Parallelizing Dinic’s Algorithm for Max Flow

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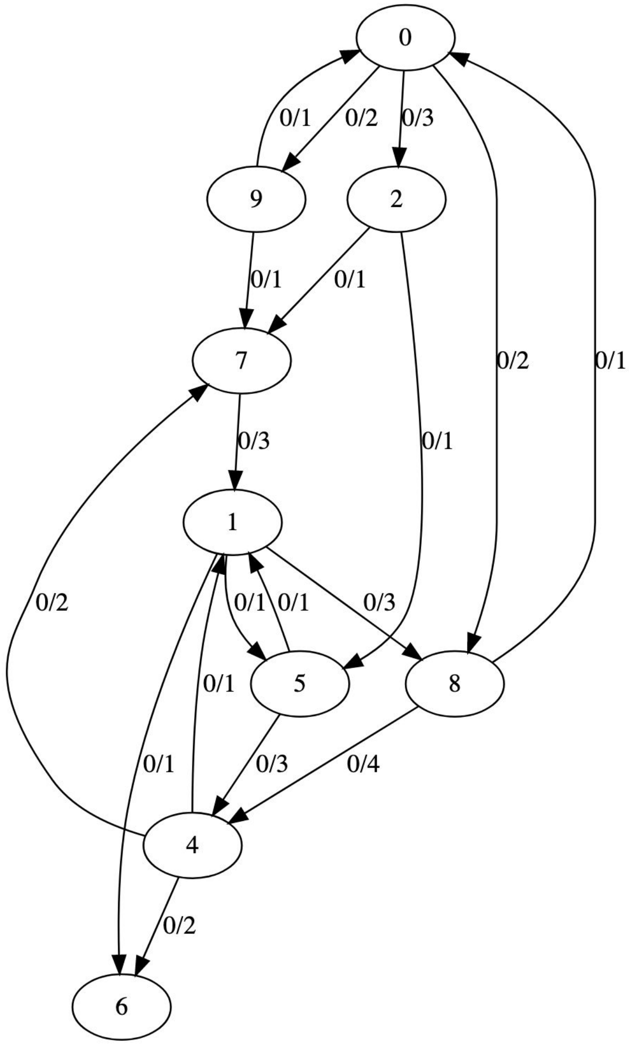
We attempted several different approaches to parallelizing Dinic’s algorithm for max flows in flow networks, which is primarily composed of depth-first search and breadth-first search.

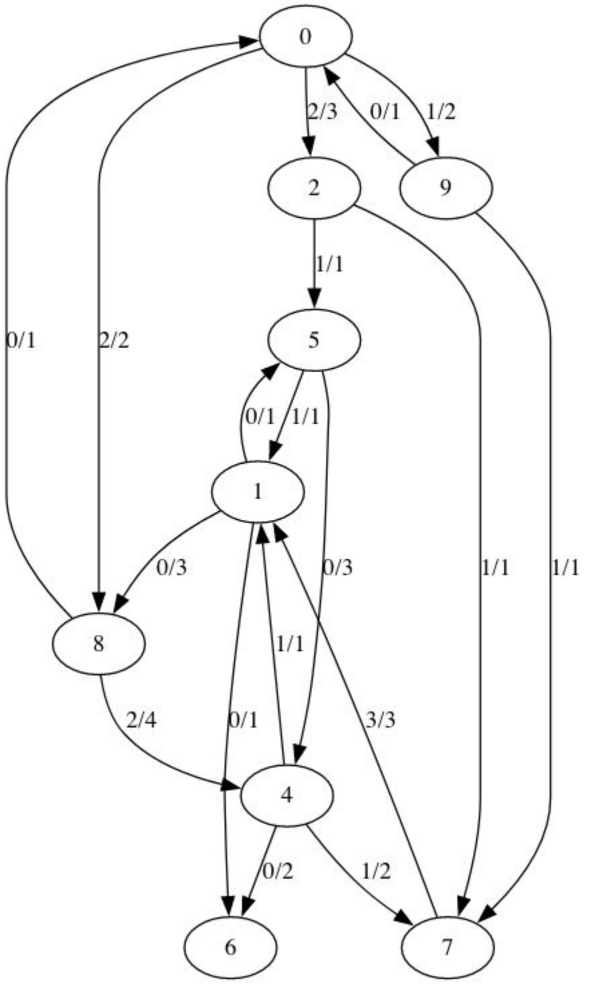
We found that BFS was by far the most time consuming part of the algorithm, and took several approaches to parallelizing it.

Some included using a per-vertex parallelism approach, and using GPU acceleration with CUDA.

