Assignment_#6

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1. Source Code

< 5.1 >

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

np.random.seed(seed=1)
X_min = 4
X_max = 30
X_n = 16
X = 5 + 25 * np.random.rand(X_n)
Prm_c = [170, 108, 0.2]
T = Prm_c[0] - Prm_c[1] * np.exp(-Prm_c[2] * X) \
+ 4 * np.random.randn(X_n)
np.savez('ch5_data.npz', X=X, X_min=X_min, X_max=X_max, X_n=X_n, T=T)
```

```
print(X)
```

```
[15.42555012 23.00811234  5.00285937 12.55831432  8.66889727  7.30846487
  9.65650528 13.63901818 14.91918686 18.47041835 15.47986286 22.13048751
 10.11130624 26.95293591  5.68468983 21.76168775]
```

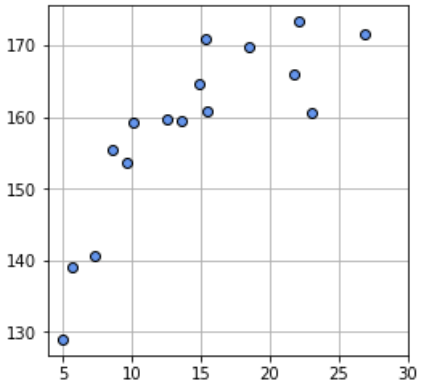
```
print(np.round(X, 2))
```

```
[15.43 23.01  5.   12.56  8.67  7.31  9.66 13.64 14.92 18.47 15.48 22.13
 10.11 26.95  5.68 21.76]
```

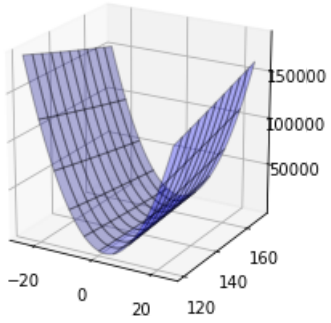
```
print(np.round(T, 2))
```

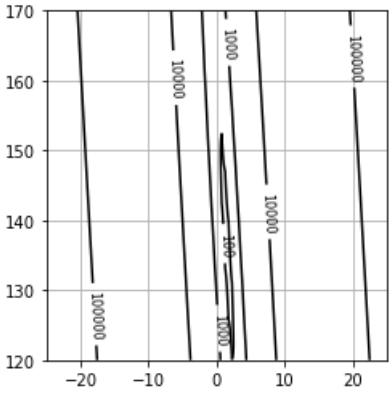
```
[170.91 160.68 129.   159.7  155.46 140.56 153.65 159.43 164.7  169.65
 160.71 173.29 159.31 171.52 138.96 165.87]
```

```
plt.figure(figsize=(4, 4))
plt.plot(X, T, marker='o', linestyle='None',
         markeredgecolor='black', color='cornflowerblue')
plt.xlim(X_min, X_max)
plt.grid(True)
plt.show()
```




```
from mpl_toolkits.mplot3d import Axes3D
def mse_line(x, t, w):
    y = w[0] * x + w[1]
    mse = np.mean((y - t)**2)
    return mse

xn = 100
w0_range = [-25, 25]
w1_range = [120, 170]
x0 = np.linspace(w0_range[0], w0_range[1], xn)
x1 = np.linspace(w1_range[0], w1_range[1], xn)
xx0, xx1 = np.meshgrid(x0, x1)
J = np.zeros((len(x0), len(x1)))
for i0 in range(xn):
    for i1 in range(xn):
        J[i1, i0] = mse_line(X, T, (x0[i0], x1[i1]))

plt.figure(figsize=(9.5, 4))
plt.subplots_adjust(wspace=0.5)

ax = plt.subplot(1, 2, 1, projection='3d')
ax.plot_surface(xx0, xx1, J, rstride=10, cstride=10, alpha=0.3,
                color='blue', edgecolor='black')
ax.set_xticks([-20, 0, 20])
ax.set_yticks([120, 140, 160])
ax.view_init(20, -60)

plt.subplot(1, 2, 2)
cont = plt.contour(xx0, xx1, J, 30, colors='black',
                   levels=[100, 1000, 10000, 100000])
cont.clabel(fmt='%1.0f', fontsize=8)
plt.grid(True)
plt.show()
```






