The Environmental Implications of Renting The Runway

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Abstract

I study the impact on clothing production of subscription-based leasing arrangements: I adopt a stylized set-up that features two competitive fringes of retailers and an Access Based Service Provider (ABSP) that offers a two-period rental contract; each fringe supplies a horizontally differentiated good, and the subscription to the ABSP gives access to both products. Consumers display a taste for variety: their gross surplus from the purchase of both products is super-additive. The green credential of the ABSP stems from its efficiency in serving the fraction of users that buys both items; with only one piece per subscriber, the ABSP can provide two distinct outfits, swapping items among its clients between periods. I show that the entrance of the ABSP reduces total production if and only if users exhibit a sufficiently strong preference for variety and if the unit production costs are low enough. Subsequently, I analyze the competitive interaction between the ABSP and the retail side in a Stackelberg game: a retailer that sells both products holds a first-mover advantage against an ABSP. In this set-up, if the demand side is characterized by very low acceptance of rental vis-à-vis ownership, then entrance necessarily implies a larger production volume.

JEL: D16, D21, D43, L13, Q56

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1 Introduction

The clothing sector accounts for an important chunk of the overall greenhouse gas (GHG) output of the planet[1]. Several studies conducted using the life-cycle assessment methodology show that the lion's share of the environmental impact of a fashion product belongs to the production phase (van der Velden et al. 2014, Diekel et al., 2021 Sohn et al., 2021, Luo et al., 2022): for instance, Luo et al. (2022) document that the industrial manufacturing process of a pair of cotton jeans accounts for more than 95% of its carbon footprint. Recycling opportunities for textiles are limited: in most cases, garments are complex blends of fabrics and fibre materials. The process of recovery involves mechanical shredding, which results in shorter fibres; these will be weaker than their virgin counterpart, and, in many instances, not suitable for the production of new clothes[2]. Insulation panels and carpets are two of the most common applications for salvaged materials. The bottom line is that of the 53 million tons of textiles used to produce new apparel in 2015, less than 1% were converted into new pieces (EMF, 2017).

Extended producer responsibility (EPR) schemes are the most popular tool under the lens of regulators to cope with the problem: these consist of provisions aimed at holding the manufacturers directly accountable for the collection, sorting and recycling of the goods they sell. Currently, among the 27 members of the European Union, only France (since 2020), Sweden (since 2022) and the Netherlands (from July 2023) have implemented EPR for textiles. As an example, France, as set forth in Article L541-10-3 of the Code de l'environnement, charges importers and retailers of fashion products an eco-contribution up to 20% of the pre-VAT retail price that depends on the environmental performance of the item[3]. This payment is meant to finance a collective entity that fulfils the waste management tasks of the undertakings subject to the regulation[4] (see Vernier, 2021). The main limitation of these

[1] No reliable estimate on the environmental impact of fashion exists; anecdotal evidence, in CO_2 equivalent, spans between 2% to 8% of the global GHG production. This interval comprises the overall GHG produced by aviation and twice aviation and maritime shipping combined. The vertically disintegrated nature of the fashion supply chain, its global reach and the lack of comprehensive data are the main hurdles to an accurate assessment. See `https://hbr.org/2022/01/the-myth-of-sustainable-fashion`, last accesses 16-Jan-2023. For a discussion on the transparency of the available studies and on their trustworthiness, see `https://www.vox.com/the-goods/2020/1/27/21080107/fashion-environment-facts-statistics-impact` and `https://ecocult.com/now-know-fashion-5th-polluting-industry-equal-livestock/`, last accessed 16-Jan-2023.

[2] On top of this, as clothes are dyed chemically, recycling also requires bleaching, which further weakens the fibres. See `https://www.bbc.com/future/article/20200710-why-clothes-are-so-hard-to-recycle`, last accessed 18-Jan-2023.

[3] See `https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000043974919#:~:text=Les%20signal%C3%A9tiques%20et%20marquages%20pouvant,%C3%A0%20la%20gestion%20des%20d%C3%A9chets` (in French), last accessed 19-Jan-2023.

[4] Firms can in principle avoid this obligation by taking care of the end-of-life treatment of their products themselves, but this circumstance is rather rare.

programmes is their lack of bite in terms of demand-side abatement: while they incentivize firms and customers to turn to more sustainable materials, consumers internalize the harm of excessive production only to the extent that such injury is reflected in the manufacturing cost differential. In other words, there are no additional levies on the items that abide by the standards of the regulation. However, policymakers have proved sceptical with respect to the environmental taxation of garments: the reason lies in the high sensitivity of the fashion market to cross-border trade, and hence its susceptibility toward issues related to intellectual property such as parallel imports.

A popular guideline invoked by several advocates of the circular economy is to scale up business models (OECD, 2018, EMF, 2021, WRAP, 2021) that provide access-based services (ABS) (Fritze et al., 2020), namely product sharing and rental. The former arrangement, also known as collaborative consumption, is operated by peer-to-peer intermediaries that connect lenders and one-time renters: the platforms earn from a transaction fee on the rental price agreed by the parties and operate the service generally through smartphone applications (such as Tulerie, By Rotation and Hurr). These businesses are pure matchmakers: the users bear every expense, from transport to laundry. The latter practice is performed by firms (like Rent The Runway, Armoire Style, Flyrobe, Nuuly or Le Tote) that lease pieces supplied by a gamut of upstream fashion designers[5]. These lessors are also in charge of shipping and washing; they differ from a traditional retailer only in that they rent out garments rather than sell them. They are *rentailers*. Their core business revolves around their wardrobe in the cloud: customers choose from it a contractually determined number of articles at a fixed rental fee and keep them for a given amount of time. Commonly, these subscriptions are monthly, and the total amount of items rented can be delivered either in one or two shipments[6]. For example, Rent The Runway's standard plan guarantees eight pieces per month at 144 $, which are delivered in batches of four: the first at the beginning of the month, and the second after the first two weeks, conditionally on the return of the first bundle.

The main advantage of ABS is in terms of savings; on the one hand, when consumers lend clothes on peer-to-peer marketplaces, they can earn an income stream from a product sitting idle, and hence recoup a fraction of the purchase price; on the other hand, the aforementioned rentailers offer access to medium-high quality garments that would retail for a considerably

[5]Most of the designers charge the lessors a unit price per garment. Nonetheless, other types of covenants are possible: for example, Levi Strauss operates under a revenue-sharing agreement with Rent The Runway. See `https://www.reuters.com/article/retail-renting-idUKL5N2604W1`, last accessed 24-01-2022.