What is a max flow

- Directed graph
- Edges have capacities of flow they can carry
- How much flow can be pushed between the source and sink vertex?



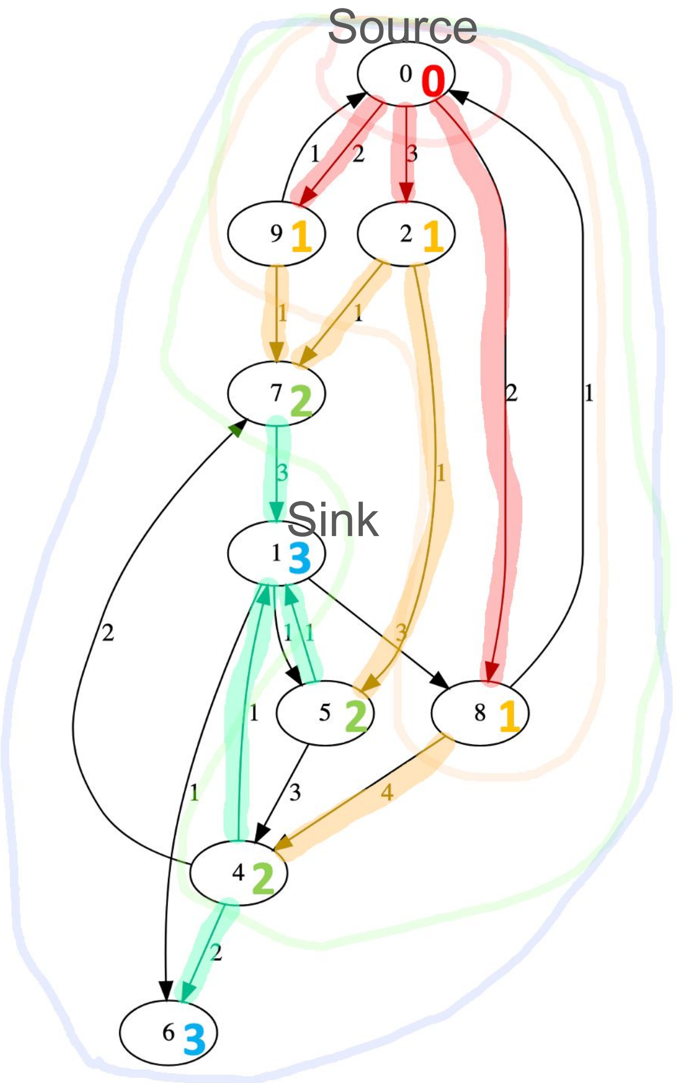


Dinic’s algorithm for Max Flow

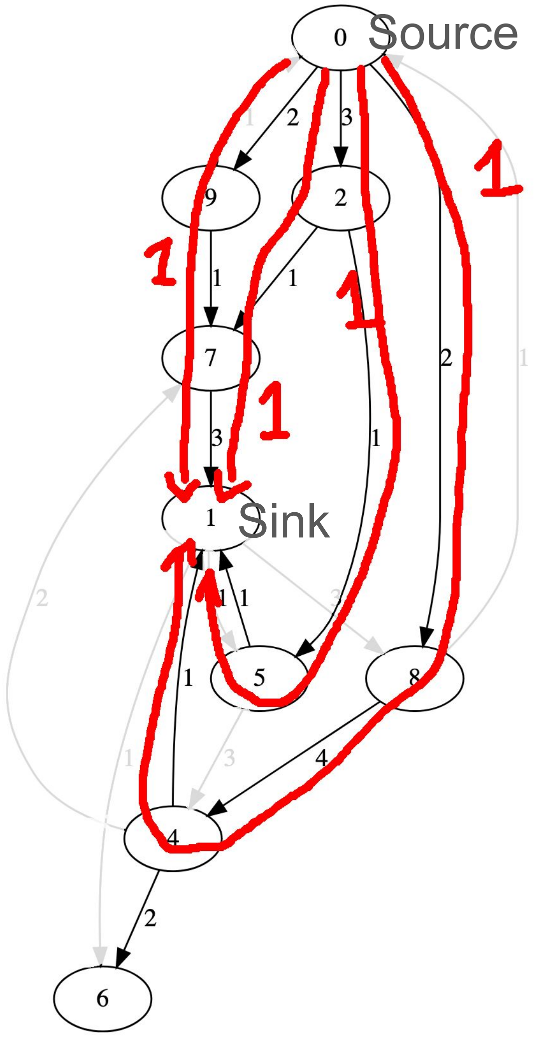
An $O(mn^2)$ algorithm with 2 repeating steps:

