矩阵、 Alexander 多项式、 Conway 多项式、Jones 多项式等。最后把 各种不变量的关系叙述得非常清楚。
本书图文并茂，习题非常丰富。内容安排从浅入深，章节的衔接紧凑。对初学者容易忽视的要点讲得很清楚。大部分定理有严格的证明，但又不拘泥于一些繁琐的证明细节，容易使读者掌握要点。
由于所用的准备知识少，所有的证明几乎都基于平面上的 Reidemeister 变换，纽结的基本群只是简单介绍一下，代数拓扑的工具没有使用。从这点来看对于具有较深数学基础的读者可以较快浏览本书后再选择更加高深的纽结理论的书籍阅读。
下面两段国外的评论中第一篇是一个数学教授写的，第二篇是自学过这本书的一个研究生写的，颇有代表性。（杨劲根）

国外评论摘选
1) This book is an excellent introduction to knot theory for the serious, motivated undergraduate students, beginning graduate
students,mathematicains in other disciplines, or mathematically oriented scientists who want to learn some knot theory. Prequisites are a bare minimum: some linear algebra and a course in modern algebra should suffice, though a first geometrically oriented topology course (e. g., a course out of Armstrong, or Guillemin/Pollack) would be helpful.
Many different aspects of knot theory are touched on, including some of the polynomial invariants, knot groups, Alexander polynomial and related abelian invariants, as well as some of the more geometric invariants.
This book would serve as a nice complement to C. Adams "Knot Book" in that Livingston covers fewer topics, but goes into more mathematical detail. Livingston also includes many excellent exercises. Were an undergraduate to request that I do a reading course in knot theory with him/her, this would be one of the two books I'd use (Adam's book would be the other).

This book is intentionally written at a more elementary level than, say Kaufmann (On Knots), Rolfsen (Knots and Links), Lickorish (Introduction to Knot Theory) or Burde-Zieshcang (Knots), and would be a good "stepping stone" to these classics.
2) I really do enjoy this book - but picked it up as a means of teaching myself Knot Theory... as was the case with many of my text books in college, brevity (for the sake of publishing costs) makes some concepts more of a challenge to grasp. Overall, the illustrations are great, and if you do the exercizes, the material tends to flow more easliy. It seemed to me the book worked backwards a bit - first covering a subject, than introducing it comprehensively later on - not what I'm used to. Keep in mind, I'm not a Mathematician, merely a graduate student of mathematics, who is interested in learning about this subject on my own.

书名： Riemannian Geometry, 3rd ed.

作者： M.P. Do Carmo

出版商：Springer Verlag (2004) ISBN-13: 978-3540204930

页数： 322

适用范围：数学专业研究生

预备知识：微积分，线性代数

习题数量：较大

习题难度： 适中

推荐强度: 10

书评：本书是一本标准的黎曼几何教材和参考书，其作者是巴西著名几何学家 Do Carmo 教授。作者写作风格清晰明了，全书共十三章，前四章介绍了黎曼几何的基本概念， 如黎曼度量、黎曼联络、测地线和曲率等；第五章介绍了 Jacobi 场这个重要的工具， 阐明了测地线与曲率的关系；第六章对等距浸入介绍了第二基本形式及相关的基本公式。 该书从第七章开始， 主要介绍了整体问题，涉及曲率与拓扑和比较几何中的一些基本结果。该书自成体系， 是目前黎曼几何最好的入门书之一。 对于想从事研究整体微分几何及相关领域的读者，该书也适合自学。 （东瑜昕）

国外评论摘选

i) "This book based on graduate course on Riemannian geometry covers the topics of differential manifolds, Riemannian metrics, connections, geodesics and curvature, with special emphasis on the intrinsic features of the subject. Classical results are treated in detail. contains numerous exercises with full solutions and a series of detailed examples which are picked up repeatedly to illustrate each new definition or property introduced. For this third edition, some topics have been added and worked out in the same spirit." (L'ENSEIGNEMENT MATHEMATIQUE, Vol. 50, (3-4), 2004)

ii) "This book is based on a graduate course on Riemannian geometry and analysis on manifolds that was held in Paris . Classical results on the relations between curvature and topology are treated in detail. The book is almost self-contained, assuming in general only basic calculus. It contains nontrivial exercises with full solutions at the end. Properties are always illustrated by many detailed examples." (EMS Newsletter, December 2005)