```
def dmse_line(x, t, w):
    y = w[0] * x + w[1]
    d_w0 = 2 * np.mean((y - t) * x)
    d_w1 = 2 * np.mean(y - t)
    return d_w0, d_w1
```

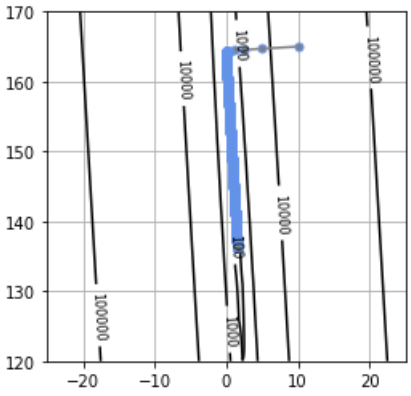
```
d_w = dmse_line(X, T, [10, 165])
print(np.round(d_w, 1))
```

```
[5046.3  301.8]
```

```
def fit_line_num(x, t):
    w_init = [10.0, 165.0]
    alpha = 0.001
    i_max = 100000
    eps = 0.1
    w_i = np.zeros([i_max, 2])
    w_i[0, :] = w_init
    for i in range(1, i_max):
        dmse = dmse_line(x, t, w_i[i - 1])
        w_i[i, 0] = w_i[i - 1, 0] - alpha * dmse[0]
        w_i[i, 1] = w_i[i - 1, 1] - alpha * dmse[1]
        if max(np.absolute(dmse)) < eps:
            break
    w0 = w_i[i, 0]
    w1 = w_i[i, 1]
    w_i = w_i[:i, :]
    return w0, w1, dmse, w_i


plt.figure(figsize=(4, 4))
xn = 100
w0_range = [-25, 25]
w1_range = [120, 170]
x0 = np.linspace(w0_range[0], w0_range[1], xn)
x1 = np.linspace(w1_range[0], w1_range[1], xn)
xx0, xx1 = np.meshgrid(x0, x1)
J = np.zeros((len(x0), len(x1)))
for i0 in range(xn):
```

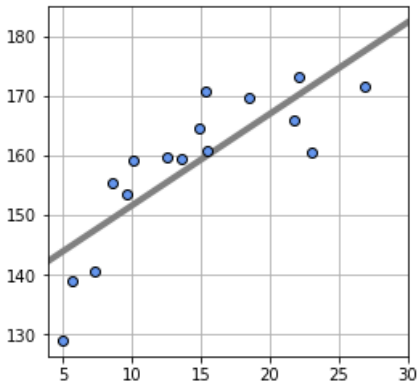
```
    for i1 in range(xn):
        J[i1, i0] = mse_line(X, T, (x0[i0], x1[i1]))
cont = plt.contour(xx0, xx1, J, 30, colors='black',
                   levels=(100, 1000, 10000, 100000))
cont.clabel(fmt='%1.0f', fontsize=8)
plt.grid(True)
W0, W1, dMSE, W_history = fit_line_num(X, T)
print('반복 횟수 {0}'.format(W_history.shape[0]))
print('W=[{0:.6f}, {1:.6f}]'.format(W0, W1))
print('dMSE=[{0:.6f}, {1:.6f}]'.format(dMSE[0], dMSE[1]))
print('MSE={0:.6f}'.format(mse_line(X, T, [W0, W1])))
plt.plot(W_history[:, 0], W_history[:, 1], '.-',
         color='gray', markersize=10, markeredgecolor='cornflowerblue')
plt.show()
```

```
반복 횟수 13820
W=[1.539947, 136.176160]
dMSE=[-0.005794, 0.099991]
MSE=49.027452
```



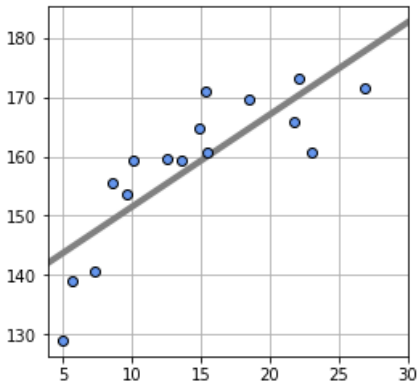