- Create a directed acyclic level graph with a BFS
- Repeatedly run DFS through the level graph to find paths from source to sink
    - Push flow through these paths
    - Stop when there are no more paths - a blocking flow

The level graph after BFS

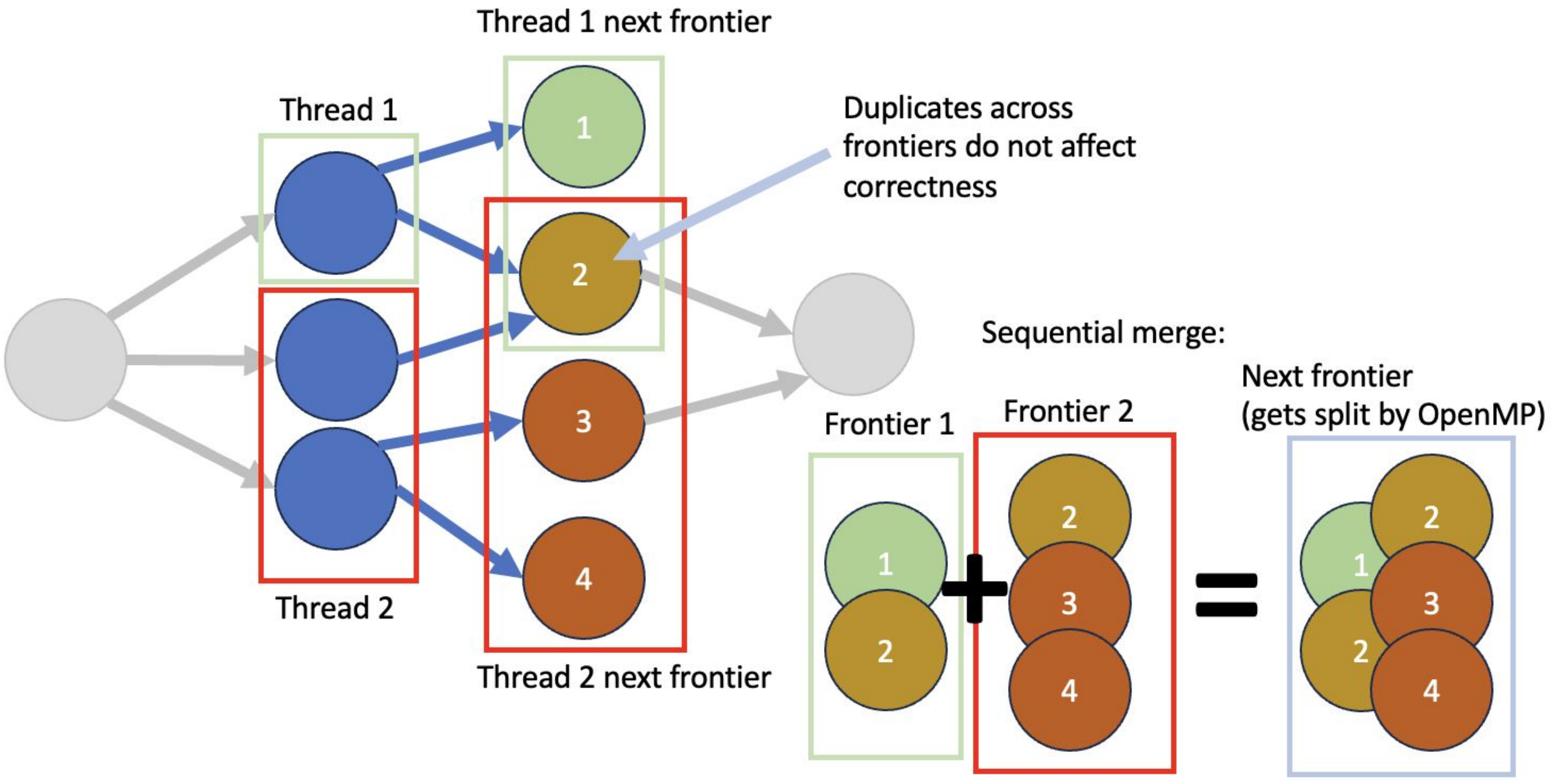


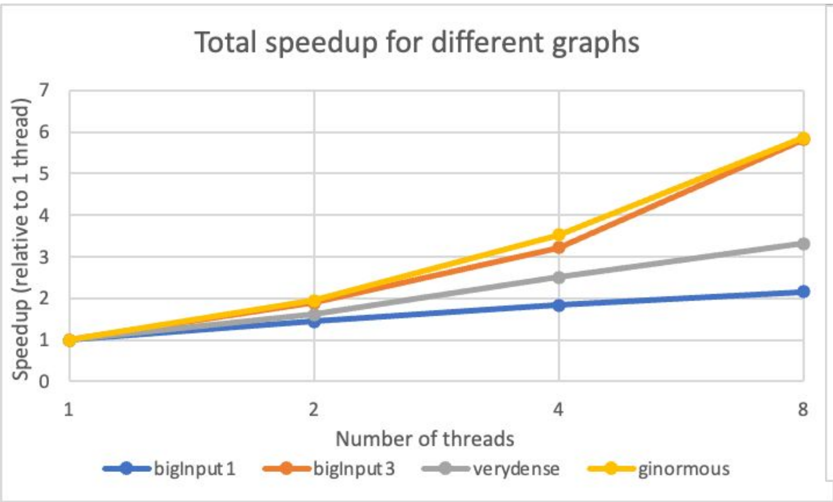
A blocking flow in that level graph (from 4 DFS)

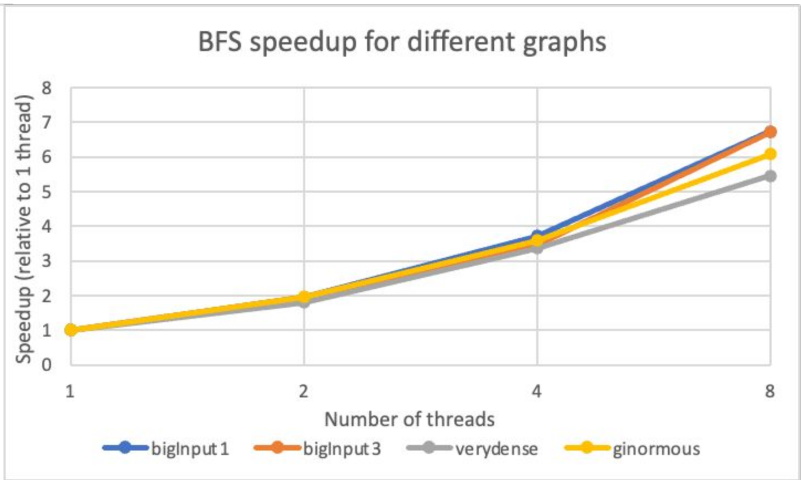


FastBFS (frontier + merge)

- Parallelize per vertex in frontier
- Each thread gets its own frontier
- Frontiers are merged sequentially (not a bottleneck)
- No locks prevent the same vertex to appear in multiple frontiers, duplicates caught later
- Separate layered-graph neighbors per thread, loop through all in sequential DFS

