Implement models in Stan

Call center data set — exponential likelihood with gamma prior

In [1]:
```
# import libraries
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import pystan

# load call center data set
waiting_times_day = np.loadtxt('call_center.csv')
```

In [2]:
```
# data pre-processing
current_time = 0
waiting_times_per_hour = [[] for _ in range(24)]  # Make 24 empty lists,
 one per hour
for t in waiting_times_day:
    current_hour = int(current_time // 60)
    current_time += t
    waiting_times_per_hour[current_hour].append(t)

# get data for 11th hour
waiting_times_hour = waiting_times_per_hour[11]
print("{:d} call during 11th hour.".format(len(waiting_times_hour)))
```

```
892 call during 11th hour.
```

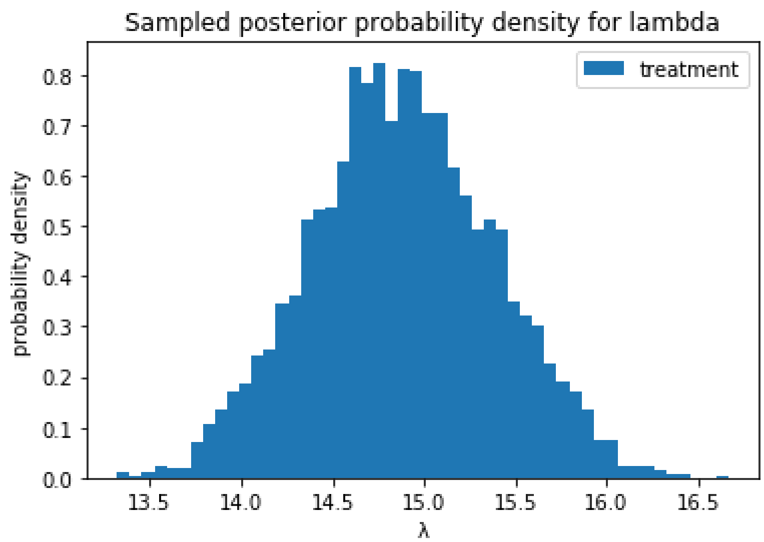
In [3]:
```
stan_code = """

data {
    real<lower=0> alpha;   // prior hyperparameters for gamma
    real<lower=0> beta;   // prior hyperparameters for gamma
    int<lower=0> n;   // number of call during the hour
    vector[n] waiting_times_hour;   // call waiting times
}

parameters {
    real lambda;  // the probability of call
}

model {
    lambda ~ gamma(alpha, beta);
    waiting_times_hour ~ exponential(lambda);
}

"""

exponential_gamma_stan_model = pystan.StanModel(model_code=stan_code)
```

```
INFO:pystan:COMPILING THE C++ CODE FOR MODEL anon_model_c0ad0931853126a
0e6315f43bcb9f559 NOW.
/usr/local/lib/python3.7/site-packages/Cython/Compiler/Main.py:367: Fut
ureWarning: Cython directive 'language_level' not set, using 2 for now
(Py2). This will change in a later release! File: /var/folders/k8/dkych
j2n5c98xy85t13b1md40000gn/T/tmp9iwjqu7c/stanfit4anon_model_c0ad09318531
26a0e6315f43bcb9f559_7142811583325279888.pyx
  tree = Parsing.p_module(s, pxd, full_module_name)
```

In [4]:
```
stan_data = {
    "alpha": 1,
    "beta": 0.25,
    "n": len(waiting_times_hour),
    "waiting_times_hour": waiting_times_hour,
}

results = exponential_gamma_stan_model.sampling(data=stan_data)
print(results)
```

```
Inference for Stan model: anon_model_c0ad0931853126a0e6315f43bcb9f559.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

         mean se_mean     sd   2.5%    25%    50%    75%  97.5%  n_eff
  Rhat
lambda  14.88    0.01    0.5  13.92  14.55  14.88  15.22  15.86   1216
   1.0
lp__   1513.7    0.01   0.68 1511.8 1513.5 1514.0 1514.1 1514.2   2047
   1.0

Samples were drawn using NUTS at Fri Oct 19 21:22:31 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

In [5]:
```
samples = results.extract()