```
def show_line(w):
    xb = np.linspace(X_min, X_max, 100)
    y = w[0] * xb + w[1]
    plt.plot(xb, y, color=(.5, .5, .5), linewidth=4)


plt.figure(figsize=(4, 4))
W=np.array([W0, W1])
mse = mse_line(X, T, W)
print("W0={0:.3f}, w1={1:.3f}".format(W0, W1))
print("SD={0:.3f} cm".format(np.sqrt(mse)))
show_line(W)
plt.plot(X, T, marker='o', linestyle='None',
         color='cornflowerblue', markeredgecolor='black')
plt.xlim(X_min, X_max)
plt.grid(True)
plt.show()
```

```
w0=1.540, w1=136.176
SD=7.002 cm
```




```
def fit_line(x, t):
    mx = np.mean(x)
    mt = np.mean(t)
    mtx = np.mean(t * x)
    mxx = np.mean(x * x)
    w0 = (mtx - mt * mx) / (mxx - mx**2)
    w1 = mt - w0 * mx
    return np.array([w0, w1])

W = fit_line(X, T)
print("w0={0:.3f}, w1={1:.3f}".format(W[0], W[1]))
mse = mse_line(X, T, W)
print("SD={0:.3f} cm".format(np.sqrt(mse)))
plt.figure(figsize=(4, 4))
show_line(W)
plt.plot(X, T, marker='o', linestyle='None',
         color='cornflowerblue', markeredgecolor='black')
plt.xlim(X_min, X_max)
plt.grid(True)
plt.show()
```

```
w0=1.558, w1=135.872
SD=7.001 cm
```



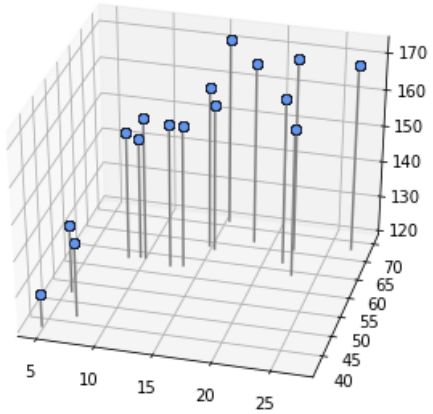
< 5.2 >

```
X0 = X
X0_min = 5
X0_max = 30
np.random.seed(seed=1)
X1 = 23 * (T / 100)**2 + 2 * np.random.randn(X_n)
X1_min = 40
X1_max = 75
```

```
print(np.round(X0, 2))
print(np.round(X1, 2))
print(np.round(T, 2))
```

```
[15.43 23.01  5.   12.56  8.67  7.31  9.66 13.64 14.92 18.47 15.48 22.13
 10.11 26.95  5.68 21.76]
[70.43 58.15 37.22 56.51 57.32 40.84 57.79 56.94 63.03 65.69 62.33 64.95
 57.73 66.89 46.68 61.08]
[170.91 160.68 129.   159.7  155.46 140.56 153.65 159.43 164.7  169.65
 160.71 173.29 159.31 171.52 138.96 165.87]
```

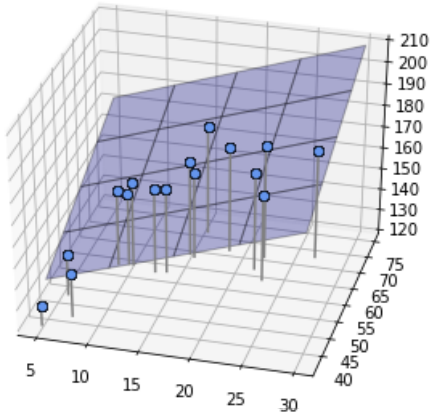
```
def show_data2(ax, x0, x1, t):
    for i in range(len(x0)):
        ax.plot([x0[i], x0[i]], [x1[i], x1[i]],
                [120, t[i]], color='gray')
        ax.plot(x0, x1, t, 'o',
                color='cornflowerblue', markeredgecolor='black',
                markersize=6, markeredgewidth=0.5)
        ax.view_init(elev=35, azim=-75)

plt.figure(figsize=(6, 5))
ax = plt.subplot(1,1,1,projection='3d')
show_data2(ax, X0, X1, T)
plt.show()
```