书名： Foundations of Differential Geometry (in two volumes)

作者： Shoshichi Kobayashi & Katsumi Nomizu

出版商：John Wiley & Sons, Inc. (1996)

页数： Vol.I : 329 , Vol.II: 468

适用范围：数学专业研究生
预备知识：微积分，线性代数，微分流形、 Lie 群基础知识
习题数量：无
推荐强度：10
书评：
本书共两卷，旨在系统介绍微分几何的基础内容，其作者是著名的几何学家 S. Kobayashi 和 K. Nomizu 。第一卷首先概要地介绍了微分流形、李群和纤维丛的概念，然后主要介绍了主丛上的联络论、向量丛上的线性联络和仿射联络、黎曼流形上的黎曼联络，还涉及空间形式、仿射联络或黎曼度量的自同构群等。第二卷主要介绍了一些经典的专题，如子流形理论、Morse 指标理论，复流形、齐性空间和对称空间、示性类理论等。本书内容翔实、处理严谨，行文精练， 自二十世纪六十年代问世以来，一直被认为是经典的微分几何参考书。 1996 年John Wiley & Sons 出版社将其选入经典图书系列重印了其第三版，可见其影响。 对于想从事微分几何和相关领域研究的读者，这是一本很好的参考书。（东瑜昕）

国外评论摘选
1) The two-volume set by Kobayashi and Nomizu has remained the definitive reference for differential geometers since their appearance in 1963(volume 1) and 1969 (volume 2). Over the decades, many readers have developed a love/hate relationship with these difficult, challenging texts. For example, in a 2006 edition of a competing text, the author remarked that "every differential geometer must have a copy of these tomes," but followed this judgment by observing that "their effective usefulness had probably passed away," comparing them to the infamously difficult texts of Bourbaki.
As a practicing differential geometer, I would argue that Kobayashi and Nomizu remains an essential reference even today, for a number of reasons. Volume 1 still remains unrivalled for its concise, mathematically rigorous presentation of the theory of connections on a principal fibre bundle---material that is absolutely essential to the reader who desires to understand gauge theories in modern physics. The essential core of Volume 1 is the development of connections on a principal fibre bundle, linear and affine connections, and the special case of Riemannian connections, where a connection must be "fitted" to the geometry that results from a pre-existing metric tensor on the underlying manifold, M. Volume 2 offers thorough introductions to a number of classical topics, including submanifold theory, Morse index theory, homogeneous and symmetric spaces, characteristic classes, and complex manifolds.
The influence of the texts by Kobayashi and Nomizu can be seen in most of the subsequent differential geometry texts, both in organization and content, and especially in the adoption of notation. If there was a particularly fine point in your favorite introductory differential geometry text that you never completely understood, the odds are good that you will find the answer, fully developed and presented at an entirely different mathematical level, in Kobayashi and Nomizu. It is not an unreasonable analogy to say that learning differential geometry without having your own copy of Kobayashi/Nomizu is like studying literature in the complete ignorance of Shakespeare.
Let there be no mistake about the advanced level of these texts. The Preface to Volume 1 clearly states that the authors presume the reader to be familiar with differentiable manifolds, Lie groups, and fibre bundles, as developed in the (now classical) texts by Chevalley, Montgomery-Zippin, Pontrjagin, and Steenrod. Today's reader is far more likely to have studied these subject from more recent books like those by Boothby, Hall, and Husemoller, but whatever the source, a familiarity IS presumed. The "lightning review" provided in Chapter I of Volume 1 will be extremely tough going for the reader who is new to these topics. It should also be noted that in 329 pages of Volume 1 and 470 pages of Volume 2, not a single diagram or

picture is to be found! Those drawn to geometry for its visual aspects will find Kobayashi/ Nomizu totally lacking in visual aids.
As with so many classic references in mathematics, the hardbound edition of Kobayashi and Nomizu is no longer in print. Copies appear sporadically on the used book market at absolutely obscene prices. The Classics Library paperback edition is still available, but the serious student will find that the paperbacks simply do not fare well under serious, sustained use.

