CEE 6300 – Homework 1

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1 Problem 1

1.1

1.1.1 a

Figure 1

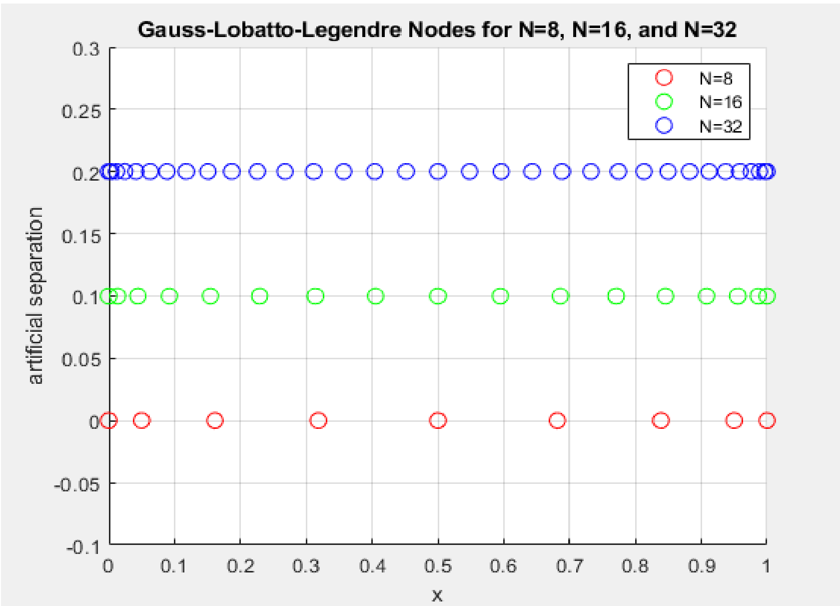

Figure 1: G.L.L. grid nodes

1.1.2 b

The mathematical expression of GLL is the roots of the derivative of a Legendre polynomial, which is naturally biased toward the endpoint. This bias helps us better capture boundary behavior and reduce interpolation errors across the domain since for fixed boundary conditions, it's harder to capture behaviors near the boundary compared to the center.

1.2

1.2.1 a

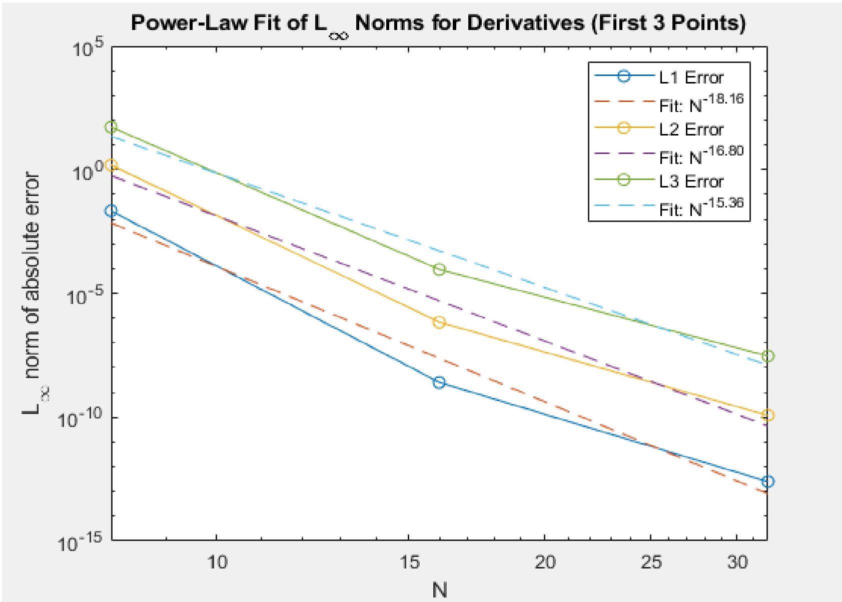

Figure 2: L = 1

The power law exponent for each derivative is shown above in the graph. The error is a monotonic decrease for the first three N.