[6]The UK-based Hirestreet is a notable exception: the company offers a bundle of either two or three pieces at a fixed price, and the return date is set only 4 days after the consignment. Every aforementioned rentailer also offers one-time rentals and conversion purchases. Rent The Runway's CEO Jennifer Hyman, in a recent interview, claims that "[subscription] has become the majority of our revenue". See `https://www.theverge.com/23277843/rent-the-runway-ceo-jennifer-hyman-fashion-rental-decoder-podcast-interview`, last accessed 26-Jan-2023. Rent The Runway is currently the biggest ABS by revenue, at around 100 USD million per year.

higher amount. Many sustainability experts also envisage an important environmental gain from production abatement: since both practices allow to serve different consumers with the same unit, given the steady drop in the utilization levels of clothes over the last 20 years and soaring global demand, the latter should shrink with the penetration of ABS. Benjaafar et al. (2019) illustrate the argument for collaborative consumption analytically, in a single product setup: for goods with moderate ownership costs, a sharing market offers to consumers with low willingness to pay for ownership the option to simply rent the article rather than purchasing it. If the rental price is not excessively high, hence if the prospective income from leasing does not generate a market expansion effect[7], the number of owners in the industry, and thus aggregate production, declines. A few theoretical works in the sustainable supply chains literature consider the green credentials of leasing vis-à-vis selling (Agrawal et al., 2012 and 2021, Gülserliler et al., 2022). These papers consider rental and purchase of a durable good in isolation, and examine the effects on the secondary market of the business model choice. In the canonical framework, a pure retailer loses control of the sold units: it faces thus competitive pressure from the used market, which is generally assumed to be frictionless. The main insight is that a manufacturer can choose to rent out its product rather than sell it to profitably re-market the off-lease units and avoid cannibalization: this implies that a firm might find it convenient to scrap some old products, if disposal is not excessively costly. When the use impact of the depreciated units is very large, a rental-based business model can thus outperform one relying uniquely on sales in terms of sustainability (see Agrawal et al., 2012).

These articles fall short of depicting the environmental trade-offs of renting clothes through fixed quantity bundles in several ways: first, they all assume that consumers can observe whether the product they rent has been used or not. This is not the case for the leasing-based ABS described previously. Given the degree of efficiency professed by these undertakings in cleaning and repairing their off-lease units[8] and considering that each firm employs a rating system for every single piece it rents out to prove such efficiency to its customer base, one might conjecture that wear-and-tear does not play a role at all in a consumer's decision-making process. Second, none of these works considers the interplay between bundling, leasing and the aggregate volume of sales, as they adopt a single product framework; nevertheless, fashion rentailers' main business is to offer subscription plans where the number of items is fixed. Third, these works do not consider competition *between* business models;

[7]This assumption might not be innocent for peer-to-peer lending of fashion products; in the UK, different sources document a blossoming of users that behave as professionals on platforms such as By Rotation or Hurr. See, for example, `https://www.vogue.co.uk/fashion/article/fashion-rental-tips`, last accessed 24-Jan-2023.

[8]See `https://www.theverge.com/23277843/rent-the-runway-ceo-jennifer-hyman-fashion-rental-decoder-podcast-interview`, last accessed 26-Jan-2023.

as clothing rental accounts worldwide for roughly 0.3% of the global apparel market[9], it is crucial to assess the environmental consequences of potential penetration *given* the existing prevalence of the retail channel. Fourth, the existing body of analytical literature on leasing assumes preferences à la Mussa and Rosen (1978); yet, clothing subscription plans are inherently bundles of horizontally differentiated goods, and grant subscribers multiple garments. Fifth, the plan purchasers are individuals who value substantially the opportunity of wearing different outfits for distinct occasions. This might be suggestive of a taste for variety: a degree of super-additivity in the consumption of multiple products (see, for example, Gabszewicz et al., 2001). Within the sustainable supply chain literature on leasing, no study considers this feature. Two questions follow naturally:

1. *Is leasing clothes by means of subscription plans more sustainable than selling, if we consider competition between business models?*
2. *How does the competitive landscape on the retail side affect the equilibrium outcome?*

In this paper, I study analytically the impact on aggregate production of leasing products through bundles. I adopt a setup that features two periods and two horizontally differentiated products, in which consumers display a taste for variety: their gross surplus is super-additive. I first develop a toy framework, characterized by two competitive fringes of retailers and a rental-based ABS provider (hereafter ABSP) that leases both products through a two-period contract. I show that the entrance of the ABSP in the industry results in lower aggregate production if and only if the unit production cost of the items is sufficiently low and/or consumers exhibit a sufficiently strong preference for variety. Subsequently, I extend the setting to imperfect competition: a retailer who holds a first-mover advantage and an ABSP compete on prices in a Stackelberg game. I find that if consumers have a bad perception of rental, and discount it strongly with respect to purchasing, then the entrance of the ABSP necessarily results in a higher volume of production, and this circumstance is more likely the lower the degree of superadditivity of preferences. The rest of the article is structured as follows: section 2 presents a stylized setup that features two competitive markets of retailers and an ABSP; in section 3, a monopoly retailer that sells both items faces competition à la Stackelberg from an ABSP. section 4 connects the result to the literature. Section 5 concludes.

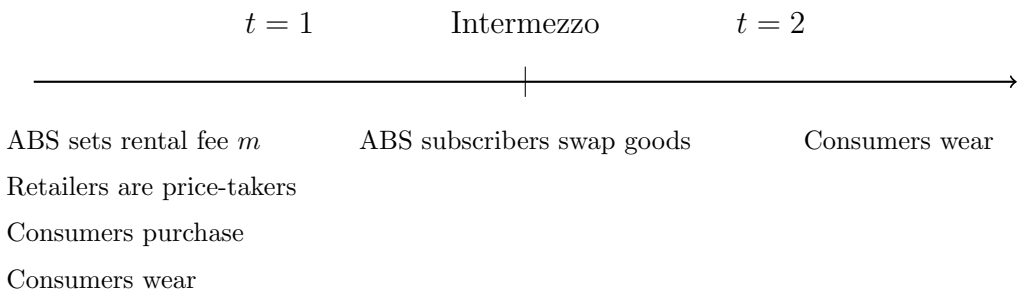
2 Toy Version: Competitive Retail Market

I consider a stylized setting with no strategic interactions between two competitive fringes of retailers and a rental-based ABS. I compare a baseline scenario where consumers can only purchase the products with the case where they can also opt for a rental subscription plan; I restrict the environmental assessment of the two scenarios uniquely to the production impact,

[9]See `https://www.rentle.io/blog/rental-business/online-clothing-rental-market`, last accessed

comparing aggregate demands in equilibrium. The model features two periods and two goods: in the first period, the ABS (if present) and consumers make pricing and purchasing decisions simultaneously; additionally, consumers wear one of at most two pieces of clothing they purchased. In the second period, only consumers play: they can only take usage decisions, and choose whether they want to use one product, or the other, if they purchased it. The contract offered by the ABS grants consumers access to both goods, for only one period of play: between period 1 and period 2, consumers have to handle back the good they used in the previous period, and then receive the other good. I assume that every consumer who subscribes to the ABS first wears the piece that grants her the highest gross surplus, which I define in the next paragraph. Additionally, I abstract away from wear and tear. Figure 1 represents the timeline of the game.

Figure 1: Timeline with two competitive fringes of retailers.



