3.26齐次变换矩阵

In[1]:=
```
homoT =
{{Cos[theta], -Sin[theta], 0, a},
 {Sin[theta] * Cos[alpha], Cos[theta] * Cos[alpha], -Sin[alpha], -Sin[alpha] * d},
 {Sin[theta] * Sin[alpha], Cos[theta] * Sin[alpha], Cos[alpha], Cos[alpha] * d},
 {0, 0, 0, 1}};
T[α_, aa_, dd_, θ_] := homoT /. {theta → θ, alpha → α, a → aa, d → dd};
```

每个连杆上的齐次变换矩阵分别为：

In[24]:=
```
T0to1 = T[0 Degree, 0, 795.5, theta1]
T1to2 = T[90 Degree, 250, 0, theta2 + 90 Degree]
T2to3 = T[0 Degree, 950, 0, theta3]
T3to4 = T[90 Degree, 300, 1550, 90 Degree + theta4]
T4to5 = T[-45 Degree, 0, 98.7 * √2, theta5]
T5to6 = T[45 Degree, 0, 180 - 98.7, theta6]
```

Out[24]=
$$\begin{pmatrix} \cos(\text{theta1}) & -\sin(\text{theta1}) & 0 & 0 \\ \sin(\text{theta1}) & \cos(\text{theta1}) & 0 & 0. \\ 0 & 0 & 1 & 795.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[25]=
$$\begin{pmatrix} -\sin(\text{theta2}) & -\cos(\text{theta2}) & 0 & 250 \\ 0 & 0 & -1 & 0 \\ \cos(\text{theta2}) & -\sin(\text{theta2}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[26]=
$$\begin{pmatrix} \cos(\text{theta3}) & -\sin(\text{theta3}) & 0 & 950 \\ \sin(\text{theta3}) & \cos(\text{theta3}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[27]=
$$\begin{pmatrix} -\sin(\text{theta4}) & -\cos(\text{theta4}) & 0 & 300 \\ 0 & 0 & -1 & -1550 \\ \cos(\text{theta4}) & -\sin(\text{theta4}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[28]=
$$\begin{pmatrix} \cos(\text{theta5}) & -\sin(\text{theta5}) & 0 & 0 \\ \frac{\sin(\text{theta5})}{\sqrt{2}} & \frac{\cos(\text{theta5})}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 98.7 \\ -\frac{\sin(\text{theta5})}{\sqrt{2}} & -\frac{\cos(\text{theta5})}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 98.7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[29]=
$$\begin{pmatrix} \cos(\text{theta6}) & -\sin(\text{theta6}) & 0 & 0 \\ \frac{\sin(\text{theta6})}{\sqrt{2}} & \frac{\cos(\text{theta6})}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -57.4878 \\ \frac{\sin(\text{theta6})}{\sqrt{2}} & \frac{\cos(\text{theta6})}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 57.4878 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

于是正运动学解为

In[30]:=
```
T0to6 = T0to1.T1to2.T2to3.T3to4.T4to5.T5to6 // Simplify
```
化简

Out[30]=

$$\frac{1}{2}(\cos(\text{theta4})(\cos(\text{theta1})\sin(\text{theta2}+\text{theta3})(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6})+\sin(\text{theta6}))+\cos$$

$$\frac{1}{2}(\sin(\text{theta1})(\cos(\text{theta3})(\cos(\text{theta5})(2\sin(\text{theta2})\sin(\text{theta4})\cos(\text{theta6})+\sin(\text{theta6})(\sin(\text{theta2})\cos(\text{the}$$

$$\frac{1}{2}(\sin(\text{theta2})(\cos(\text{theta}$$

上面那种形式显示不全，下面是完全的结果：

In[31]:=
```
Grid[T0to6]
```
格子

$$\frac{1}{2}(\cos(\text{theta4})(\cos(\text{theta1})\sin(\text{theta2}+\text{theta3})(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6})+\sin(\text{theta6}))+\cos(\text{theta5})(\cos(\text{theta1})\sin(\text{theta6})\sin(\text{theta2}+\text{theta3})+2\sin(\text{theta1})\cos(\text{theta6}))-\sqrt{2}\sin(\text{theta1})\sin(\text{theta5})\sin(\text{theta6}))+\cos(\text{theta1})(\sin(\text{theta3})(\cos(\text{theta2})\sin(\text{theta4})(2\cos(\text{theta5})\cos(\text{theta6})-\sqrt{2}\sin(\text{theta5})\sin(\text{theta6}))+\sin(\text{theta2})(\sqrt{2}$$