1.2.2 b

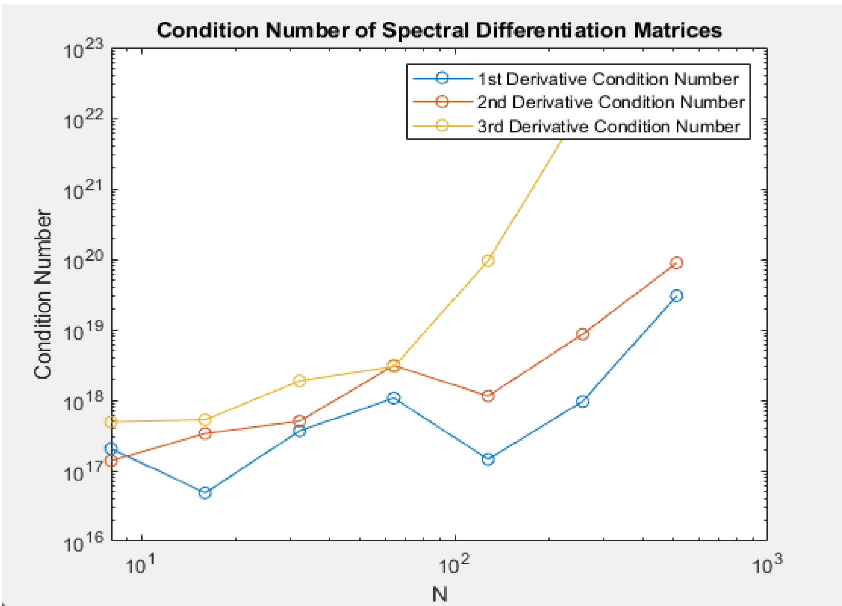

Figure 3: L = 1

The errors are about the same for N = 32 and N = 64, it starts to increase at N = 128. From my result, the critical value of N does not vary with different P. As shown in the plot, the condition number also increases around N = 100.