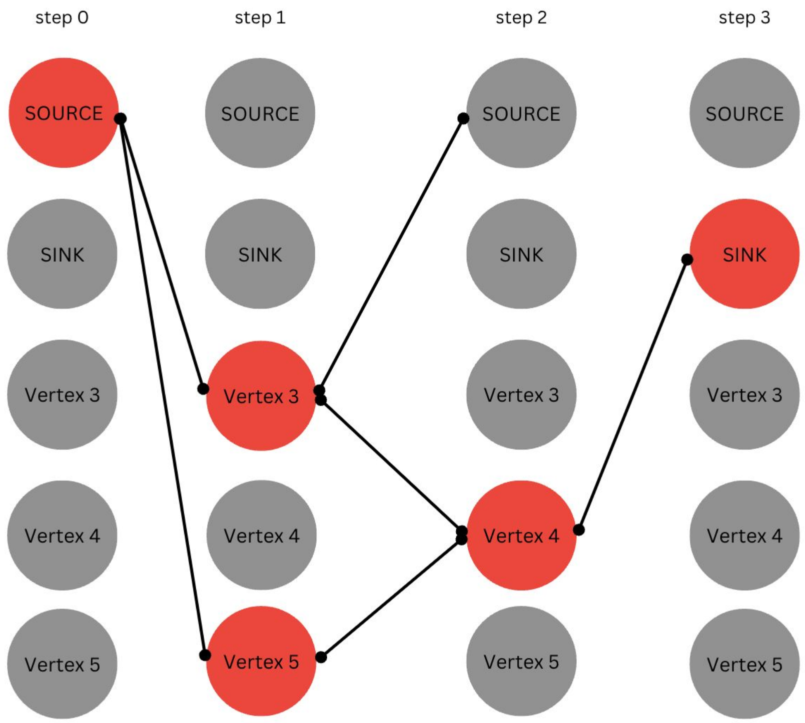


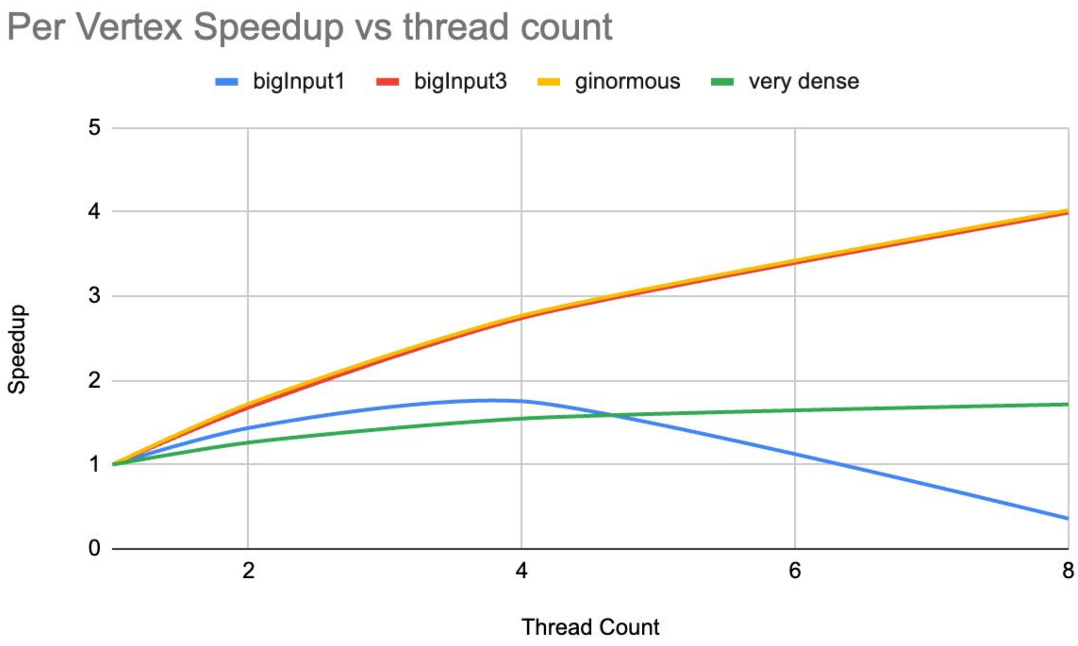




Per-Vertex