2.1 Baseline

Two competitive fringes of retailers sell each a horizontally differentiated product; fringe A sells product A, and fringe B sells product B. A competitive fashion industry designs and manufactures both goods at the same marginal production cost $c \in (0, 1)$. Consumers are represented by a continuum of mass one, and their heterogeneous preferences are defined by the parameter θ, which is uniformly distributed over the $[0, 1]$ support. Consumer θ has willingness to pay θ for good A and $1 - \theta$ for good B, and they buy at most one unit of *each* good. This implies that they can also purchase both A and B; the gross surplus from this option is the sum of the willingness to pay for each product, plus a benefit $\xi > 0$, which conveys a taste for variety. The key idea encapsulated by this parameter is that for garment and fashion products consumers appreciate varying their outfits; they reward dressing differently for different occasions, independently from their preferences for a given piece. This complementarity between products can also be related to the frequency of usage: for example, a buyer should display a lower taste for variety for ski clothing or ceremony suits rather than for casual clothing. To rule out the uninteresting case where the entrance of the ABS in the market unambiguously reduces aggregate production, I assume that $\xi < c < 1$ in every specification. Finally, the consumers can also purchase the outside good, whose net

surplus I normalize to 0. The utility of consumer θ is thus:

$$U(\theta) = \begin{cases} \theta - c & \text{if she buys } A \\ 1 - \theta - c & \text{if she buys } B \\ 1 + \xi - 2c & \text{if she buys } \{A, B\} \\ 0 & \text{if she buys } O \end{cases}$$

It follows that if $c \leq C(\xi) \equiv (1+\xi)/2$, the option of purchasing both goods will dominate the outside option, otherwise, the opposite will hold true. I first consider the former case, and I denominate it Saturation (S) under Perfect Competition (PC): under the premise $c \leq C(\xi)$, the consumer $\Theta_{B,\{A,B\}} \equiv c - \xi$ will be at the margin between buying good B or both, the consumer $\Theta_{B,A} \equiv 1/2$ will be at the margin between buying good B or good A, and the consumer with $\Theta_{\{A,B\},A} \equiv 1 + \xi - c$ will be at the margin between buying both goods or good A. It is easy to verify that for $c \leq C(\xi)$, $\Theta_{B,\{A,B\}} \leq \Theta_{B,A} \leq \Theta_{\{A,B\},A}$, namely the consumer indifferent between good A and good B is not marginal. Therefore, consumers $\theta \in [0, \Theta_{B,\{A,B\}})$ buy good B, consumers $\theta \in [\Theta_{B,\{A,B\}}, \Theta_{\{A,B\},A}]$ buy both goods and consumers $\theta \in (\Theta_{\{A,B\},A}, 1]$ buy good A. Aggregate production is thus:

$$\begin{aligned} Q_{S,PC} &= \underbrace{1 - \Theta_{B,\{A,B\}}}_{\text{good } A} + \underbrace{\Theta_{\{A,B\},A}}_{\text{good } B} \\ &= 2(1 + \xi - c) \end{aligned}$$

Notice that if $c < \xi$, then $c > (1+\xi)/2 \iff \xi > 1$, but if $\xi > 1$, then every consumer strictly prefers to buy both products to *any* other option: hence, the only equilibrium admissible is Saturation.

Next, I consider the Uncovered (U) market case, which arises if $c > C(\xi)$; given this necessary condition, the consumer $\Theta_{BO} \equiv 1 - c$ will be indifferent between buying B or the outside option, and the consumer $\Theta_{OA} \equiv c$ will be indifferent between the outside option or A. Under the necessary condition $c > C(\xi)$, following similar reasoning as in the Saturation case, the consumer indifferent between good A and good B will not be marginal. Hence, consumers with $\theta \in [0, \Theta_{BO}]$ will buy B, consumers with $\theta \in (\Theta_{BO}, \Theta_{OA})$ will purchase the outside good, and consumers with $\theta \in [\Theta_{OA}, 1]$ buy good A. Recalling that in the Uncovered market case we have that $c > 1/2$, aggregate production is:

$$\begin{aligned} Q_{U,PC} &= \underbrace{1 - \Theta_{OA}}_{\text{good } A} + \underbrace{\Theta_{BO}}_{\text{good } B} \\ &= 2(1 - c) < 1 \\ &< Q_{S,PC} \end{aligned}$$

Intuitively, if costs are sufficiently low and the demand side exhibits a sufficiently strong taste for variety, there will be a fraction of consumers with preferences not excessively skewed

toward a single product that will purchase every piece; conversely, if production costs are high enough and consumers are not prone to pay a great premium for a wider gamut of clothing items, then a fraction of the market will not be covered, and the most polarized consumers will purchase only the piece they prefer the most. The former case will clearly result in higher production than the latter.

2.2 ABSP

I introduce in the former setup an ABSP that, upon the payment of a membership fee m, allows consumers to have access to its wardrobe in the cloud, which comprises both good A and good B. In line with standard industry practices, the contract stipulates that consumers can rent at most one item per period, which they need to return at the end of this temporal window. I assume that each consumer wears first the piece she prefers the most in gross surplus terms: given the uniform distribution of preferences, this implies that every consumer that purchases a plan from the ABS will wear first product A if she has $\theta > 1/2$ and B otherwise. The rentailer is supplied by the same competitive industry that provides the garment to the markets of retailers, at the same unit production cost of c. In this framework, renting a product is perceived as a qualitatively inferior alternative to ownership, since the fraction of consumers that purchase the subscription can only hold the good for one period. I capture this discount in valuation with the parameter $\delta \in (0,1)$, which can be also interpreted as the acceptance rate of rental across consumers. Hence, a consumer subscribing to the ABS obtains gross benefit $\delta\theta$ for good A and $\delta(1-\theta)$ for good B. Since the plan offered by the ABS allows users to change their outfits, the subscription plan grants the additional extra surplus ξ that consumers who purchase both products enjoy. Labelling as L the option of purchasing the ABS plan, the utility function of consumer θ reads:

$$U(\theta) = \begin{cases} \theta - c & \text{if she buys } A \\ 1 - \theta - c & \text{if she buys } B \\ \delta + \xi - m & \text{if she buys } L \\ 1 + \xi - 2c & \text{if she buys } \{A, B\} \\ 0 & \text{if she buys } O \end{cases}$$

Thus purchasing the subscription plan is not a dominated strategy if and only if $m < \min\{\delta + \xi, 2c + \delta - 1\}$. The consumer $\Theta_{BL} \equiv 1 + m - c - \delta - \xi$ is indifferent between buying B or the rentailer's plan, the consumer $\Theta_{LA} \equiv \delta + \xi + c - m$ is indifferent between buying A or the rentailer's plan, and again the consumer $\Theta_{BA} \equiv 1/2$ is indifferent between buying B or A. Notice that if $\Theta_{LA} < \Theta_{BA} < \Theta_{BL}$, no consumer will subscribe to the ABS; however, since $\min\{\delta + \xi + c - 1/2, \delta + \xi, 2c + \delta - 1\} = \min\{\delta + \xi, 2c + \delta - 1\}$, the constraints $m < \min\{\delta + \xi, 2c + \delta - 1\}$ are necessary and sufficient to guarantee entry. Hence, consumers with $\theta \in [0, \Theta_{BL})$ buy B, consumers with $\theta \in [\Theta_{BL}, \Theta_{LA}]$ rent on the ABS, and consumes

with $\theta \in (\Theta_{LA}, 1]$ purchase A.

