Problem 1:

a.

Hypercube:

The cost of all to all broadcast is $t_s \log(p) + t_w * \frac{n}{n} * (p-1)$.

Total cost is
$$T_p = \frac{n^2}{p} + t_s \log(p) + t_w (n - \frac{n}{p})$$

Torus:

The cost of all for all broadcast is $2*t_s(\sqrt{p}-1)+t_w*\frac{n}{p}*(p-1)$.

Total cost:
$$T_p = \frac{n^2}{p} + 2 * t_s \log(\sqrt{p} - 1) + t_w(n - \frac{n}{p})$$

b.

For Matrix Transposition, they just exchange data between each pair of processors that are not on the principle diagonal.

So the total cost for communication and local transpose is

$$T_p = \frac{n^2}{2p} + \left(t_s + t_w * \frac{n^2}{p}\right) * \frac{p}{2}$$

Problem 2:

Algorithm:

In this project, we are supposed to bisect the dataset into 4 parts with equal number of points. Our method firstly finds the pivot of x which can separate points into two parts, then continue to divide those 2 parts in y. For each step, we need to sort the array index for both x and y when we find the pivot. We want to make those points that smaller than pivot move to its left and others to its right. Tasks are divided into processors which decrease the runtime.

64 quadrants:

1 processor:

elapsed time = 20.523767

Total cost: 300480231229406912.000000

2 processors:

elapsed time = 10.404246

Total cost: 300480231229774976.000000

4 processors:

elapsed time = 5.466761

Total cost: 300480231229662976.000000

8 processors:

elapsed time = 3.133327

Total cost: 300480231229646656.000000

16 processors:

elapsed time = 1.879128

Total cost: 300480231229634944.000000

32 processors:

elapsed time = 1.829047

Total cost: 300480231229586688.000000

64 processors:

elapsed time = 1.349933

Total cost: 300480231229580032.000000

128 processors:

elapsed time = 2.030502

Total cost: 300480231229580032.000000

256 processors:

elapsed time = 6.293851

Total cost: 300480231229580032.000000

128 quadrants:

1 processor:

elapsed time = 10.421792

Total cost: 115984963747418896.000000

2 processors:

elapsed time = 5.365972

Total cost: 115984963747440704.000000

4 processors:

elapsed time = 2.909101

Total cost: 115984963747389440.000000

8 processors:

elapsed time = 1.851383

Total cost: 115984963747373248.000000

16 processors:

elapsed time = 1.133237

Total cost: 115984963747341120.000000

32 processors:

elapsed time = 1.279584

Total cost: 115984963747342624.000000

64 processors:

elapsed time = 0.880695

Total cost: 115984963747354576.000000

128 processors:

elapsed time = 1.412000

Total cost: 115984963747355152.000000

256 quadrants:

1 processor:

elapsed time = 5.387627

Total cost: 37546465352678760.000000

2 processors:

elapsed time = 2.862995

Total cost: 37546465352637520.000000

4 processors:

elapsed time = 2.031409

Total cost: 37546465352637056.000000

8 processors:

elapsed time = 1.071243

Total cost: 37546465352634288.000000

16 processors:

elapsed time = 0.753612

Total cost: 37546465352636328.000000

32 processors:

elapsed time = 0.941824

Total cost: 37546465352637792.000000

64 processors:

elapsed time = 0.819788

Total cost: 37546465352637576.000000

128 processors:

elapsed time = 1.950089

Total cost: 37546465352637928.000000