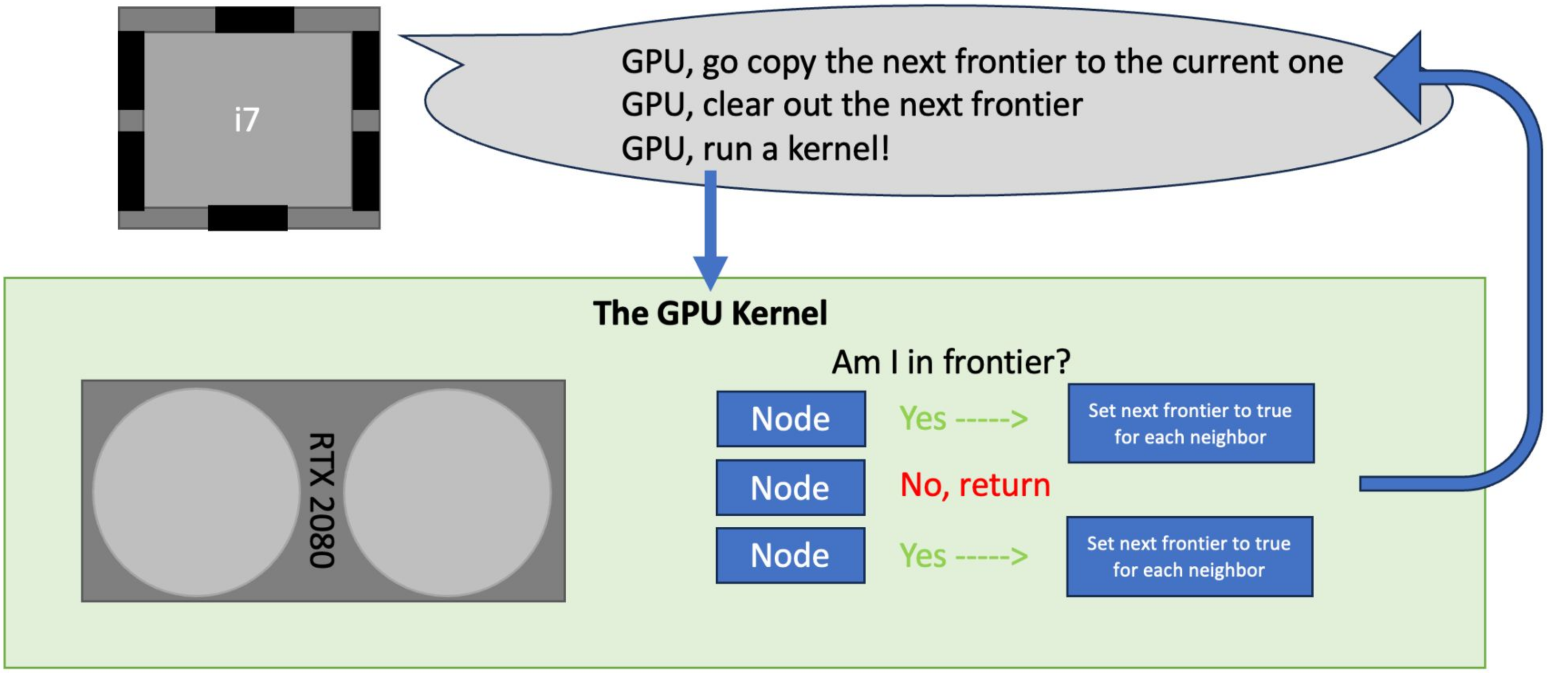
- Loop over all vertices in parallel
- See if node in step before advancing its neighbors