# plot posterior histograms
plt.hist(samples['lambda'], bins=50, density=True, label='treatment')
plt.title('Sampled posterior probability density for lambda')
plt.xlabel('λ')
plt.ylabel('probability density')
plt.legend()
plt.show()

# posterior 95% confidence interval
print('Posterior 95% interval over lambda:', np.percentile(samples['lamb
da'], [2.5, 97.5]))
```




```
Posterior 95% interval over lambda: [13.91888387 15.86209869]
```

Normal likelihood with normal-inverse-gamma prior

In [6]:
```python
# import libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy import stats

# load data
data = np.array([3.54551763569501, 4.23799861761927, 4.72138425951628, -
0.692265320368236, 3.04473513808788, 3.10721270732507, 3.42982225852764,
 3.12153903971176, 3.60532628639808, 2.46561737557325, 1.64059465916131,
 2.4621623937158, 2.76744495617481, 2.11580054750407, 5.14077208608354,
4.90288499104252, 1.43357579078348, 4.78997817363558, 1.93633438207439,
2.43698838097178, 3.95389148701877, 2.4242295507716, 2.90256268679023,
2.90931728045901, 0.658072819386888, 3.05946763895983, 3.42615331539605,
 2.68842833004417, 2.35850130765166, 2.20014998540933, 4.73846511350084,
 4.19839721414451, 2.11805510171691, -0.572742936038015, 0.3894139820106
23, 3.87846130744249, 1.34057656890858, 0.7235748351719, 5.1104236984017
4, 4.00747556696571, 3.18080956726965, 3.24677964069676, 5.1154659863626
, 1.80276616697155, 0.305877679021404, -0.449168307882718, 4.63705561194
774, 1.37783714058301, 4.9608149859515, 6.7764195802069, 1.7551552292239
9, 7.04457337435215, 0.625185284955128, 2.25130734369064, 2.197701781192
55, 2.16858257249432, 6.25367644481438, 0.116081323476489, 2.06315857864
341, 1.82409781471718, 5.15226741230987, 2.03408231293173, -1.1245085433
7596, 5.03511270642234, 2.03841989653263, 5.80911741751597, 2.3171812878
3245, 4.97575010580997, 3.34262752222776, -0.786983904253601, 0.77736235
9850013, 0.975825009321195, 3.76354577515958, 7.27215002907876, 1.354040
89480189, 3.76567940257157, 3.48573993343334, 1.85976988586156, 1.935670
61960716, 5.31071812003942, 2.96832987672751, 3.32378908637275, 2.616319
60054551, 5.80897964052825, 4.95215217171488, 1.32036772796131, 3.799325
42233371, 3.08108492766309, 2.6734110081666, -0.14251851138521, 2.487443
75588965, 3.98463042123415, 6.32781680028, 4.0029172024315, 4.2321036945
9457, 1.71412938967325, 5.16492114963802, 2.53409673107906, 4.7734696397
3334, 3.34088878725551, 4.77681472750664, 3.81135755590976, 1.1405426998
3137, 1.42057452397702, 0.132142311125433, 7.12577254064672, 4.854220127
81764, 4.15745720676399, 4.48763147363348, 1.56060322283629, 2.648217615
42887, 1.26655351354548, 4.48497722937931, 4.3286302403783, 4.2615767951
2625, 4.0597558651364, 5.14051109132496, 2.5660348362221, 1.107640138186
17, 0.386889523012303, 3.54150473246237, 3.57480214382351, 1.95150869584
847, 2.70688970563118, 2.47971849820016, 6.50838037000679, 4.01511556826
974, 1.11562740835344, 5.02637639472439, 4.38184491686864, 5.60423144047
386, 2.40067408379298, 5.7849941378344, 2.37225791084559, 6.860314659102
73, 4.09214858239736, 6.85994063692621, 3.62202415158781, -1.11220646958
158, 3.73920971696866, 3.24533871512216, 1.28724203643002, 0.29115254177
3164, 0.368630935755111, 6.71607270510525, 5.42278455200833, 5.351884161
19281, 2.305874586163, -1.85878097203032, 2.69877382351447, 4.8412186055
0417, 4.40973060799391, 5.04399320650774, 2.68632252661298, 6.0653161065
9912, 3.11881325011993, 3.45532087005125, 3.08442259840346, 4.4356442413
6733, 2.84252623135804, 1.50536798885106, 1.48868622407603, 2.0732283761
5663, 2.5476910210998, 5.66941808257884, 2.16731067416426, 2.49843958833
905, 3.94586413879977, 0.316433764679541, -0.608937441815983, 2.59434365
58557, 1.05516869528337, 2.1447601332725, 6.65846634141906, 2.1771555267
834, 5.23953812029442, 3.53629759842647, 6.03263538017003, 3.85739159396
599, 5.95093453004638, 1.12856987160476, 3.5559912886093, 2.219748642444
89, 3.38471394882135, -1.90805399279409, 3.5113699258973, 4.493199554123
46, 5.10507952638867, 1.08277895384184, 4.58403638422759, 1.373049944268
24, 4.17566975753523, 3.36454182510378, 0.177136582644021, 2.91337423388
405, 3.22796455457526, 2.80124198378441, 1.95189718582788, 3.37659263896
```