书名： Introduction to Lie groups and Lie algebras

作者： A.A.Sagle & R.E.Walde

出版商： Academic Press (1973)

页数： 361

适用范围：大学数学系研究生低年级教材

预备知识：抽象代数

习题数量：较小

习题难度： 中等

推荐强度： 9.5

书评： 本书详细介绍了李群和李代数的基本知识以及半单李代数的结构。本书的特点是起点很低 , 对欧氏空间中的微分、张量积、模及其表示以及微分流形和 Riemann 流形的基本内容都作了一定的介绍， 学生只需要有最基本的群论知识就可学习李群和李代数，而不需要事先掌握较多的几何基础 , 同时本书对于李群和李代数的介绍又是相当完全的。 （周子翔）

10 偏微分方程¶

书名： Hyperbolic Partial Differential Equations
作者： Peter D. Lax
出版商： American Mathematical Society, Providence, Rhode Island
页数： 217
适用范围：数学专业研究生
预备知识：泛函分析，常微分方程
习题数量：无
推荐强度： 10
书评： 本书是柯朗研究所系列讲义丛书之一。其作者是美国著名数学家 Peter D. Lax 教授。他在双曲型方程尤其是拟线性双曲型守恒律方程组及其计算，孤立子理论， 拟微分算子理论等诸多方面作出了开创性及里程碑式的工作。 本书恰是作者对双曲型方程理论知识方面的讲解。 全书包含十章及五个附录，介绍了线性双曲型方程及方程组的一些基本概念，能量估计， 解的存在性以及解的其他性质，还介绍了线性双曲型方程及方程组的差分格式及其差分格式的稳定性， 以及散射理论和拟线性双曲型守恒律方程组一些基本理论。 附录中最后一章由美国著名数学家 Morawetz 教授执笔。对于想从事双曲型方程研究的读者来说， 这是一本很好的入门书。（张永前）

书名： Partial Differential Equations, An Introduction
作者： Walter A. Strauss
出版商： John Wiley ＆ Son Inc.
页数： 425
适用范围：大学数学系本科高年级学生或低年级研究生
预备知识：微积分，线性代数，常微分方程
习题数量：较大
习题难度： 较难
推荐强度： 9
书评： Walter A. Strauss 是美国 Brown 大学数学系教授，著名的偏微分方程专家。本书自出版以来， 被美国多所著名大学作为本科生的偏微分方程课程的教科书。 全书包含十四章及一个附录，介绍了几类重要的偏微分方程的来源和基本性质以及基本的研究方法， 并在最后两章中介绍了一些来自物理学的非线性偏微分方程方面的进一步课题。 书中习题类型甚广，而且还配有部分习题的答案供读者参考。 本书适合作为偏微分方程的教材及参考书。（张永前）

国外评论摘选
1） This 1992 title by Strauss (professor at MIT) has become a standard for teaching PDE theory to junior and senior applied maths and engineering students in many American universities. Last year, being an informal teaching assistant for the class, I found many of the

students struggling with the concepts and exercises in the book. Admittedly the style of writing here is very dense and if the reader does not have a very strong background in the topic, chances are high he or she will face a grand level of frustration with the exposition and the subject as a whole. One would need perseverance and dedication working numerous hours with this text before things start to settle in. After about the second or third chapter onward, those who were still taking the class had an easier time understanding the material and doing the excercises.
Contentwise, after a brief and important introductory chapter (which should not be skipped by any reader!) the book first focuses on the properties and methods of solutions of the one-dimensional linear PDEs of hyperbolic and parabolic types. Then after two separate chapters, one on the trio of Dirichlet, Neumann, and Robin conditions and the other on the Fourier series, the author embarks upon the discussion of elliptic PDEs via the methods of harmonic analysis and Green's functions. Subsequently there is a brief introduction to the numerical techniques for finding approximate solutions to the three types of PDEs, mostly centered on the finite differences methods.
The beginning of roughly the second half of the text is devoted to the higher-dimensional wave equations and boundary conditions in plane and space, utilizing the machinery of Bessel and Legendre functions, and ending up with a section on angular momentum in quantum mechanics. In the following, Dr. Strauss brings up the discussion of the general eigenvalue problems, and then proceeds with a treatment of the advanced subject of weak solutions and distribution theory. (This topic is normally skipped in an undergraduate course.) The last two chapters are a pure delight to read, dealing with the PDEs from physics as well as a survey of the nonlinear phenomena (shocks, solitons, bifurcation theory). A few appendixes at the end, summarize the analysis background needed for the course and must be consulted before and during the first reading.
All in all this is a very splendid source for all the applied maths and engineering students, that can be used in conjuction with other references to help break through the conceptual barriers. In fact, I recommended the book by Stanley Farlow to our students and many found the presentation there very modular and accessible. For example, some of the Strauss' homework problems, such as solving the Poisson equation on an annulus, were subjects of a single chapter in Farlow. In any event, I am very much hoping to see a new and more student-friendly edition of the Strauss' text be prepared and issued in the near future.

