

$$1. \quad m_1(\mu) = \text{Normal}(\mu \mid 0, 10^2)$$

$$3. \quad m_3(\mu) = m_1(\mu) = \text{Normal}(\mu \mid 0, 10^2)$$

$$10. \quad m_{10}(\sigma^2) = \text{Inverse-Gamma}(\sigma^2 \mid 1, 1)$$

$$8. \quad m_8(\sigma^2) = m_{10}(\sigma^2) = \text{Inverse-Gamma}(\sigma^2 \mid 1, 1)$$

$$6. \quad m_6(x) = 1$$

$$\begin{aligned} 5. \quad m_5(x) &= \int_0^\infty \int_{-\infty}^\infty \text{factor} \times m_3(\mu) m_8(\sigma^2) d\mu d\sigma^2 \\ &= \int_0^\infty \int_{-\infty}^\infty \text{Normal}(x \mid \mu, \sigma^2) \text{Normal}(\mu \mid 0, 10^2) \text{Inverse-Gamma}(\sigma^2 \mid 1, 1) d\mu d\sigma^2 \\ &= \int_0^\infty \text{Normal}(x \mid 0, \sigma^2 + 10^2) \text{Inverse-Gamma}(\sigma^2 \mid 1, 1) d\sigma^2 \end{aligned}$$

(Can't perform integral.)

$$\begin{aligned} 4. \quad m_4(\mu) &= \int_0^\infty \int_{-\infty}^\infty \text{factor} \times m_6(x) m_8(\sigma^2) dx d\sigma^2 \\ &= \int_0^\infty \int_{-\infty}^\infty \text{Normal}(x \mid \mu, \sigma^2) \text{Inverse-Gamma}(\sigma^2 \mid 1, 1) dx d\sigma^2 \\ &= 1 \end{aligned}$$

$$2. \quad m_2(\mu) = m_4(\mu) = 1$$

$$\begin{aligned} 7. \quad m_7(\sigma^2) &= \int_{-\infty}^\infty \int_{-\infty}^\infty \text{factor} \times m_6(x) m_3(\mu) dx d\mu \\ &= \int_{-\infty}^\infty \int_{-\infty}^\infty \text{Normal}(x \mid \mu, \sigma^2) \text{Normal}(\mu \mid 0, 10^2) dx d\mu \\ &= 1 \end{aligned}$$

$$9. \quad m_9(\sigma^2) = m_7(\sigma^2) = 1$$