```
246, -1.6463045238231])
print(len(data), "data")
```

```
200 data
```

In [7]:

```
stan_code = """

data {
    real<lower=0> mu;
    real<lower=0> nu;
    real<lower=0> alpha;   // prior hyperparameters for gamma
    real<lower=0> beta;   // prior hyperparameters for gamma
    int<lower=0> n;   // length of data
    vector[n] data_set;   // data
}

parameters {
    real sigma2;
    real x;
}

model {
    sigma2 ~ inv_gamma(alpha, beta);
    x ~ normal(mu, sigma2/nu);
    data_set ~ normal(x, sqrt(sigma2));
}

"""

normal_inverse_gamma_stan_model = pystan.StanModel(model_code=stan_code)
```

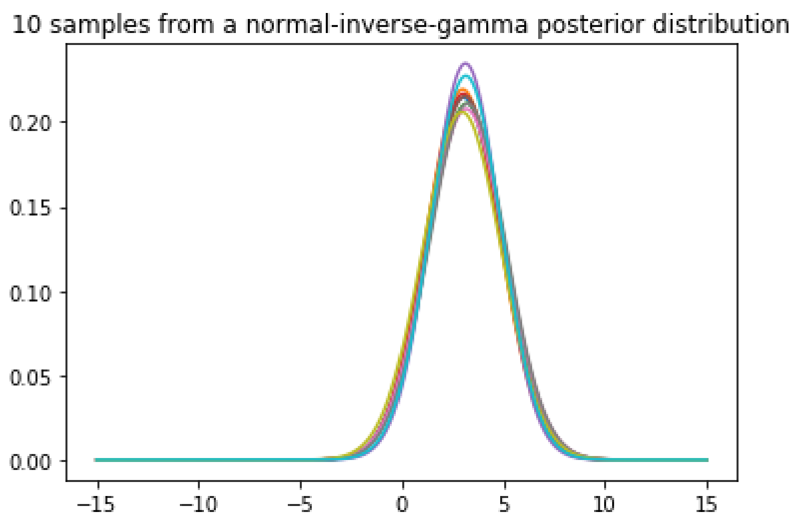
```
INFO:pystan:COMPILING THE C++ CODE FOR MODEL anon_model_c6f7bd945788c72
a51d3425ce712ba75 NOW.
/usr/local/lib/python3.7/site-packages/Cython/Compiler/Main.py:367: Fut
ureWarning: Cython directive 'language_level' not set, using 2 for now
(Py2). This will change in a later release! File: /var/folders/k8/dkych
j2n5c98xy85t13b1md40000gn/T/tmphtfut0kz/stanfit4anon_model_c6f7bd945788
c72a51d3425ce712ba75_6437476439639438699.pyx
  tree = Parsing.p_module(s, pxd, full_module_name)
```

In [8]:
```
stan_data = {
    "mu": 0,
    "nu": 0.054,
    "alpha": 1.12,
    "beta": 0.4,
    "n": len(data),
    "data_set": data,
}