2） I used this book in a tough applied math course, and the quality of this book did not help matters much. There are a couple of good things about this book. The material chosen is appropriate and reasonably comprehensive for an intro PDE text. In other words, the table of contents is a nice read. The notation is very clean and concise throughout, as is the typesetting. The bibliography was also useful, pointing me to some great supplementary texts.
Now for the bad parts. An intro PDE book should explain clearly the basic concepts behind PDEs, including how certain famous equations (wave, heat, Laplace , etc.) arise in physical modeling. It should explain in detail the various computational techniques for finding analytical solutions to these equations. It should explain relevant elementary theorems needed for these computational techniques. This book attempts to do all of these things, but does so poorly. The basic problem is that the book's explanations and examples are too terse and incomplete for an introductory text. Analytically solving a PDE is a relatively difficult task, involving several computational steps and techniques. Examples of these techniques should be worked in detail, but in this book, they frequently omit steps or fail to explain where or how a particular technique is being applied. Theorems are often not stated, or if they are, proofs are either omitted or partially sketched. This makes the book difficult for beginners, but it is not a terrible reference if you have already been exposed to the material.
My advice: given the price of this book and its mediocre quality, you would do better by looking elsewhere for an intro PDE text.

书名： Partial Differential Equations

作者： Lawrence C. Evans
出版商： American Mathematical Society, Providence, Rhode Island
页数： xviii+662
适用范围：数学专业研究生
预备知识：微积分，线性代数，常微分方程
习题数量：较大
习题难度：适中
推荐强度： 10

书评： Lawrence C. Evans 是著名的偏微分方程专家，美国加州大学伯克利分校的教授。本书 被美国多所著名大学采用作为研究生偏微分方程课程的教科书或者参考书。 作者在这本教材中介绍了偏微分方程的许多重要课题，重点介绍了偏微分方程各种现代处理方法。 内容主要分为三部分。第一部分介绍了一些偏微分方程解的表示， 其中包含了一阶非线性偏微分方程的特征线方法， Hamilton-Jacobi 方程和双曲守恒律方程组解的表示方法 以及一些特殊非线性方程的行波解等。 第二部分主要介绍了处理二阶线性椭圆型方程及抛物型方程的现代方法。 第三部分介绍处理非线性方程的变分和非变分方法以及处理 Hamilton-Jacobi 方程和双曲守恒律方程组 的一些现代方法。每章之后都给出了相关内容的出处的说明以及与正文内容紧密配合的习题。 本书极适合作为研究生的偏微分方程教科书。（张永前）

国外评论摘选
1) This is a textbook for a first-year graduate course in PDE (for mathematics students). You should take courses in analysis (on the level of Rudin) and measure theory before you expect to understand everything in this book. This is by far the best book on PDE. The text is extremely clear, and most of the rather technical proofs are prefaced with "heuristic" calculations to help the reader understand what is going on. The chapter on the calculus of variations is the best exposition I have found of the subject, and Evans completely dispenses with the awful "delta" notation which never made any sense.
The text doesn't make much use of the Fourier transform and doesn't even mention distributions, and this gives his book a definite nonlinear flavor (which is a good thing). This should become the standard introduction to PDE on the graduate level.
2) I have taught a one-year course in PDE based on Evans' book and found it extremely cogent and stimulating both for myself and for the students. The treatment is up-to-date, with a definite nonlinear flavor. Beyond that, the exercises are very good, and the treatment is sufficiently detailed to make class preparation fairly fast. It does demand mathematical dexterity and maturity of the students right from the start, though.
3） I've seen a lot of positive reviews of this text, and I feel the need to explain some cons of this book. Before that, I will say this is probably the best introduction to PDE theory out there. This is NOT a book for people looking for a dissertation on undergraduate methods of solution (separation of variables, fourier series, etc.). If that is what you are looking for, go to Haberman or perhaps Strauss.
Ok, so here are the problems I see with this text. First, there is no mention of distributions in this book. Evans addresses this in the intro., saying it's not necessary. I find that hard to swallow, given that fundamental solutions play a big part in the text. Despite this, Evans devotes parts of the book to going into very esoteric subjects like mean value theorems for the heat equation. The other glaring gap in this text is the absence of Schauder estimates; a corner-stone for linear elliptic theory. On a note of personal preference, I would have like to