Since preferences are uniformly distributed, consumers with $\theta \in [\Theta_{BL}, 1/2]$ will rent B in period 1 and then return it to get good A in period 2, whereas consumers with $\theta \in [1/2, \Theta_{LA}]$ will first wear A and then swap it for B. This stylized representation conveniently portrays the environmental potential of rental-based subscriptions in terms of production abatement: if the fraction of ABS subscribers were to purchase both goods, for these consumers, two units per head would be produced. However, rental through subscriptions allows a novel indirect swap of items between users, which allows every subscriber to wear two goods, producing only one unit per capita. It is thus immediate to see that aggregate production will be $Q_{ABS} = 1$, whenever the ABS enters the market; as a consequence, the environmental credentials of the ABS hinge crucially on whether the market was saturated (since $Q_{ABS} < Q_{S,PC}$) or uncovered (as $Q_{ABS} > Q_{U,PC}$) before its entry. Hence, the assumption $\xi < c < 1$ excludes cases where entrance yields unambiguously a reduction in the total number of clothes produced.
The problem of the ABS is thus:

$$\begin{aligned} \max_{m} \quad & \Pi_{ABS}(m) = (m - c)[\Theta_{LA} - \Theta_{BL}] \\ s.t. \quad & m \leq \delta + \xi \qquad (1) \\ & m < 2c + \delta - 1 \qquad (2) \end{aligned}$$

Which yields the equilibrium

$$\begin{aligned} m^*_{PC} &= c + \frac{\delta + \xi}{2} - \frac{1}{4} \qquad (3) \\ \Pi^*_{ABS,PC} &= \frac{[2(\delta + \xi) - 1]^2}{8} \end{aligned}$$

Plugging (3) into the constraints (1) and (2) yields the restriction $c \in (\underline{c}(\delta, \xi), \bar{c}(\delta, \xi)]$, with $\underline{c}(\delta, \xi) \equiv 3/4 + (\xi - \delta)/2$ and $\bar{c}(\delta, \xi) \equiv 1/4 + (\delta + \xi)/2$, which imply the additional restriction $\delta > \underline{\delta} \equiv 1/2$. The latter in turn implies that, whenever the ABS enters the market, $\underline{c}(\delta, \xi) < C(\xi) < \bar{c}(\delta, \xi)$; hence, the sustainability claims of renting through an ABS, in this stylized setup, depend on whether the goods rented out is sufficiently cheap and on the extent to which consumers exhibit a strong love for variety, so that the ABS penetrates in a saturated market. The following findings are summed up in Lemma 1.

Lemma 1. *If $c \leq C(\xi)$, then the entrance of an ABS in the market diminishes aggregate production. Conversely, if $c > C(\xi)$, entry produces a market expansion effect.*

Proof. In the text. ■

Figure 2 provides a graphical intuition: the green area outlines the cases where the ABS would divert consumption away from double purchases; the reed area represents instead the region of parameters where the rentailer fosters the production of new clothing items, capturing

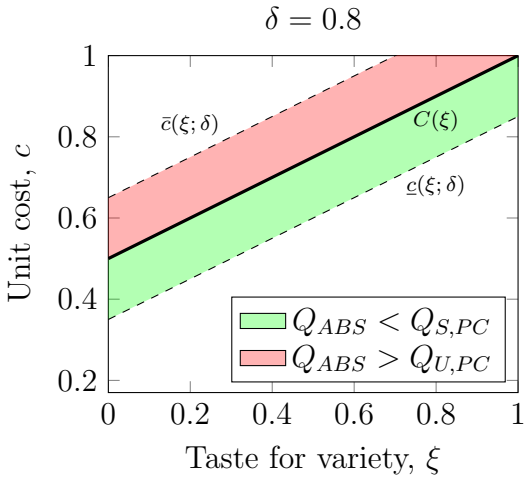


Figure 2: Sustainability of the ABS with a competitive fringe of retailers.

a fraction of the market that was previously unserved. The main message from this toy framework is that, in the absence of strategic interactions between actors, the environmental credentials of a rental-based ABS that offers subscription plans are uniquely a function of the cost of the products for the buyers, and of the extent to which they appreciate variety: how consumers' perceive rental only affects entry decisions, namely a change in δ either shrinks (if negative) or enlarges (if positive) the area between the $\underline{c}(\delta, \xi)$ and $\bar{c}(\delta, \xi)$ curves. In the next section, I examine how competition between a retailer that sells the two items and the ABS affects the result.

3 The Model

I modify the previous setup by assuming that good A and good B are now sold by a monopoly retailer, who charges the same price p for one unit of either of the two. Then, I model the strategic interaction between the ABS and the vendor as a Stackelberg game, where the latter player holds a first-mover advantage. I focus on accommodation cases; therefore, I rule out parameter ranges that result either in blockaded entry or deterrence. As in the stylized version, the sustainability of leasing multiple items through subscription plans depends on whether the ABS enters the industry in the Saturation or in the imperfect coverage scenario; however, the existence of market power on the retail side defines an important relationship between the rental acceptance rate and the environmental credentials of the rentailer.

3.1 Baseline

Under the same set of assumptions that characterizes the demand side in the baseline of the toy version, the utility of consumer θ reads:

$$U(\theta) = \begin{cases} \theta - p & \text{if she buys } A \\ 1 - \theta - p & \text{if she buys } B \\ 1 + \xi - 2p & \text{if she buys } \{A, B\} \\ 0 & \text{if she buys } O \end{cases}$$

Notice that if $\xi > 1$, again every consumer prefers to buy $\{A, B\}$. If $p \leq (1+\xi)/2$, the option of purchasing both products dominates the outside good, whereas if $p < (1+\xi)/2$, the opposite holds true. The problem of the retailer in the Saturation case under Monopoly (subscript M) is to maximize $\Pi_{R,S}(p) = (p-c)D_{S,M}$ subject to $p \leq (1+\xi)/2$, whereas in the Uncovered market scenario, the objective is to maximize $\Pi_{R,U}(p) = (p-c)D_{U,M}$ such that $p > (1+\xi)/2$. The following lemma characterizes the equilibrium.

Lemma 2. *If* $c \leq \tilde{C}(\xi) \equiv \sqrt{2\xi^2 + 2\xi} - \xi$*, the equilibrium is Saturation:*

$$p_S^* = \frac{1+\xi}{2} \tag{4}$$

$$\Pi_{R,S}^* = \left(\frac{1+\xi}{2} - c\right)(1+\xi) \tag{5}$$

$$Q_{S,M}^* = 1 + \xi \tag{6}$$

If $c > \tilde{C}(\xi)$*, the equilibrium is Uncovered market:*

$$p_U^* = \frac{1+c}{2} \tag{7}$$

$$\Pi_{R,U}^* = \frac{(1-c)^2}{2} \tag{8}$$

$$Q_{U,M}^* = 1 - c \tag{9}$$

Proof. See the Appendix. ■

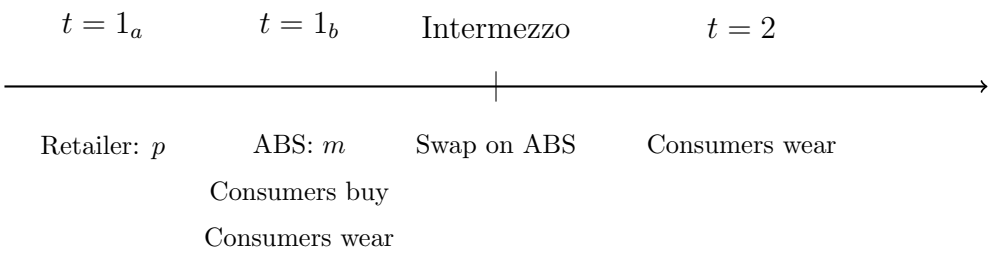
As in the toy version, since $\tilde{C}'(\xi) > 0$ and $Q_{S,M}^* > Q_{U,M}^*$, lower production costs and stronger preference for variety entail a higher chance that a fraction of consumers will buy both items; however, endogenizing the retailer leads to a decision frontier $\tilde{C}(\xi)$ on its pricing policy that is now a strictly concave function that passes from the origin. The latter feature will be the source of a fundamental difference from the perfectly competitive case.