```
def show_plane(ax, w):
    px0 = np.linspace(X0_min, X0_max, 5)
    px1 = np.linspace(X1_min, X1_max, 5)
    px0, px1 = np.meshgrid(px0, px1)
```

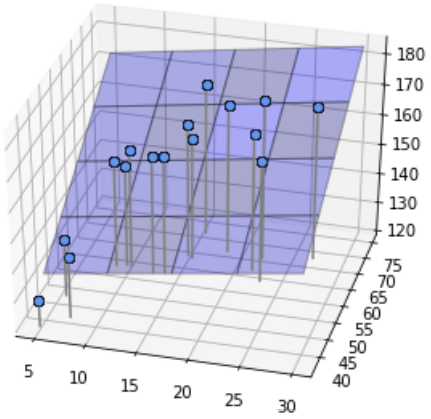
```
    y = w[0]*px0 + w[1] * px1 + w[2]
    ax.plot_surface(px0, px1, y, rstride=1, cstride=1, alpha=0.3,
                    color='blue', edgecolor='black')

def mse_plane(x0, x1, t, w):
    y = w[0] * x0 + w[1] * x1 + w[2] # (A)
    mse = np.mean((y - t)**2)
    return mse

plt.figure(figsize=(6, 5))
ax = plt.subplot(1, 1, 1, projection='3d')
W = [1.5, 1, 90]
show_plane(ax, W)
show_data2(ax, X0, X1, T)
mse = mse_plane(X0, X1, T, W)
print("SD={0:.3f} cm".format(np.sqrt(mse)))
plt.show()
```

```
SD=12.876 cm
```




```
def fit_plane(x0, x1, t):
    c_tx0 = np.mean(t * x0) - np.mean(t) * np.mean(x0)
    c_tx1 = np.mean(t * x1) - np.mean(t) * np.mean(x1)
    c_x0x1 = np.mean(x0 * x1) - np.mean(x0) * np.mean(x1)
    v_x0 = np.var(x0)
    v_x1 = np.var(x1)
    w0 = (c_tx1 * c_x0x1 - v_x1 * c_tx0) / (c_x0x1**2 - v_x0 * v_x1)
    w1 = (c_tx0 * c_x0x1 - v_x0 * c_tx1) / (c_x0x1**2 - v_x0 * v_x1)
    w2 = -w0 * np.mean(x0) - w1 * np.mean(x1) + np.mean(t)
    return np.array([w0, w1, w2])

plt.figure(figsize=(6, 5))
ax = plt.subplot(1, 1, 1, projection='3d')
W = fit_plane(X0, X1, T)
print("w0={0:.1f}, w1={1:.1f}, w2={2:.1f}".format(W[0], W[1], W[2]))
show_plane(ax, W)
show_data2(ax, X0, X1, T)
mse = mse_plane(X0, X1, T, W)
print("SD={0:.3f} cm".format(np.sqrt(mse)))
plt.show()
```

```
w0=0.5, w1=1.1, w2=89.0
SD=2.546 cm
```

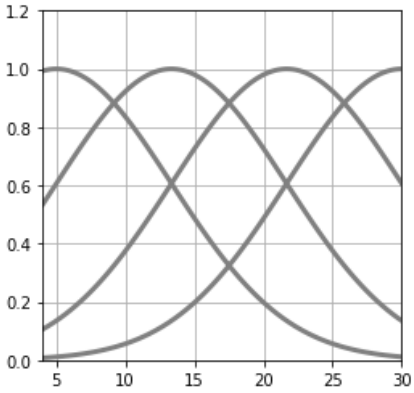


< 5.4 >

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

outfile = np.load('ch5_data.npz')
X = outfile['X']
X_min = outfile['X_min']
X_max = outfile['X_max']
X_n = outfile['X_n']
T = outfile['T']
```

```
def gauss(x, mu, s):
    return np.exp(-(x - mu)**2 / (2 * s**2))
```

```
M = 4
plt.figure(figsize=(4, 4))
mu = np.linspace(5, 30, M)
s = mu[1] - mu[0] # (A)
xb = np.linspace(X_min, X_max, 100)
for j in range(M):
    y = gauss(xb, mu[j], s)
    plt.plot(xb, y, color='gray', linewidth=3)
plt.grid(True)
plt.xlim(X_min, X_max)
plt.ylim(0, 1.2)
plt.show()
```




```
def gauss_func(w, x):
    m = len(w) - 1
    mu = np.linspace(5, 30, m)
    s = mu[1] - mu[0]
    y = np.zeros_like(x)
    for j in range(m):
        y = y + w[j] * gauss(x, mu[j], s)
    y = y + w[m]
    return y
```