have seen more of the book dedicated to a functional analytic foundation; the appendicies that are present are simply not enough.
Overall, the book gives a decent introduction; but is far from being self-contained and is not enough of a foundation for people wishing to pursue research in PDE. Evans does acknowledge this in his introduction, but I think its something that is frequently overlooked in reviews of this text.

书名： Partial Differential Equations

作者： Fritz John

出版商： Springer-Verlag, New York-Berlin

页数： x+249

适用范围：大学数学系本科

预备知识：微积分，线性代数，常微分方程

习题数量：中等

习题难度：适中

推荐强度： 10

书评： 本书是系列丛书《 Applied Mathematical Sciences 》的第一卷。其作者是已故著名数学家、美国纽约大学 Courant 研究所 Fritz John 教授。其内容包括：一阶偏微分方程， Cauchy-Kovalevskaya 定理 , Holmgren 定理 , Lewy 的著名反例，波动方程及 Poisson 方程、热传导方程和相应高阶方程及对称双曲组的性质及 相应定解问题的基本求解方法。既介绍了特征线方法等经典方法， 也介绍了差分方法及 Hilbert 空间理论等近代方法。本书已被四次再版， 作者在每个新版中加入了一些新的内容，如在第四版中加入了关于 Cauchy-Kovalevskaya 定理解的存在区域大小的讨论等。 1991 年 Springer-Verlag 出版社又重印了其第四版， 可见其影响。我国科学出版社也在 1986 年翻译出版了其第四版。该书由朱汝金教授翻译。 书后习题与正文内容紧密配合，有助于对所介绍方法的理解。 适合偏微分方程的初学者作为入门及参考书。（张永前）

11 概率论¶

书名： An Introduction to probability theory and its applications, Vol 1
作者： William Feller
出版商： John Wiley & Sons
页数： 509
适用范围：大学数学系本科
预备知识：微积分
习题数量：中等
习题难度：一般
推荐强度： 10

书评： 本书是概率界公认的经典教材，作者是现代概率论的大师，本教材多年来曾经多次再版多次重印，内容包括古典概率 随机序列，极限理论，更新过程，随机游动与马氏链等，此书之所以成为经典是因为 其中包含有大量的直观的概率模型，涵盖物理生物经济等几乎所有科学分支，直到今天，许多研究论文都可以在本书的例子中捕捉到其背景．本书虽然经典，但使用它作为教材对教师和学生都需要勇气，它要求授课者有非常广泛的知识背景，它也要求读者有通过现象看清 本质的能力，另外本教材并不象传统的教材那样具有 Bourbaki 风格，也就是说不是通常的教材那么系统，所以采用本书作为概率论教材的 学校并不很多，但它的确是一本富有宝藏的参考书．(应坚刚)