3.2 ABS: Stackelberg Competition

I model competition between the ABS and the traditional vendor à la Stackelberg, and I assume that the retailer has a first-mover advantage. I look for sub-game perfection.

Figure 3 illustrates the timing: again, firms take their pricing decision in period 1; in period 1_a, the Retailer sets p; in period 1_b, the ABS sets m and consumers purchase and wear one

Figure 3: Timeline



good; between periods, the subscribers to the rental service return the piece they wore in period 1 and receive the other; finally, in $t = 2$, consumers wear one of the two pieces. The utility function of consumer θ is:

$$U(\theta) = \begin{cases} \theta - p & \text{if she buys } A \\ 1 - \theta - p & \text{if she buys } B \\ \delta + \xi - m & \text{if she buys } L \\ 1 + \xi - 2p & \text{if she buys } \{A, B\} \\ 0 & \text{if she buys } O \end{cases}$$

Before proceeding with the analysis it is worth recalling from the previous section that the entrance of the ABS always results in aggregate production equal to unity: the environmental benefit of leasing thus depends again on whether entry happens in the region of parameters that features a Saturation equilibrium.

Next, notice that L dominates $\{A, B\}$ and O if and only if $m < \min\{\delta + \xi, 2p + \delta - 1\}$; these constraint imply again that $\tilde{\Theta}_{BL} < \Theta_{BA} < \tilde{\Theta}_{LA}$, where $\tilde{\Theta}_{BL} \equiv 1 + m - \delta - \xi - p$ and $\tilde{\Theta}_{BL} \equiv \delta + \xi + p - m$ are the consumers indifferent between purchasing B or between the subscription and A, respectively. Solving the game by backward induction, in stage 1_b, the ABS problem is

$$\begin{aligned} \max_m \quad & \Pi_{ABS}(m, p) = (m - c)[\tilde{\Theta}_{LA} - \tilde{\Theta}_{BL}] \\ s.t. \quad & m \leq \delta + \xi \qquad (10) \\ & m < 2c + \delta - 1 \qquad (11) \end{aligned}$$

The first-order conditions of the problem yield:

$$m(p) = \frac{\delta + \xi + c + p}{2} - \frac{1}{4} \qquad (12)$$

Using (12) into the constraints (10)-(11), and defining $P_a \equiv 1/2 + (c + \xi - \delta)/3$ and $P_b \equiv$

$1/2 + \delta + \xi - c$, the best response function of the ABS is:

$$R_{ABS}(p) = \begin{cases} +\infty & \text{if } p \leq P_a \\ \frac{\delta+\xi+c+p}{2} - \frac{1}{4} & \text{if } P_a < p \leq P_b \\ \delta + \xi & \text{if } p > P_b \end{cases} \tag{13}$$

Hence, for $p \leq P_a$, entry is either blockaded or deterred depending on the unit production cost; to rule out this set of equilibria, I impose the following restrictions on the model's parameters:

1. *No Blockaded Entry*: $P_a < P_b \iff c < \bar{k}(\xi;\delta) \equiv \delta + \xi/2$.
2. *No Deterrence*: $P_a < c \iff c > \underline{c}(\xi;\delta) \equiv 3/4 + (\xi - \delta)/2$.
3. $\underline{c}(\xi;\delta) < \bar{k}(\xi;\delta) \iff \delta > \underline{\delta} \equiv 1/2$

Thus, (13) boils down to:

$$R_{ABS}(p) = \begin{cases} \frac{\delta+\xi+c+p}{2} - \frac{1}{4} & \text{if } p \leq P_b \\ \delta + \xi & \text{if } p > P_b \end{cases} \tag{14}$$

In stage 1_a, the Retailer maximizes $\Pi_R(p,m) = (p-c)(1 - \tilde{\Theta}_{LA} + \tilde{\Theta}_{BL})$ subject to $m = R_{ABS}(p)$. Proposition 1 characterizes the equilibrium, which involves four unique Subgame Perfect Nash Equilibria (S.P.N.E.): an Interior (I) equilibrium where the ABS and the Retailer choose interior prices; a Corner equilibrium ($C(1)$) where the solution of the ABS problem is a corner fee and the Retailer sets a corner price; a Corner equilibrium ($C(2)$) where the solution of the ABS problem is a corner fee and the Retailer accommodates at an interior price; and a Leapfrogging (L) equilibrium where the Retailer exits the market and the ABS serves the whole industry. The expressions that define the thresholds on the parameters δ and c are provided in the Appendix.

Proposition 1. *There exists three thresholds on δ, labelled $\hat{\delta}(\xi)$, $\delta'(\xi)$ and $\delta''(\xi)$, where $1/2 \equiv \underline{\delta} < \hat{\delta}(\xi) < \delta'(\xi) < \delta''(\xi)$, and $\partial\delta''(\xi)/\partial\xi < 0$, that uniquely identify four regions of parameters that define the S.P.N.E. of the game;*

1. *If $\underline{\delta} < \delta < \hat{\delta}(\xi)$*

   (i) Whenever $\underline{c}(\xi;\delta) < c < \bar{k}(\delta;\xi)$, the S.P.N.E. is $C(2)$:

$$p^*_{C(2)} = \frac{1+c}{2} \qquad \Pi^*_{R,C(2)} = \frac{(1-c)^2}{2} \tag{15}$$

$$m^*_{C(2)} = \delta + \xi \qquad \Pi^*_{ABS,C(2)} = c(\delta + \xi - c) \tag{16}$$

2. *If $\hat{\delta}(\xi) < \delta < \delta'(\xi)$:*

(ii) If $\underline{c}(\xi;\delta) < c < c_b(\xi;\delta) < \bar{k}(\xi;\delta)$*, the S.P.N.E. is* $C(1)$*:*

$$p^*_{C(1)} = P_b \qquad \Pi^*_{R,C(1)} = \frac{[1+2(\delta+\xi)-4c][1-2(\delta+\xi)+2c]}{2} \tag{17}$$

$$m^*_{C(1)} = \delta+\xi \qquad \Pi^*_{ABS,C(1)} = 2(\delta+\xi-c)^2 \tag{18}$$

(iii) If $c_b(\xi,\delta) < c < \bar{k}(\xi;\delta)$*,* $C(2)$ *is the unique S.P.N.E.*

