In short, choosing $(i,j) = (1,2)$ w.l.o.g. and the covariance matrix $\Sigma = \begin{pmatrix} \Sigma_{1,1} & 0 & \vdots \\ 0 & \Sigma_{2,2} & \vdots \\ \cdots & & \cdots \end{pmatrix}$ means that $\Sigma_{1,2} = \Sigma_{2,1} = 0$

Therefore using the equation from part b yields

$$p(x_1|x_2) = \mathcal{N}(x_1 ; \mu_1 + \Sigma_{12} \cdot \Sigma_{2,2}^{-1} (x_2 - \mu_2), \Sigma_{1,1} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$$

$$= \mathcal{N}(x_1 ; \mu_1 + 0 \; \Sigma_{2,2}^{-1} (x_2 - \mu_2), \Sigma_{1,1} - 0 \, \Sigma_{2,2}^{-1} 0)$$

$$= \mathcal{N}(x_1 ; \mu_1, \Sigma_{1,1})$$

$p(x_1|x_2) = p(x_1) \rightarrow$ marginal

and x_1 and x_2 are marginally independent. Since this can be applied to any $i \neq j$ pair (by permuting), we say that the statement in the first bullet point always hold.

In short, for a precision matrix $\Lambda = \Sigma^{-1}$ and a zero entry on the $(i,j)=(1,2)$th index of the Λ means that x_i and x_j are condinitionally independent where other variables are x_k with $k \neq i,j$ needs to be proven.

Using the equation from part b,

$$p(x_1 \mid x_2, x_{k \neq x_1 x_2}) = \mathcal{N}\left(x_1 ; \mu_1 - \Lambda_{1,1}^{-1} (\underbrace{\Lambda_{1,2}}_{0} \cdots \Lambda_{1k}) \begin{pmatrix} x_2 - \mu_2 \\ \vdots \\ x_k - \mu_k \end{pmatrix}, \Lambda_{11}^{-1} - (\underbrace{\Lambda_{12}}_{0} \cdots \Lambda_{1k}) \begin{pmatrix} \overset{0}{\Lambda_{22} \Lambda_{2,1}} \\ \vdots \quad \vdots \\ \Lambda_{kk} \Lambda_{k,1} \end{pmatrix} \right)$$

Setting $\Lambda_{1,2} = \Lambda_{2,1} = 0$

$$= \mathcal{N}\left(x_1 ; \mu_1 - \Lambda_{11}^{-1} (\cdots \Lambda_{1,k} \cdots) \begin{pmatrix} \vdots \\ x_k - \mu_k \\ \vdots \end{pmatrix}, \Lambda_{11}^{-1} - (\cdots \Lambda_{1,k} \cdots) \begin{pmatrix} \vdots \\ \Lambda_{kk} \Lambda_{k,1} \\ \vdots \end{pmatrix} \right)$$

$$= p(x_1 \mid x_{k \neq 1,2})$$

Thus, we have proven that the last term does not involve x_2 and x_1 and x_2 are conditionally independent given $\{x_k\}$.

By permuting the indices, this result can also be generalized for arbitrary (i,j) in the statement.