国外评论摘选
1) Although people often recommend K.L. Chung at our math department as an introduction to probability theory, i think that Feller is just another view of the problem. If you prefer a concise writing style then Chung is better. On the other hand, Feller's books are full of examples so that you cannot go through this book without having an accurate picture of the historical developments of probability theory and its many applications (even if sometimes applications are driving the need for theory...). This is anyway something you must have read if you want to get an intuitive understanding of probability theory. Whatever your preferred writing style is, Feller is probably a "must-read" if you're involved on probability theory, just because of its importance in the literature, not because you like it. Maths are not just about formalism, they're also a matter of culture.
2) I came across Vol 1 as a maths student in the 1970s. Indeed, the book was suggested to me by a quantum physicist recommended for the Nobel Prize in 1965 (John Ward, now deceased)- Feynman, Schwinger and Tomonaga shared the prize.
This is a difficult book and was not widely used even in the 70s as a textbook. I can recall the word "idiosyncratic" being used by someone to describe the book. The problem is that the book seeks to address deep issues and that requires hard work. It is not the sort of book a struggling student will find helpful. As one matures as a mathematician one can appreciate the incredible depth of the material. As a practical example - about 30 years after I first touched this book a Head of Quant approached me in relation to a paper by Marsaglia on distributions of ratios of normal variates. The verification of Marsgalia's derivation (which is non-trivial) is to be found as a series of 3 problems in Vol 1.

With the development of stochastic calculus in the finance world Feller can look a bit outdated but if you can understand the core material you are doing well. Stochastic calculus would be a push over.
Vols 1 and 2 present a treasure trove for those who want to delve into the area. I still use Feller's coin tossing example from Vol 1 to demonstrate to those in the finance world that their understanding of the "law of averages" is imperfect.
The funny thing is that Vol 2 (which I could never afford as a student) is so hard to get. I think that was because Vol 2 was regarded as even more obscure than Vol 1. I got a copy from Amazon second hand and it is now united with its twin in my study.
Peter Haggstrom, Bondi Beach Australia

书名： A course in Probability Theory
作者： Kai Lai Chung
出版商： Academic Press
页数： 353
适用范围：大学数学系本科
预备知识：实变函数
习题数量：较大
习题难度：难
推荐强度： 9

书评： Chung 的这本概率论教材是 Stanford 大学数学系用的一学年课程的教材，从严格的测度论开始，内容非常广泛并且深入，包括收敛性，大数定律，强大数定律，特征函数，中心极限定理，重对数律，无穷可分分布，随机游动理论，条件期望，鞅与马氏 链等，虽然不需要很多的预备知识，但要求读者有很好的数学素养和对纯粹数学 的兴趣，与 Chung 一贯风格一样，教材本身写得非常严谨，某些部分也可以说 非常难，侧重于概率的纯理论方面，对概率的应用和直观背景说得不多．此书约共有 500 个习题，其中一些习题需要很高的技巧．
本教材比现在大多数高校数学系使用的教材要难，但是使用于那些对数学真正 有兴趣的人，也是那些程度较好的学生一本很好的参考书．(应坚刚)

国外评论摘选
1) This text by Chung was one of the texts that I used when I was taking a graduate course in probability at Stanford in 1975. It is carefully written but challenging. It provides good coverage of the central limit theorem, the law of large numbers and the law of the iterated logarithm. It also covers stable laws very well. The style is one of rigorous mathematics with theorems, and lemmas given with their mathematical proofs. The book was recently revised. The revised text does not change much but new material on measure and integration that is now commonly included in the first graduate course in probability has been added. In the 1970s at Stanford a course in measure theory was a prerequisite for the course in advanced probability although some student took it concurrently.
If you plan to get this text, the revised edition is probably worth it. If you already have this edition and know your measure theory, it may not be worth it to get the new edition.
2) "A course in probability theory", written by Kai Lai Chung, has been referred by not only mathematicians but also mathematical economists.This book is written very rigorously, but almost all of the theorems have easy-to-understand proofs. So it is not difficult to follow. Moreover, there are lots of exercises in this book. So I do recommend this book.