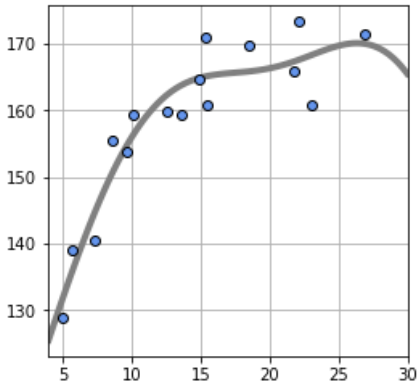
```
def mse_gauss_func(x, t, w):
    y = gauss_func(w, x)
    mse = np.mean((y - t)**2)
    return mse
```

```
def fit_gauss_func(x, t, m):
    mu = np.linspace(5, 30, m)
    s = mu[1] - mu[0]
    n = x.shape[0]
    psi = np.ones((n, m+1))
    for j in range(m):
        psi[:, j] = gauss(x, mu[j], s)
    psi_T = np.transpose(psi)

    b = np.linalg.inv(psi_T.dot(psi))
    c = b.dot(psi_T)
    w = c.dot(t)
    return w
```

```
def show_gauss_func(w):
    xb = np.linspace(X_min, X_max, 100)
    y = gauss_func(w, xb)
    plt.plot(xb, y, c=[.5, .5, .5], lw=4)

plt.figure(figsize=(4, 4))
M = 4
W = fit_gauss_func(X, T, M)
show_gauss_func(W)
plt.plot(X, T, marker='o', linestyle='None',
         color='cornflowerblue', markeredgecolor='black')
plt.xlim(X_min, X_max)
```

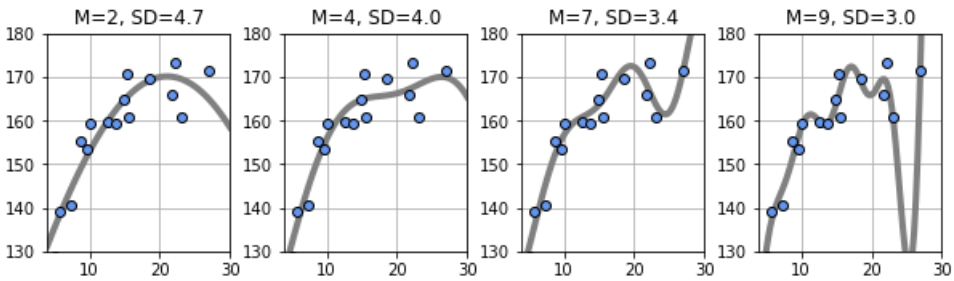
```
plt.grid(True)
mse = mse_gauss_func(X, T, W)
print('W='+ str(np.round(W,1)))
print("SD={0:.2f} cm".format(np.sqrt(mse)))
plt.show()
```

```
W=[29.4 75.7  2.9 98.3 54.9]
SD=3.98 cm
```