CUDA

- Each vertex gets its thread
- Each kernel advances the frontier one level



GPU, go copy the next frontier to the current one
GPU, clear out the next frontier
GPU, run a kernel!

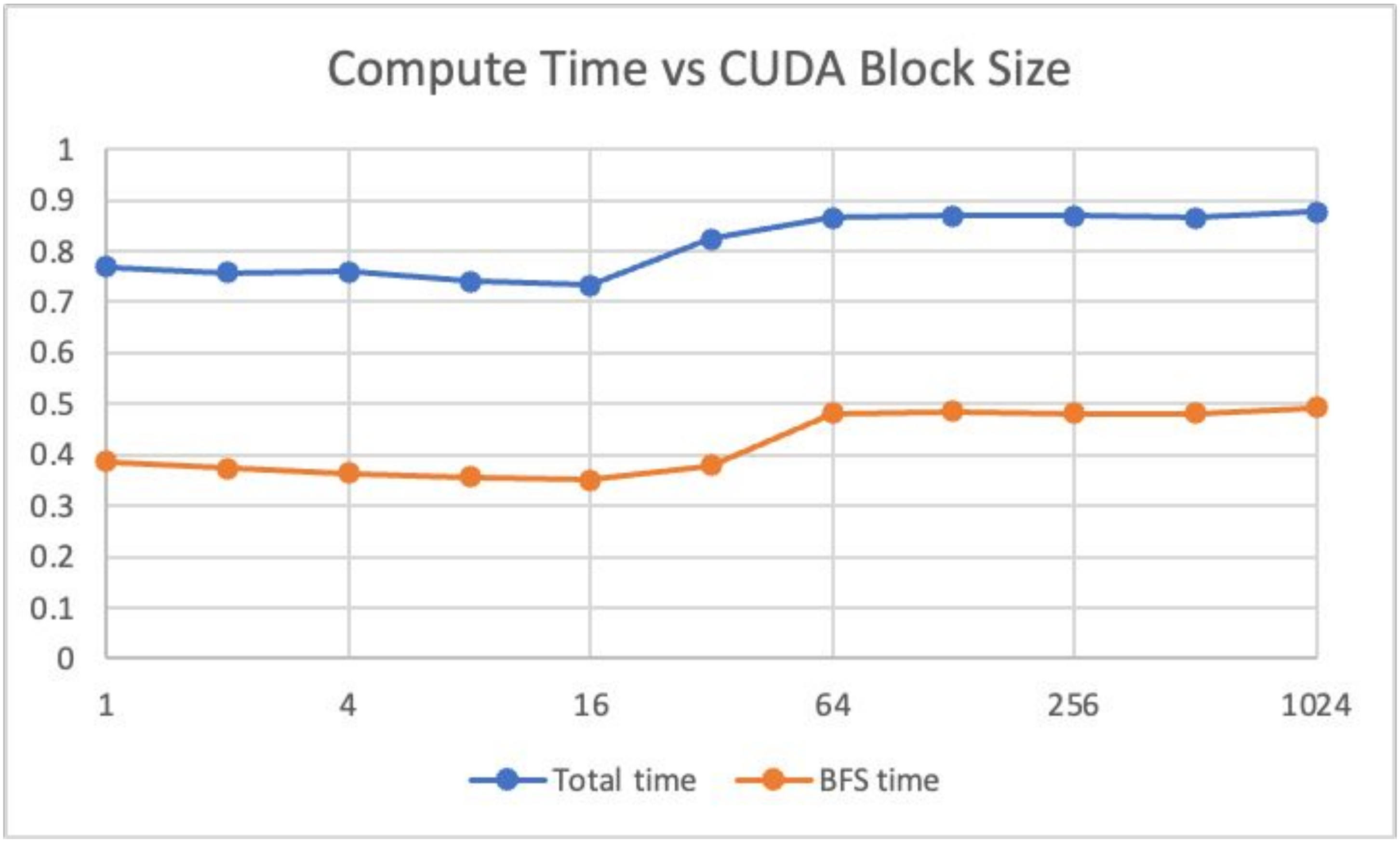
The GPU Kernel

Am I in frontier?