12 计算数学¶

书名：Numerical Optimization
作者： J. Nocedal \& S. Wright
出版商： 科学出版社, 2006
页数：636
适用范围：计算数学、运筹学、工业工程、计算机专业高年级本科或者研究生教材
预备知识：数学分析、高等代数、数值线性代数
习题数量：一般
习题难度：一般
推荐强度：10
书评： 本书是由 Northwestern 大学电子工程和计算机科学系教授、数值最优化最著名的专家 Nocedal 和 Wisconsin 大学 Madison 分校 (UW-Madison) 计算机科学系教授、优化著名专家 Wright 合写的一本数值最优化教材。深受读者欢迎，国外很多院系也把这本书作为最优化或者非线性规划课程的教材。本书内容十分丰富，包括求解无约束问题的最速下降法、牛顿法、拟牛顿法、共轭梯度法、信赖域算法；非线性最小二乘问题、求解非线性方程组的数值计算方法；线性规划的单纯形法和内点算法、二次规划；求解约束问题的罚函数法、乘子法、增广拉格朗日法、序列二次规划算法。关于这本书和其它非线性规划专著的比较可以参考国外评论部分 Michael Ferris 做的精彩评价。 (杨卫红)
国外评论摘选
I find the book under review (hereafter N-W) to be a well-written treatment of continuous nonlinear optimization and I would recommend its use for upper level undergraduate or graduate level courses in nonlinear optimization. The book is sufficiently detailed to be useful to researchers, but its real merit is as an educational resource.
In reaching this conclusion, I asked the following questions of the text: What is the competition for this text? Will this text replace the one I use? Does it cover all the topics I cover, or enough of them? Is it readable?
I believe there are five books that are possible competitors to (N-W), namely (1) D. Bertsekas, Nonlinear programming, Athena Scientific, Belmont, MA, 1995; (2) J. E. Dennis, Jr. and R. B. Schnabel, Numerical methods for unconstrained optimization and nonlinear equations, Corrected reprint of the 1983 original, SIAM, Philadelphia, PA, 1996; (3) R. Fletcher, Practical methods of optimization, Second edition, Wiley, Chichester, 1987; (4) P. E. Gill, W. Murray and M. H. Wright, Practical optimization, Academic Press, London, 1981; (5) S. Nash and A. Sofer, Linear and nonlinear programming, McGraw-Hill, New York, 1996.
The book by Fletcher is the classical reference; I believe\break (N-W) will supplant Fletcher's for two reasons. Firstly, the new book uses a more modern typeset and is much easier to read and understand. Such clarity enables students to learn the material more quickly, and, furthermore, to read the relevant literature more easily. Secondly, the topics covered in the new book are more current and balanced (the one exception being nonsmooth optimization). In particular, the section on automatic derivatives is an important and welcome addition. Clearly, the (N-W) text benefits from being written a good decade later.

Similar comments can be made comparing (N-W) to the Dennis and Schnabel book. While the latter is much more specialised than Fletcher's text, it is still the book of choice for many who are interested in unconstrained and nonlinear equation software.
I believe (N-W) is much more up-to-date and will be a much better choice for engineers and practitioners interested in knowing some mathematical background for the algorithms they need to use. The treatment of the same topics is more thorough but eminently readable in (N-W); furthermore, (N-W) covers linear and constrained optimization as well, without becoming too long.
While the treatment of line search methods in the book by Gill, Murray and Wright is very good, (N-W) covers these in adequate detail, and furthermore gives an up-to-date view of trust region methods as well. The treatment of interior point methods in the description given in (N-W) also benefits from the research carried out over the past decade, and is preferable to that of Gill et al.
The Nash and Sofer book is somewhat similar to (N-W) and may be preferable for lecturers who wish to teach nonlinear optimization appended to a course on linear programming. I believe, however, that the approach given in (N-W) is better, and enables a clearer understanding of how linear problems fit within the big picture. Again, the treatment of trust region methods is much more complete in (N-W).
The Bertsekas book is very different from (N-W). It is a much more theoretical text, and covers aspects of convex analysis and nonsmooth optimization in much greater detail than (N-W). For more theoretical courses, I would prefer to use the Bertsekas text. However, for courses that stress algorithms for smooth problems, the (N-W) text covers all the necessary ground concisely, clearly and in an eminently readable fashion. These two books complement each other nicely.
Is the book well organised? For the most part, I believe it is. The only caveat to this is that the trust region and conjugate gradient chapters seem to be the wrong way around, with many forward references in Chapter 4. What's missing? Several things are missing from the book, in my opinion. Some treatment of nonsmooth optimization would have been useful in this text, as many of the ideas from that literature can be used to supplement our understanding of algorithms and enhance the problem classes that can be treated. (N-W) also does not cover derivative free methods at all, unless one counts finite differences. Given the huge variety of applications that use such methods in practice, I believe some comments on their applicability, strengths and weaknesses would strengthen the book. Finally, I would be remiss if I did not complain about the omission of complementarity. The unifying viewpoint and newly emerging problem classes that could benefit from overlap would draw these two fields more closely together again. (by Michael C. Ferris)