results = normal_inverse_gamma_stan_model.sampling(data=stan_data)
print(results)
```

```
Inference for Stan model: anon_model_c6f7bd945788c72a51d3425ce712ba75.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

         mean se_mean     sd   2.5%    25%    50%    75%  97.5%  n_eff
  Rhat
sigma2   3.59  6.3e-3   0.36   2.95   3.34   3.57   3.81   4.37   3240
   1.0
x        3.07  2.2e-3   0.13   2.81   2.98   3.07   3.15   3.33   3499
   1.0
lp__   -236.5    0.02   0.99 -239.1 -236.9 -236.1 -235.7 -235.5   1669
   1.0

Samples were drawn using NUTS at Fri Oct 19 21:23:06 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

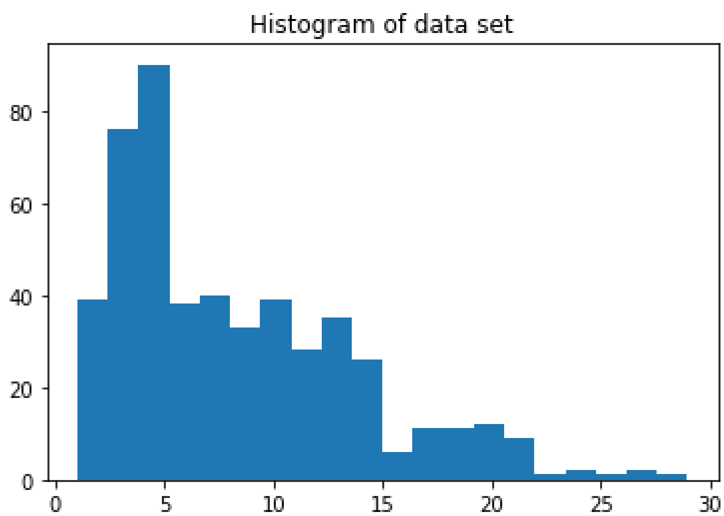
In [9]:
```
samples = results.extract()

# posterior 95% confidence interval
print('Posterior 95% interval over x:', np.percentile(samples['x'], [2.5
, 97.5]))
print('Posterior 95% interval over sigma2:', np.percentile(samples['sigm
a2'], [2.5, 97.5]))

# Plot the normal distributions corresponding to the samples
num_samples = 10
plot_x = np.linspace(-15, 15, 500)
for i in range(num_samples):
    plot_y = stats.norm.pdf(plot_x, loc=np.random.choice(samples['x'], 1
), scale=np.sqrt(np.random.choice(samples['sigma2'], 1)))
    plt.plot(plot_x, plot_y)
plt.title("{:d} samples from a normal-inverse-gamma posterior distributi
on".format(num_samples))
plt.show()
```

```
Posterior 95% interval over x: [2.80565101 3.32521065]
Posterior 95% interval over sigma2: [2.95328909 4.36981581]
```