3. If $\delta'(\xi) < \delta < \delta''(\xi)$*:*

(iv) If $\underline{c}(\xi;\delta) < c < c_a(\xi;\delta)$*, the S.P.N.E. is I:*

$$p^*_I = c + \frac{3}{4} - \frac{\delta+\xi}{2} \qquad \Pi^*_{R,I} = \frac{[3-2(\delta+\xi)]^2}{16} \tag{19}$$

$$m^*_I = c + \frac{1}{8} + \frac{\delta+\xi}{4} \qquad \Pi^*_{ABS,I} = \frac{[1+2(\delta+\xi)]^2}{32} \tag{20}$$

(v) If $c_a(\xi;\delta) < c < c_b(\xi;\delta)$*, the S.P.N.E. is* $C(1)$*.*

(vi) If $c_b(\xi;\delta) < c < \bar{k}(\xi;\delta)$*, the S.P.N.E. is* $C(2)$*.*

4. If $\delta > \delta''(\xi)$*:*

(a) Whenever $\underline{c}(\xi;\delta) < c < \bar{k}(\delta;\xi)$*, the S.P.N.E. is L:*

$$p^*_L = +\infty \qquad \Pi^*_{R,L} = 0 \tag{21}$$

$$m^*_L = \delta+\xi \qquad \Pi^*_{ABS,L} = \delta+\xi-c \tag{22}$$

Proof. See the Appendix. ■

Proposition 1 highlights that if consumers have a sufficiently positive perception of leasing, the *total* cost advantage of the ABS in supplying both goods allows it to offer a deal that is so enticing that every consumer prefers to subscribe rather than buy from the Retailer, which would be forced to charge below marginal cost pricing to gain some market share. Since I assumed that renting and owning clothes grants the same variety benefit, higher ξ lowers the threshold necessary for the market to tip: the ABS is indeed more efficient in serving the buyers that benefit the most from variety, and hence a strong preference for a diversified wardrobe combined with a mild discount for the inconvenience of not owning the pieces gives the rentailer an insurmountable comparative advantage. Notice the first-order effect that the complementarity premium plays on the existence of a leapfrogging equilibrium: the threshold $\delta''(\xi)$ exists if and only if $\xi > 1/2$. The equilibria (15)-(20) are more traditional in interpretative terms: if costs are high enough to support entry but below a given level, firms set interior prices; as the unit costs grow, the ABS and the Retailer first charge each a corner price; as costs increase even further, given the total cost advantage of the ABS in serving the consumers closer to the median of the distribution, the Retailer decides to soften

competition and raise prices.

The characterization of the equilibrium establishes an interesting relationship between the rental acceptance rate, δ, and the environmental credentials of the ABS.

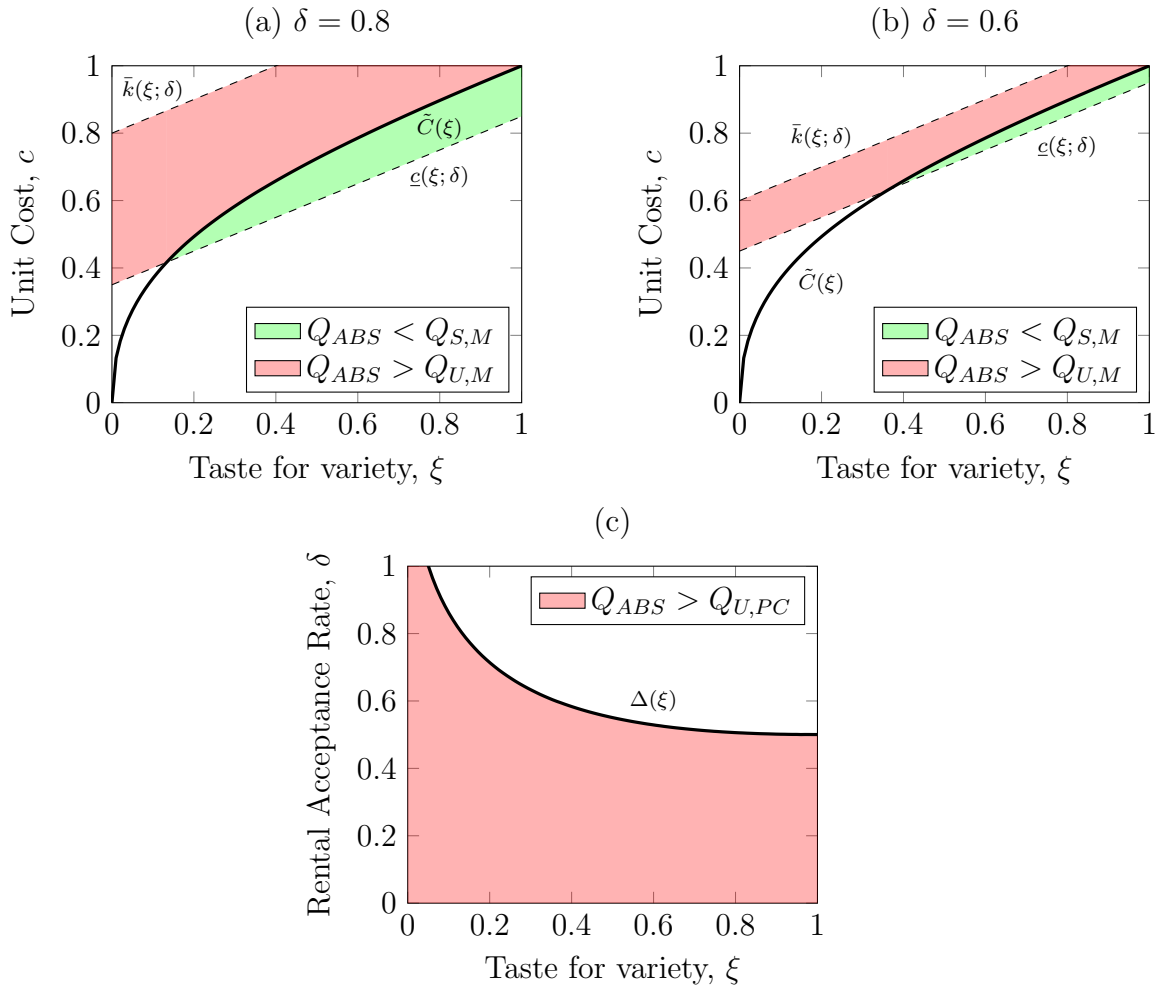


Figure 4: Sustainability of the ABS with a monopoly retailer.