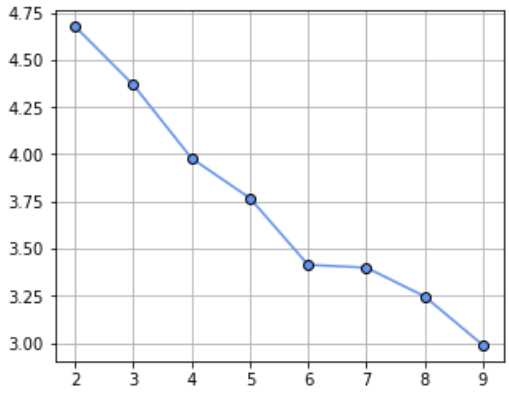


< 5.5 >

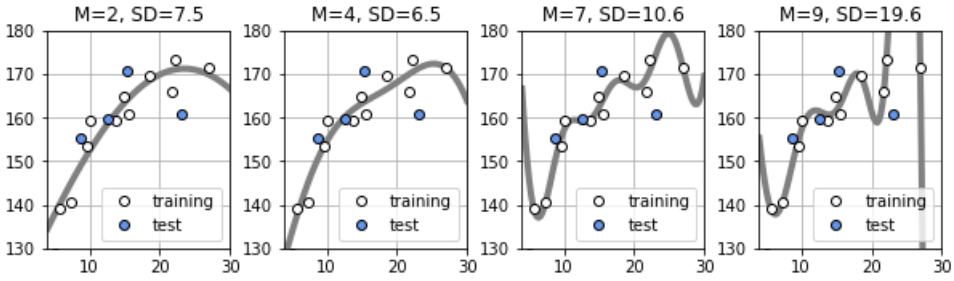
```
plt.figure(figsize=(10, 2.5))
plt.subplots_adjust(wspace=0.3)
M = [2, 4, 7, 9]
for i in range(len(M)):
    plt.subplot(1, len(M), i + 1)
    W = fit_gauss_func(X, T, M[i])
    show_gauss_func(W)
    plt.plot(X, T, marker='o', linestyle='None',
             color='cornflowerblue', markeredgecolor='black')
    plt.xlim(X_min, X_max)
    plt.grid(True)
    plt.ylim(130, 180)
    mse = mse_gauss_func(X, T, W)

    plt.title("M={0:d}, SD={1:.1f}".format(M[i], np.sqrt(mse)))
plt.show()
```



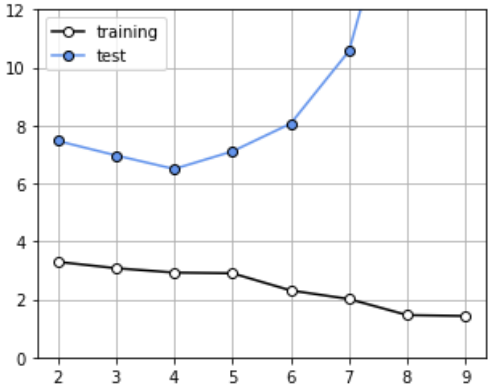
```
plt.figure(figsize=(5, 4))
M = range(2, 10)
mse2 = np.zeros(len(M))
for i in range(len(M)):
    W = fit_gauss_func(X, T, M[i])
    mse2[i] = np.sqrt(mse_gauss_func(X, T, W))
plt.plot(M, mse2, marker='o',
         color='cornflowerblue', markeredgecolor='black')
plt.grid(True)
plt.show()
```




```
X_test = X[:int(X_n / 4 + 1)]
T_test = T[:int(X_n / 4 + 1)]
X_train = X[int(X_n / 4 + 1):]
T_train = T[int(X_n / 4 + 1):]

plt.figure(figsize=(10, 2.5))

plt.subplots_adjust(wspace=0.3)
M = [2, 4, 7, 9]
for i in range(len(M)):
    plt.subplot(1, len(M), i + 1)
    W = fit_gauss_func(X_train, T_train, M[i])
    show_gauss_func(W)
    plt.plot(X_train, T_train, marker='o',
             linestyle='None', color='white',
             markeredgecolor='black', label='training')
    plt.plot(X_test, T_test, marker='o', linestyle='None',
             color='cornflowerblue',
             markeredgecolor='black', label='test')
    plt.legend(loc='lower right', fontsize=10, numpoints=1)
    plt.xlim(X_min, X_max)
    plt.ylim(130, 180)
    plt.grid(True)
    mse = mse_gauss_func(X_test, T_test, W)
    plt.title("M={0:d}, SD={1:.1f}".format(M[i], np.sqrt(mse)))
plt.show()
```




```
plt.figure(figsize=(5, 4))
M = range(2, 10)
mse_train = np.zeros(len(M))
mse_test = np.zeros(len(M))
for i in range(len(M)):
    W = fit_gauss_func(X_train, T_train, M[i])
    mse_train[i] = np.sqrt(mse_gauss_func(X_train, T_train, W))
    mse_test[i] = np.sqrt(mse_gauss_func(X_test, T_test, W))
plt.plot(M, mse_train, marker='o', linestyle='-',
         markerfacecolor='white', markeredgecolor='black',
         color='black', label='training')
plt.plot(M, mse_test, marker='o', linestyle='-',
         color='cornflowerblue', markeredgecolor='black',
         label='test')
plt.legend(loc='upper left', fontsize=10)
plt.ylim(0, 12)
plt.grid(True)
plt.show()
```




```
def kfold_gauss_func(x, t, m, k):
    n = x.shape[0]
    mse_train = np.zeros(k)
    mse_test = np.zeros(k)
    for i in range(0, k):
        x_train = x[np.fmod(range(n), k) != i] # (A)
        t_train = t[np.fmod(range(n), k) != i] # (A)
        x_test = x[np.fmod(range(n), k) == i] # (A)
        t_test = t[np.fmod(range(n), k) == i] # (A)
        wm = fit_gauss_func(x_train, t_train, m)
        mse_train[i] = mse_gauss_func(x_train, t_train, wm)
        mse_test[i] = mse_gauss_func(x_test, t_test, wm)
    return mse_train, mse_test
```