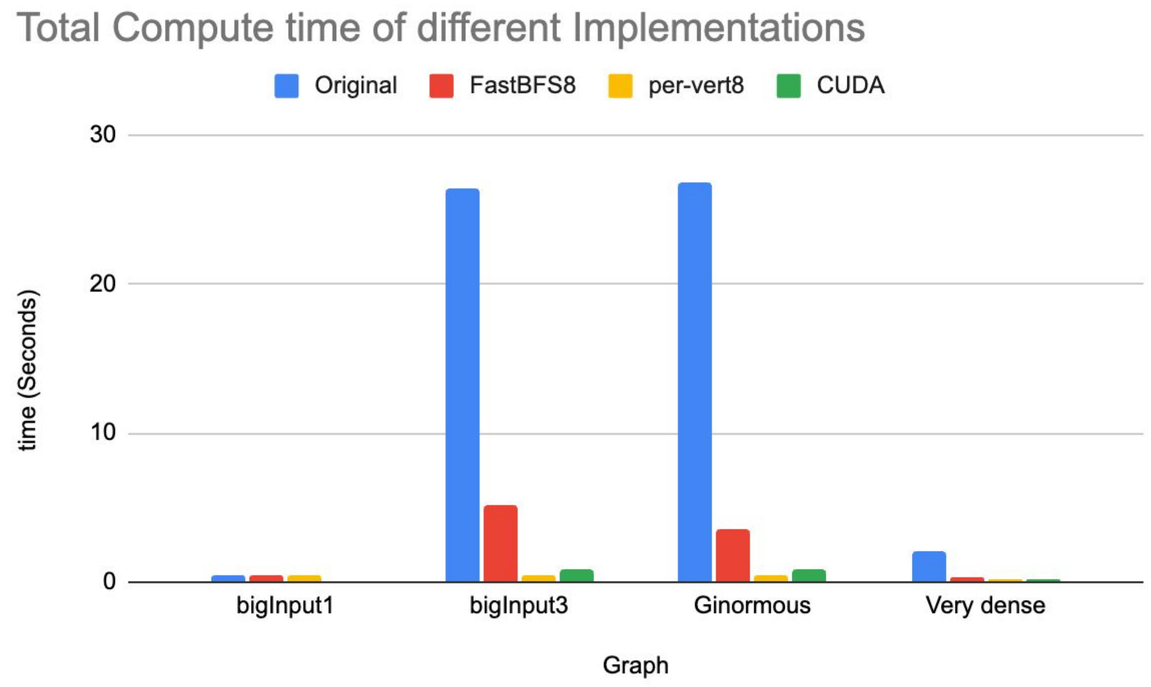
Node Yes -----> Set next frontier to true for each neighbor