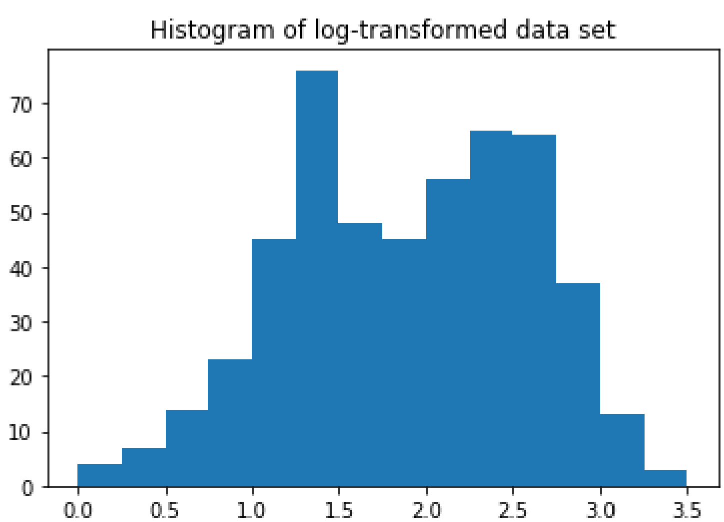


Log-normal HRTEM data

In [10]:
```
# import libraries
import numpy as np
import matplotlib.pyplot as plt
from scipy import stats

# load data
data = np.loadtxt('hrtem.csv')
plt.hist(data, bins=20)
plt.title('Histogram of data set')
plt.show()
print('%i data, min: %f, max: %f' % (len(data), min(data), max(data)))

# convert data into log data
log_data = np.log(data)
plt.hist(log_data, bins=np.linspace(0, 3.5, 15))
plt.title('Histogram of log-transformed data set')
plt.show()
print('%i data, min: %f, max: %f' % (len(log_data), min(log_data), max(l
og_data)))
```




```
500 data, min: 1.051827, max: 28.942578
```




```
500 data, min: 0.050529, max: 3.365314
```

In [11]:
```
stan_data = {
    "mu": 2.3,
    "nu": 0.1,
    "alpha": 2,
    "beta": 5,
    "n": len(log_data),
    "data_set": log_data,
}

results = normal_inverse_gamma_stan_model.sampling(data=stan_data)
print(results)
```

```
Inference for Stan model: anon_model_c6f7bd945788c72a51d3425ce712ba75.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.

         mean se_mean     sd   2.5%    25%    50%    75%  97.5%  n_eff
  Rhat
sigma2   0.49  5.2e-4   0.03   0.44   0.47   0.49   0.52   0.56   3708
   1.0
x        1.89  4.8e-4   0.03   1.83   1.87   1.89   1.91   1.96   4267
   1.0
lp__   -76.11    0.02   1.01 -78.76  -76.5  -75.8 -75.38 -75.12   1765
   1.0

Samples were drawn using NUTS at Fri Oct 19 21:23:07 2018.
For each parameter, n_eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor on split chains (at
convergence, Rhat=1).
```

In [12]:
```
samples = results.extract()

# posterior 95% confidence interval
print('Posterior 95% interval over x:', np.percentile(samples['x'], [2.5
, 97.5]))
print('Posterior 95% interval over sigma2:', np.percentile(samples['sigm
a2'], [2.5, 97.5]))

# Plot the normal distributions corresponding to the samples
num_samples = 10
plot_x = np.linspace(0, 6, 500)
for i in range(num_samples):
    plot_y = stats.norm.pdf(plot_x, loc=np.random.choice(samples['x'], 1
), scale=np.sqrt(np.random.choice(samples['sigma2'], 1)))
    plt.plot(plot_x, plot_y)
plt.title("{:d} samples from a log-data normal-inverse-gamma posterior d
istribution".format(num_samples))
plt.show()
```

```
Posterior 95% interval over x: [1.83014705 1.95525897]
Posterior 95% interval over sigma2: [0.43690563 0.56046326]
```