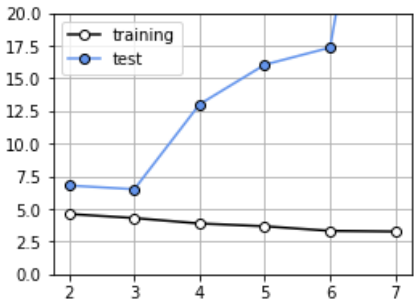
```
np.fmod(range(10),5)
```

```
array([0, 1, 2, 3, 4, 0, 1, 2, 3, 4])
```

```
M = 4
K = 4
kfold_gauss_func(X, T, M, K)
```

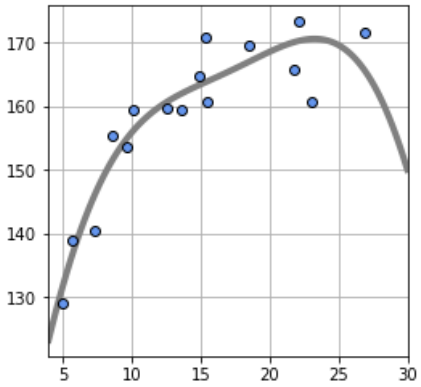
```
(array([12.87927851,  9.81768697, 17.2615696 , 12.92270498]),
 array([ 39.65348229, 734.70782018,  18.30921743,  47.52459642]))
```

```
M = range(2, 8)
K = 16
Cv_Gauss_train = np.zeros((K, len(M)))
Cv_Gauss_test = np.zeros((K, len(M)))
for i in range(0, len(M)):
    Cv_Gauss_train[:, i], Cv_Gauss_test[:, i] =\
                    kfold_gauss_func(X, T, M[i], K)
mean_Gauss_train = np.sqrt(np.mean(Cv_Gauss_train, axis=0))
mean_Gauss_test = np.sqrt(np.mean(Cv_Gauss_test, axis=0))

plt.figure(figsize=(4, 3))
plt.plot(M, mean_Gauss_train, marker='o', linestyle='-',
         color='k', markerfacecolor='w', label='training')
plt.plot(M, mean_Gauss_test, marker='o', linestyle='-',
         color='cornflowerblue', markeredgecolor='black', label='test')
plt.legend(loc='upper left', fontsize=10)
plt.ylim(0, 20)
plt.grid(True)
plt.show()
```



```
M = 3
plt.figure(figsize=(4, 4))
W = fit_gauss_func(X, T, M)
show_gauss_func(W)
plt.plot(X, T, marker='o', linestyle='None',
         color='cornflowerblue', markeredgecolor='black')
plt.xlim([X_min, X_max])
plt.grid(True)
mse = mse_gauss_func(X, T, W)
print("SD={0:.2f} cm".format(np.sqrt(mse)))
plt.show()
```

```
SD=4.37 cm
```



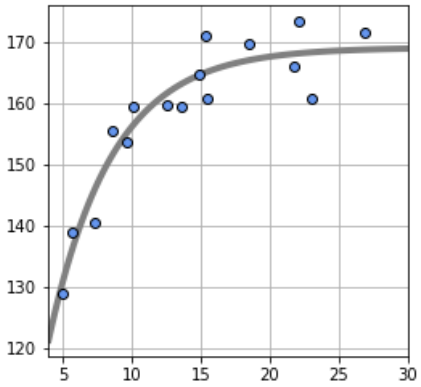
< 5.6 >

```
def model_A(x, w):
    y = w[0] - w[1] * np.exp(-w[2] * x)
    return y

def show_model_A(w):
    xb = np.linspace(X_min, X_max, 100)
    y = model_A(xb, w)
    plt.plot(xb, y, c=[.5, .5, .5], lw=4)

def mse_model_A(w, x, t):
    y = model_A(x, w)
    mse = np.mean((y - t)**2)
    return mse
```

```
from scipy.optimize import minimize

def fit_model_A(w_init, x, t):
    res1 = minimize(mse_model_A, w_init, args=(x, t), method="powell")
    return res1.x
```

```
plt.figure(figsize=(4, 4))
W_init=[100, 0, 0]
W = fit_model_A(W_init, X, T)
print("w0={0:.1f}, w1={1:.1f}, w2={2:.1f}".format(W[0], W[1], W[2]))
show_model_A(W)
plt.plot(X, T, marker='o', linestyle='None',
         color='cornflowerblue',markeredgecolor='black')
plt.xlim(X_min, X_max)
plt.grid(True)
mse = mse_model_A(W, X, T)
print("SD={0:.2f} cm".format(np.sqrt(mse)))
plt.show()
```