13 其他¶

书名： Berkeley Problems in Mathematics
作者： Souza & Silva
出版商：Springer Verlag
页数： 443
适用范围：大学数学系本科生各年级
预备知识：按各章内容而定
习题数量：大
习题难度： 中等到难
推荐强度： 8.2

书评： 本书是美国加州大学 Berkeley 分校数学系直到 90 年代的历届博士生 资格考试题集锦。作者按分析、多变量微积分、微分方程、度量空间、复分析、 抽象代数、线性代数进行了分类，总共大约 300 道题，难度参差不齐。 美国大学的数学系的研究生没有入学考试，却有资格考试，那时在入学后的第一年 里需要通过的，考试的方式各校不一样，采用笔试比较普遍，名校的题比较难。 Berkeley 的题不算特别难，和我国重点大学本科生的试题差不多。由于出题者多半是 Berkeley 数学系的教授，大部分题在别处见不到，有些考题明显是从 出题人的论文中提取的。书后附有习题解答，但不完整，遗憾的是最难的一些题 都没有解答，这也许是编书的人来不及把所有题做完。本人认为这本书是非常 好的补充题来源，值得数学系的师生选用。 （杨劲根）

国外评论摘选
This book is a rare peak inside one of the best Ph.D. programs in Mathematics in the world. It allows you to try out and test yourself on the same problems that the best young and aspiring mathematicians are testing themselves. The problems are neatly arranged by subject and in increasing level of difficulty, and the solutions, are not only beautifully written, but somewhat surprising and unexpected for a seasoned student. I pull mine out of the shelf on the rainy days and try a few more, and when I get one, I really savour it!

书名： Putnam and Beyond
作者： Gelca & Andreescu
出版商：Springer Verlag (2007) ISBN-13: 978-0-387-25765-5
页数： 798
适用范围：大学数学系本科生数学竞赛
预备知识：微积分、线性代数
习题数量：大
习题难度： 中等到难
推荐强度： 8.5

书评： 我国的中学生数学竞赛是国际领先的，各种参考书籍也很多，这多数归因于社会各界对这项活动的重视和投入。 与此相比，大学生的数学竞赛就冷清多了，相关的书籍也少。而美国对大学生和中学生的数学竞赛同样地重视。 美国的 Putnam 大学生数学竞赛有 70 年历史，每年举行一次，美国的各名校都组对参赛，是大学数学系本科生的一次盛会， 优胜者名单和试题及解答都公布在美国数学月刊上。象哈佛、麻省理工学院等校把这项荣誉看得很重，互相 较紧，就象数学中的 NBA. 美国有些著名数学家如 Milnor,Mumford,Quillen 都是当年的优胜者。
很多年前曾有过一本 Putnam 竞赛的试题和解答的中译本，收集了从 1938 年到 1980 年的全部比赛。
本书并不是 Putnam 竞赛试题集，它是关于解题方法和技巧的导引，当然也收集了大量竞赛集训题。作者先介绍了竞赛中常用的五种技巧：反证法、数学归纳法、抽屉原理、极值问题、不变量。每种方法都有精彩的示例。然后按照 代数、分析、几何、数论、组合五个专题讲解实例。例题和习题总共一千余道，书后附习题的详细解答。 从题的深度来看，有大量的题实际上是中学生竞赛题，大学生竞赛题以解析几何、微积分和线性代数为主， 抽象代数有一些，但比较浅。两位作者都有多年指导大学生和中学生竞赛的经验，他们所选的题大部分是 他们所指导的集训队的集训题。
本书是比较理想的大学生数学竞赛参考资料，也可供中学生数学奥林匹克的教练参考。对于想提高数学解题能力 的大学数学系也是很有帮助的。 （杨劲根）

国外评论摘选
Once more another panorama of amazing math problems written by two famous math problemists: Titu Andreescu and Razvan Gelca Many many congratulations to them for this invaluable treasure of math problems. I am not absolutely able to describe this excellent book; the best way is purchasing this book. I highly recommend it to all math lovers; in particular to whom are preparing themselves to mathemaical competitions of all kinds. In fact I do warmly recommand all of the books by Titu Andreescu and his colleagues without exception!!!