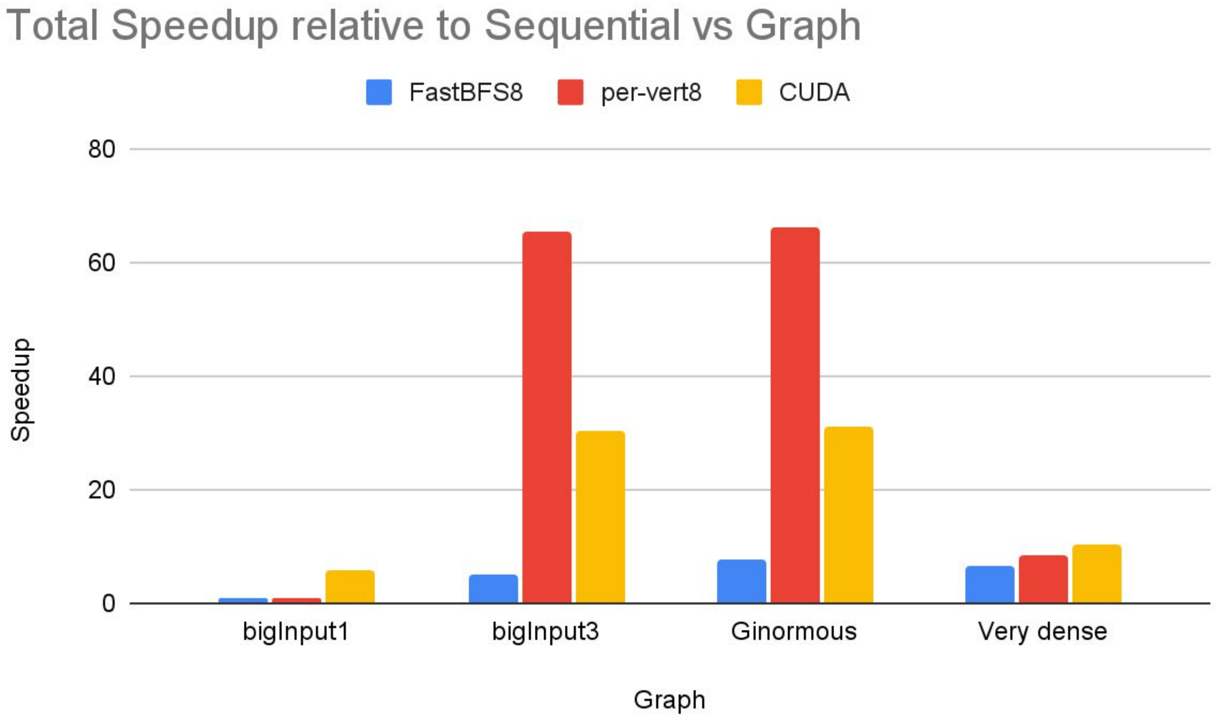
Node No, return

Node Yes -----> Set next frontier to true for each neighbor



Implementation Performance

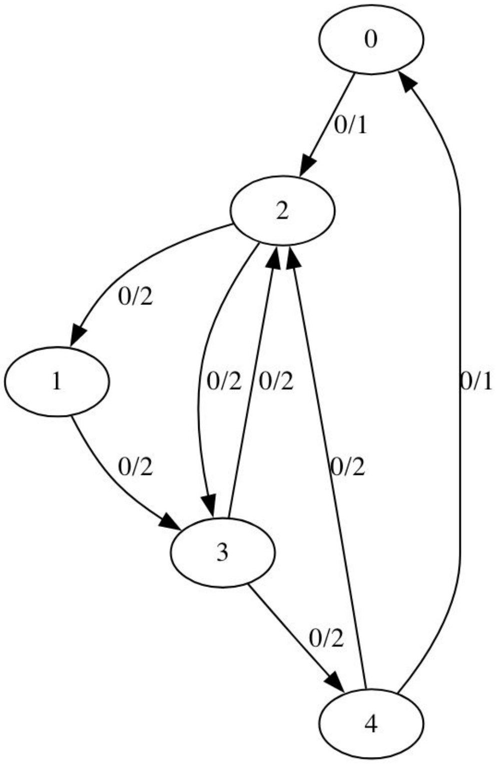




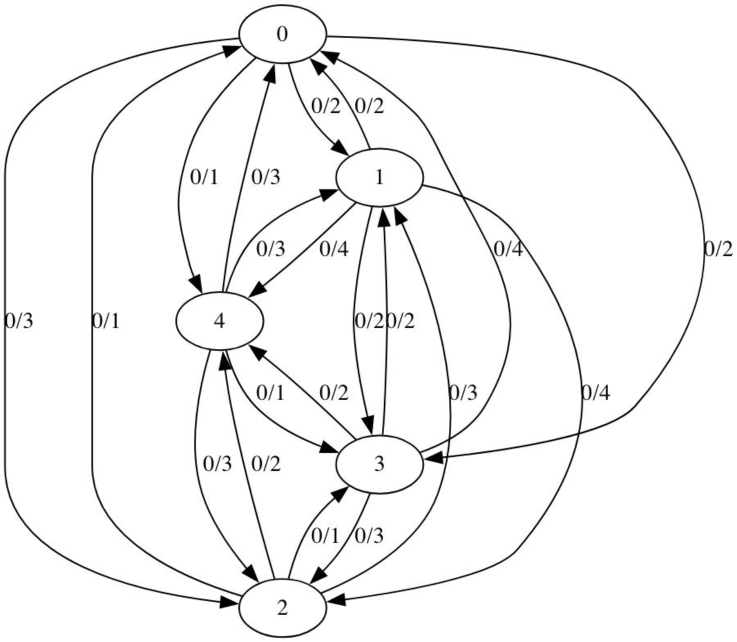
Graph Characteristics

- CUDA always outperforms FastBFS
- Per-Vertex outperforms CUDA in some cases

Sparse Graph
FastBFS does well



Dense Graph
Per-Vertex does well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