1.3

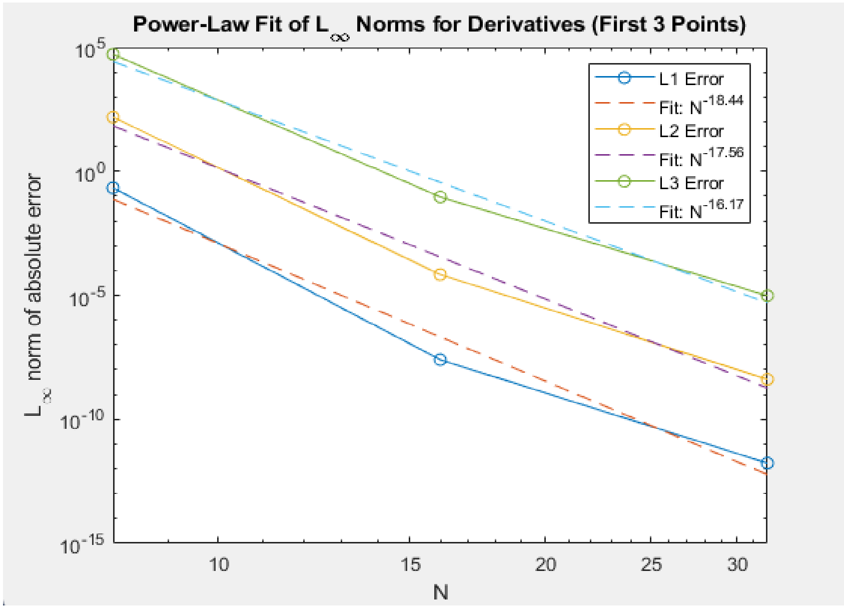

Figure 4: L = 0.1

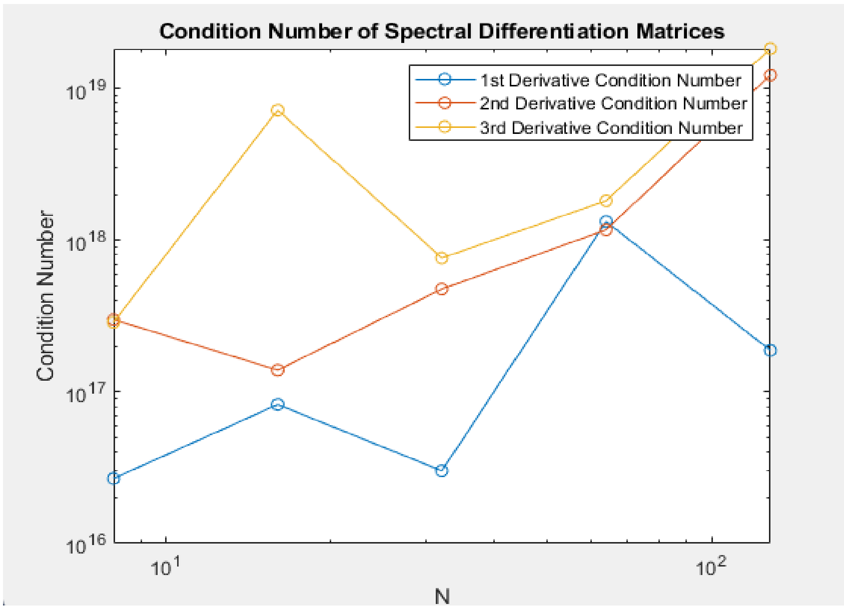

Figure 5: L = 0.1

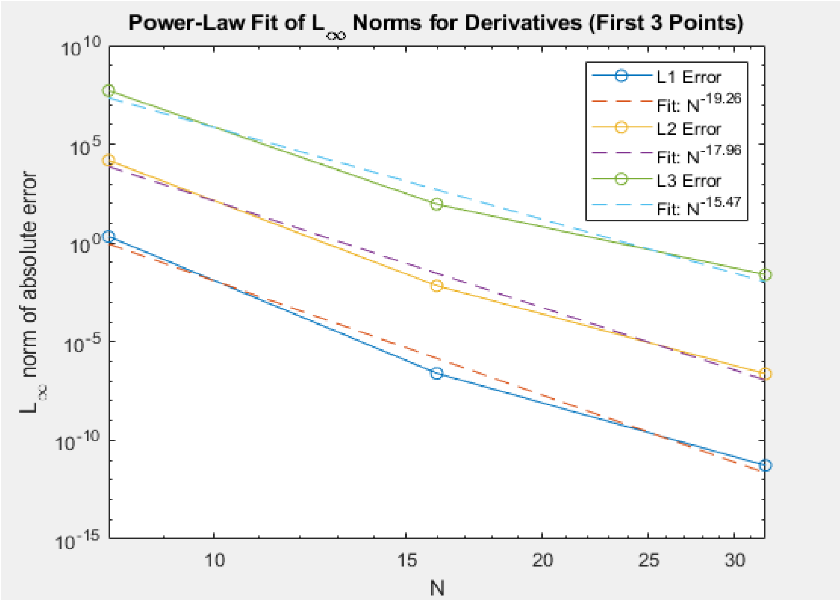

Figure 6: L = 0.01

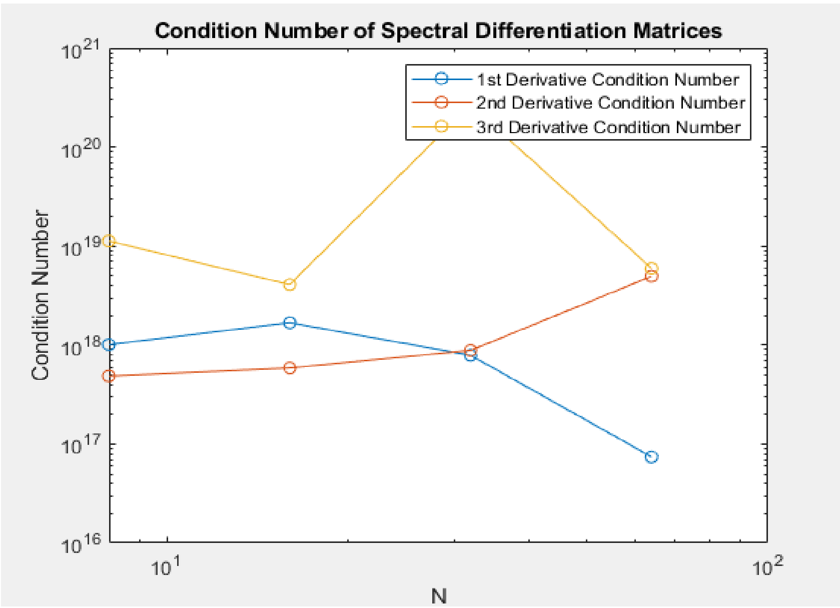

Figure 7: L = 0.01

In terms of N, the accuracy increases with N until it reaches a N that corresponds to an increase in condition number.
In terms of L, errors increase in the same order as L decreases.
In terms of P, errors increase as P increases.

2 Problem 2

2.1

$$\frac{C^{n+1} - C^n}{\Delta t} = \alpha \frac{C^{n+1}}{\Delta x^2}$$

2.2

The choice of t final allows the signal to reach equilibrium.

2.2.1 a

The advantage of using GLL nodes is that I can use the weight that corresponds to each node to calculate the integral.

2.2.2 b