b)

$$P_t^- = AP_{t-1}A^T + Q$$

$$P_t = (I - KH)P_t^- \qquad \text{and} \qquad K = P_t^- H^T (HP_t^- H^T + R)^{-1}$$

$$P_t = P_t^- - KHP_t^-$$

$$P_t = AP_{t-1}A^T + Q - (P_t^- H^T (HP_t^- H^T + R)^{-1}) HP_t^-$$

$$P_t = AP_{t-1}A^T + Q - [(AP_{t-1}A^T + Q) H^T (H(AP_{t-1}A^T + Q)H^T + R)^{-1}] HP_t^-$$

$$P_t = AP_{t-1}A^T + Q - [(AP_{t-1}A^T H^T + QH^T)(HAP_{t-1}A^T H^T + HQH^T + R)^{-1}] HP_t^-$$

$$P_t = \underbrace{A}_{C^T}P_{t-1}\underbrace{A^T}_{C} + Q - (\underbrace{A}_{C^T}P_{t-1}\underbrace{A^T H^T}_{U} + QH^T)(\underbrace{HA}_{U^T}P_{t-1}\underbrace{A^T H^T}_{U} + \underbrace{HQH^T + R}_{Z})^{-1}(\underbrace{HA}_{U^T}P_{t-1}\underbrace{A^T}_{C} + HQ)$$

From this we identify

$C = A^T$

$U = A^T H^T$

$Z = HQH^T + R$

N = Remaining terms

and the remaining terms for N is:

$$N = Q - (AP_{t-1}A^T H^T + QH^T)(HAP_{t-1}A^T H^T + HQH^T + R)^{-1} HQ$$
$$- QH^T (HAP_{t-1}A^T H^T + HQH + R)^{-1} (HAP_{t-1}A^T + HQ)$$
$$- QH^T (HAP_{t-1}A^T H^T + HQH + R)^{-1} HQ$$

Thus we have shown that Kalman Filter covariance takes on the form of a so-called DARE:

$$P_t = C^T P_{t-1} C - (C^T P_{t-1} U)(Z + U^T P_{t-1} U)^{-1} (U^T P_{t-1} C) + N$$