Proposition 2. *(i) There exists a threshold on δ, called $\Delta(\xi)$, with $\Delta'(\xi), < 0$ such that if the ABS enters the market and $\delta < \Delta(\xi)$, then entrance necessary produces a market expansion effect, $Q_{ABS} > Q_{U,PC}$. Conversely, (ii) there exists no threshold on δ, $\hat{\Delta}$, such that if $\delta > \hat{\Delta}$ or $\delta < \hat{\Delta}$, the entrance of the ABS reduces necessarily aggregate production. Finally, (iii) if entry is possible and $\delta > \Delta(\xi)$, the total demand for clothes shrinks if and only if $c < \tilde{C}(\xi)$.*

Proof. As in the toy version, if $c > \tilde{C}(\xi)$, $Q_{ABS} > Q_{U,M}$. Thus if $\tilde{C}(\xi) > \underline{c}(\xi;\delta)$, which is the case if $\delta < \Delta(\xi) \equiv 3/2 + 3\xi - 2\sqrt{2\xi^2 + 2\xi}$ (notice that $\underline{\delta} < \Delta(\xi) < 1$ and that $\Delta'(\xi) < 0$), then entrance unavoidably results in more pieces being produced. This proves point (i). Additionally, we have that $\tilde{C}(\xi) > \hat{k}(\xi;\delta) \iff \delta < \underline{\delta}$, which implies that whenever entry is possible, $\hat{k}(\xi;\delta) > \tilde{C}(\xi)$: hence, there exists no threshold on δ such that entry can happen

only in the region $c < \tilde{C}(\xi)$, namely when the ABS would have a desirable impact on the environment, which proves point (ii). Point (iii) follows naturally. ■

Proposition 2 has important policy implications: for product categories where consumers display a strong distaste for renting compared to purchasing, business models that allow leasing through subscription plans should be viewed sceptically from an environmental standpoint, since for low δ the difference in terms of surplus between purchasing both items or renting them is so high that most likely, if entry happened, it did because the ABS provided a good deal to a fraction of buyers that were not purchasing ex-ante. The result is a consequence of the endogenization of the retailer: in perfect competition, the decision on whether this middle ground will be served or not is ultimately on consumers. For a weak preference for variety, a monopolist might have no incentive to target also the consumers close to the median of the distribution, even for fairly low costs. Thus if rental is perceived poorly, and if costs are high enough so that neither deterrence nor blockaded entry are possible, it must be that the rentailer is targeting also a fraction of the market that was previously unserved. This effect is stronger the lower the taste of variety, as consumers are more likely to buy only one product in the baseline. Figure 4 represents graphically the sustainability for different values of δ (panel (a) and (b)) and the evolution of the threshold $\Delta(\xi)$ as the taste for variety increases (panel (c)). In panel (a) and (b), the intersection between $\tilde{C}(\xi;\delta)$ and $\bar{c}(\xi;\delta)$ characterizes the minimal taste for variety below which entry occurs with certainty in the Uncovered market case; panel (b) shows that for a lower level of δ, this point moves rightward, namely a much higher love for diversification is needed to support the green ambitions of the ABS. The red area in panel (c) depicts every (ξ,δ) pair were the entry of the rentailer results in a greater production volume.

4 Literature

A conspicuous body of research in the durable good stream is devoted to the choice between leasing and selling; see Waldman (2003) for a thorough review of early works. These articles have dealt with several issues: a non-extensive list includes the interplay between leasing and secondary markets (Waldman 1997, Hendel and Lizzeri, 1999, Huang et al., 2001), market segmentation (Desai and Purohit, 1998, 1999), adverse selection and moral hazard (Hendel and Lizzeri, 2002, Johnson and Waldman, 2003, 2010), business model choice with complementary products (Bhaskaran and Gilbert, 2005) and conversion sales (Jalili and Pangburn, 2020). This paper provides additional insights into the role of leasing in a spatial model with horizontal differentiation: differently from these works, I characterize a novel total cost advantage of rental-based subscriptions that stems precisely from the horizontally differentiated nature of the market.

Another bulk of recent papers treats the sharing economy: Einav et al. (2016) offer

an excellent overview. Here, I only discuss analytical works for brevity[10]. Benjafaar et al. (2019) formalize a collaborative consumption model where owners and renters with heterogeneous preferences of usage are matched asymmetrically on a sharing platform. They show that when the rental price is sufficiently high, product sharing might be conducive to higher ownership and usage rates, as the prospective income from the platform incentivizes more users to buy the good when the cost of ownership is high. In a similar spirit, Fraiberger and Sundararajan (2017) develop a dynamic model that features a sharing and a second-hand market. A few works have studied the relationship between the supply chain and product sharing (Jiang and Tian, 2018, Tian and Jiang 2018, Abishek et al., 2021). These papers highlight a complementarity between the sharing economy and the profitability of the supply chain. Filippas et al. (2020) define the preferences of consumers as a bell-shaped function of the amount of time they use a good; they define the usage supplied as the fraction of time that exceeds optimal usage. They argue that consumers are more likely to own the product the lower the predictability and portion of the time required for one use. These studies define product sharing as a leasing agreement between an owner and a renter. This paper complements this literature by formalizing a setup where products are shared among renters: in my model consumers can indirectly share items through the rentailer because they do not agree on their gross surplus for the products and because clothes are bundled.

Lastly, this chapter is related to the literature on sustainable supply chains. Following suit of Agrawal et al. (2012), other works deal with the environmental impact of a pure leasing vis-à-vis a pure selling business model considering the product architecture choice (Agrawal et al., 2021) and the RTR legislation (Gülserlier et al., 2022). This chapter contributes to this literature in two different aspects: first, it analyzes a setting where selling and leasing in the product market coexist, and second, it abstracts away from wear-and-tear to represent the underlying mechanism behind the environmental benefit of an ABS.

5 Conclusion

Several supporters of the circular economy identify business models that offer clothing subscription plans as viable solutions to abate the production of garments. In this article, I provided an economic rationale for the latter environmental benefit: an ABSP can indirectly enable trade of horizontally differentiated products between users, and hence provide more outfits with a lower amount of units. Nonetheless, policymakers and sustainability advocates should also take into account that the latter argument might fail, in the presence of market expansion effects: I proved that the latter are generally more likely to arise if consumers have a weak preference for variety and if the unit cost of the product is quite high, as there is the chance that the ABSP simply conquers a share of the market that was not previously served. I conjecture that fashion buyers should display a milder taste for diversity for garments that

[10]Examples of empirical works are Zervas et al., 2017, Fradkin, 2017 and Cullen and Farronato, 2020.

are seasonal: this implies that rental-based subscriptions might be effective in curbing the production of seasonless items that have a relatively low manufacturing cost such as T-shirts or tops. However, the argument might not hold for coats and jumpers. Endogenizing the retail side, I also showed that a sufficiently bad perception of leasing can imperil the green credentials of the ABS, especially if combined with a mild preference for variety. Intuitively, a monopoly retailer is willing to segment the market and leave a fraction of consumers unserved, if these do not value enough the option of purchasing multiple products; if this is the case and if rental is considered much inferior to owning the good in gross surplus terms, then if an ABSP can enter the market, it can only do so by capturing some users that were not buying previously. Empirical evidence on the extent to which consumers are willing to rent products rather than own them exists: for example, Lavigne and Rousseau (2022), in a survey on the acceptability of product-service-systems in Belgium, document that casual clothing is the category perceived the least favourably, whereas children's apparel performs substantially better. The result of this paper suggests that businesses that lease babywear and childhood garments are thus more likely to be sustainable than those that supply everyday outfits.

An interesting extension would involve a comparison between product-sharing and rental through subscription. I leave this and other questions for future research.