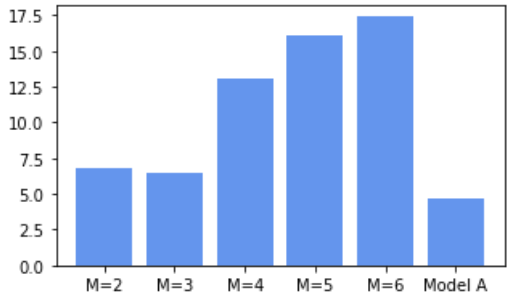
```
w0=169.0, w1=113.7, w2=0.2
SD=3.86 cm
```



< 5.7 >

```
def kfold_model_A(x, t, k):
    n = len(x)
    mse_train = np.zeros(k)
    mse_test = np.zeros(k)
    for i in range(0, k):
        x_train = x[np.fmod(range(n), k) != i]
        t_train = t[np.fmod(range(n), k) != i]
        x_test = x[np.fmod(range(n), k) == i]
        t_test = t[np.fmod(range(n), k) == i]
        wm = fit_model_A(np.array([169, 113, 0.2]), x_train, t_train)
        mse_train[i] = mse_model_A(wm, x_train, t_train)
        mse_test[i] = mse_model_A(wm, x_test, t_test)
    return mse_train, mse_test

K = 16
Cv_A_train, Cv_A_test = kfold_model_A(X, T, K)
mean_A_test = np.sqrt(np.mean(Cv_A_test))
print("Gauss(M=3) SD={0:.2f} cm".format(mean_Gauss_test[1]))
print("Model A SD={0:.2f} cm".format(mean_A_test))
SD = np.append(mean_Gauss_test[0:5], mean_A_test)
M = range(6)
label = ["M=2", "M=3", "M=4", "M=5", "M=6", "Model A"]
plt.figure(figsize=(5, 3))
plt.bar(M, SD, tick_label=label, align="center",
facecolor="cornflowerblue")
plt.show()
```

```
Gauss(M=3) SD=6.51 cm
Model A SD=4.72 cm
```



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2. 소감

이번 장에서는 수치해석의 핵심 중 하나라고 할 수 있는 지도 학습의 회귀 문제들을 함수로 구현하고 그래프로 표현해보았습니다.

수치해석 수업 시간에 배운 다변수 선형 회귀나 경사하강법 등 이론으로만 알고 있었던 부분들을 직접 소스 코드로 입력해 구현해봄으로써 해당 지식들을 이해하는데에 많은 도움이 되었습니다.

또한 그래프를 통해 구현한 모델이 새로운 데이터에 대해서 예측을 잘 하고 있는지, 데이터와 모델의 분산정도나 표준 편차를 통해 모델링이 잘 되었는지 판단하는데에 있어 흥미로움을 느꼈습니다.

모델을 구현할 때, 단순히 선형으로만 구현하는 것이 아닌 '선형 기저 함수 모델'을 통해 곡선 형태의 모델을 구현해보기도 하고, 오버 피팅에 의해 발생하는 문제점에 대비해 나온 홀드 아웃 검증을 통해 어떤 M(기저 함수의 수)에서 가장 최적의 결과가 나오는지 그래프를 통해 알아내보는 등 다소 어려운 부분들도 있었지만 소스 코드에서 나타내고 있는 함수들과 교과서의 설명란을 참고해나가면서 이해하는데 노력했습니다.

이러한 지식들과 내용들이 모여 오늘날과 앞으로의 머신러닝을 만든다는 확신과 함께 배워나감으로써 머신러닝이라는 분야에 대해서 깊게 생각해보는 시간을 가졌습니다.