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Appendix

Proof of Lemma 2. If $p \leq (1+\xi)/2$, the consumer $\tilde{\Theta}_{B,\{A,B\}} \equiv p - \xi$ is indifferent between buying B or both goods, the consumer $\Theta_{BA} \equiv 1/2$ is indifferent between buying B and A, and the consumer $\tilde{\Theta}_{\{A,B\},A} \equiv 1 + \xi - p$ is indifferent between buying both goods or A. The constraint $p \leq (1+\xi)/2$ implies that $\tilde{\Theta}_{B,\{A,B\}} \leq \Theta_{BA} \leq \tilde{\Theta}_{\{A,B\},A}$. The Retailer maximizes $\Pi_{R,S} = (p-c)(1 - \tilde{\Theta}_{B,\{A,B\}} + \tilde{\Theta}_{\{A,B\},A})$ subject to $p \leq (1+\xi)/2$; the equilibrium candidates are thus equations (4)-(6), under the necessary condition $\Pi^*_{R,S} > 0 \iff c < (1+\xi)/2$. If $p > (1+\xi)/2$, the consumer $\tilde{\Theta}_{BO} \equiv 1 - p$ is indifferent between buying B or the outside option, and the consumer $\tilde{\Theta}_{OA} \equiv p$ is indifferent between buying the outside good or A. Again $p > (1+\xi)/2 \implies \tilde{\Theta}_{BO} < \Theta_{BA} < \tilde{\Theta}_{OA}$. Thus the Retailer solves $\max_p \Pi_{R,U} = (p-c)(1 - \tilde{\Theta}_{OA} + \tilde{\Theta}_{BO})$ such that $p > (1+\xi)/2$, which yields the candidates (7)-(9) under the necessary condition $p^*_U > (1+\xi)/2 \iff c > \xi$. Thus if $c < \xi < 1$, the equilibrium is necessarily Saturation, whereas if $c > (1+\xi)/2$ the equilibrium is Uncovered market; the first bit of this claim supports the necessity of the assumption $\xi < c < 1$. Notice that $\Pi_{R,S} \leq \Pi_{R,U} \iff c \leq \tilde{C}(\xi) \equiv \sqrt{2\xi^2 + 2\xi} - \xi$. Given the assumption $\xi < c < 1$, it holds that $\xi < \tilde{C}(\xi) < (1+\xi)/2$, thus if $c \leq \tilde{C}(\xi)$, the equilibrium characterized by equations (4)-(6), whereas if $c > \tilde{C}(\xi)$, the equilibrium is characterized by equations (7)-(9). ■

Proof of Proposition 1. The problem of the retailer can be rewritten $\max\{\Pi^*_{R,I}, \Pi^*_{R,C}, 0\}$, where $\Pi^*_{R,I}$ is the value function of the problem $\max_p \Pi_{R,I} = (p-c)(1 - \tilde{\Theta}_{LA} + \tilde{\Theta}_{BL})$ subject to $p \leq P_b$ and $m = (\delta + \xi + c + p)/2 - 1/4$ and $\Pi^*_{R,C}$ is the value function of the problem $\max_p \Pi_{R,C} = (p-c)(1 - \tilde{\Theta}_{LA} + \tilde{\Theta}_{BL})$ subject to $p > P_b$ and $m = \delta + \xi$. The problems yield the equilibrium candidates in (19)-(20) and (15)-(16), respectively. From the first problem, $p^*_I \leq P_b \iff c \leq c_a(\xi;\delta) \equiv 3(\delta+\xi)/4 - 1/8$, and $c_a(\xi;\delta) > \underline{c}(\xi;\delta) \iff \delta > \delta'(\xi) \equiv 7/10 - \xi/5 > \underline{\delta}$. Moreover, since $\delta > \underline{\delta}$, we have that $c_a(\xi;\delta) < \bar{k}(\xi;\delta)$. Thus, if $\delta > \delta'(\xi)$, then $\underline{c}(\xi;\delta) < c_a(\xi;\delta) < \bar{k}(\xi;\delta)$, otherwise $c_a(\xi;\delta) < \underline{c}(\xi;\delta)$. From the second problem, $p^*_{C(2)} > P_b \iff c > c_b(\xi;\delta) \equiv 2(\delta+\xi)/3$, and $c_b(\xi;\delta) > \underline{c}(\xi;\delta) \iff \delta > \hat{\delta}(\xi) \equiv 9/14 - \xi/7$, and $c_b(\xi;\delta) > c_a(\xi;\delta) \iff \delta < \delta''(\xi) \equiv 3/2 - \xi$. The assumptions $\xi < 1$ and $\delta > \underline{\delta}$ imply that $\delta''(\xi) > \delta'(\xi) > \hat{\delta}(\xi) > \underline{\delta}$. Thus if $\delta \in [\underline{\delta}, \hat{\delta}(\xi)]$, then $c_a(\xi;\delta) < c_b(\xi;\delta) < \underline{c}(\xi;\delta) < \bar{k}(\xi;\delta)$, if $\delta \in [\hat{\delta}(\xi), \delta'(\xi)]$, then $c_a(\xi;\delta) < \underline{c}(\xi;\delta) < c_b(\xi;\delta) < \bar{k}(\xi;\delta)$, if $\delta \in [\delta'(\xi), \delta''(\xi)]$, then $\underline{c}(\xi;\delta) < c_a(\xi;\delta) < c_b(\xi;\delta) < \bar{k}(\xi;\delta)$, whereas if $\delta \in [\delta''(\xi), 1]$, then $\underline{c}(\xi;\delta) < c_b(\xi;\delta) < c_a(\xi;\delta) < \bar{k}(\xi;\delta)$. Notice from (19) that if $\delta > \delta''(\xi)$, then $\Pi^*_{R,I} < 0$. Moreover,

$$\Pi^*_{R,C(2)}\Big|_{c=c_b(\xi;\delta)} > 0 \iff \delta < \delta''(\xi)$$

It follows that if $\delta > \delta''(\xi)$, neither I nor $C(2)$ can be subgame perfect equilibria: given this result, we start by analyzing cases $\delta < \delta''(\xi)$, which implies $c_a(\xi;\delta) < c_b(\xi;\delta)$, and then derive the S.P.N.E. for the case $\delta > \delta''(\xi)$. Given $\delta < \delta''(\xi)$, we know that if $\underline{c}(\xi;\delta) < c < c_a(\xi;\delta)$, then I is the unique S.P.N.E. , whereas if $c_b(\xi;\delta) < c < \bar{k}(\xi;\delta)$, then $C(2)$ is the unique

S.P.N.E.; next, consider the case $c_a(\xi;\delta) < c < c_b(\xi;\delta)$. Within this interval, it holds that:

$$\left.\frac{\partial \Pi_{R,I}}{\partial p}\right|_{p=P_b} > 0$$
$$\left.\frac{\partial \Pi_{R,C}}{\partial p}\right|_{p=P_b} < 0$$

Hence, consider the candidate equilibrium $C(1)$, (17)-(18): in this scenario, the margin of the Retailer is positive if and only if $c > \delta + \xi - 1/2 \equiv c'$. On the other hand, the demand of the Retailer is above zero if and only if $c < (\delta + \xi)/2 + 1/4 \equiv c''$. For $\delta < \delta''(\xi)$, it holds that $c' < c_a(\xi;\delta) < c_b(\xi;\delta) < c''$, which implies that $C(1)$ is the unique S.P.N.E. . If $\delta > \delta''(\xi)$, either the demand of the Retailer or its mark-up is negative, thus the unique S.P.N.E. is L, (21)-(22). ■