$$\frac{1}{2}(-\cos(\text{theta1})(\sin(\text{theta2})(\cos(\text{theta3})(\sin(\text{theta4})(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6})+2\cos(\text{theta5})\sin(\text{theta6}))+\cos(\text{theta4})(\sqrt{2}\sin(\text{theta5})\sin(\text{theta6})-(\cos(\text{theta5})+1)\cos(\text{theta6})))+\sin(\text{theta3})(\sqrt{2}\sin(\text{theta5})\sin(\text{theta6})-\cos(\text{theta5})\cos(\text{theta6})+\cos(\text{theta6})))+\cos(\text{theta2})(\sin(\text{theta3})(\sin(\text{theta4})(\sqrt{2}$$

$$\frac{1}{2\sqrt{2}}(\cos(\text{theta1})(2\cos(\text{theta2})(\sin(\text{theta3})(\sin(\text{theta4})\sin(\text{theta5})+\sqrt{2}\cos(\text{theta4})\sin^2(\tfrac{\text{theta5}}{2}))+\sqrt{2}\cos(\text{theta3})\cos^2(\tfrac{\text{theta5}}{2}))-\sin(\text{theta2})(\sqrt{2}\sin(\text{theta3})(\cos(\text{theta5})+1)-2\cos(\text{theta3})(\sin(\text{theta4})\sin(\text{theta5})+\sqrt{2}\cos(\text{theta4})\sin^2(\tfrac{\text{theta5}}{2}))))+\sin(\text{theta1})(2\cos(\text{theta4})\sin(\text{theta5})-2\sqrt{2}\sin(\text{theta4})$$

$$\cos(\text{theta1})(\sin(\text{theta2})(\cos(\text{theta3})(\cos(\text{theta4})(139.35-40.65\cos(\text{theta5}))+57.4878\sin(\text{theta4})\sin(\text{theta5})-300.)+\sin(\text{theta3})(-40.65\cos(\text{theta5})-1689.35)-950.)+\cos(\text{theta2})(\sin(\text{theta3})(\cos(\text{theta4})(139.35-40.65\cos(\text{theta5}))+57.4878\sin(\text{theta4})\sin(\text{theta5})-300.)+\cos(\text{theta3})(40.65\cos(\text{theta5})+1689.35))+250.)+\sin(\text{theta1})(\sin(\text{theta4})(40.65\cos(\text{theta5})-139.35)+$$

` $\sin(theta5)\cos(theta6) + (\cos(theta5) - 1)\sin(theta6)\big)\big) - \cos(theta3)\big(\sqrt{2}\sin(theta2)\sin(theta4)\sin(theta5)\sin(theta6) + \cos(theta5)(\cos(theta2)\sin(theta6) - 2\sin(theta2)\sin(theta4)\cos(theta6)) + \sqrt{2}\cos(theta2)\sin(theta5)\cos(theta6) - \cos(theta2)\sin(theta6)\big)\big) - \sin(theta1)\sin(theta4)\big(\sqrt{2}\sin(theta5)\cos(theta6) + (\cos(theta5) + 1)\sin(theta6)\big)\big)$	$\sin(theta5)\cos(theta6) + 2\cos(theta5)\sin(theta6)\big) + \cos(theta4)\big(\sqrt{2}\sin(theta5)\sin(theta6) - (\cos(theta5) + 1)\cos(theta6)\big)\big) + \cos(theta3)\big((\cos(theta5) - 1)\cos(theta6) - \sqrt{2}\sin(theta5)\sin(theta6)\big)\big)\big) - \sin(theta1)\big(\cos(theta6)\big(\sin(theta4)\cos(theta5) + \sqrt{2}\cos(theta4)\sin(theta5) + \sin(theta4)\big) + \sin(theta6)\big(2\cos(theta4)\cos(theta5) - \sqrt{2}\sin(theta4)\sin(theta5)\big)\big)\big)$	$\sin^2\big(\frac{theta5}{2}\big)\big)\big)$	$57.4878\cos(theta4)\sin(theta5))$
$\frac{1}{2}\big(\sin(theta1)$	$\frac{1}{2}\big(\sin(theta1)\sin(theta2)$	$\cos(theta2)$	$\sin(theta1)(\sin(theta2)$

Out[31]=

$$(\cos(\text{theta3})\,(\cos(\text{theta5})\,(2\sin(\text{theta2})\sin(\text{theta4})\cos(\text{theta6}) + \sin(\text{theta6})\,(\sin(\text{theta2})\cos(\text{theta4}) - \cos(\text{theta2}))) + \sin(\text{theta2})\,(\cos(\text{theta4})\,(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6}) + \sin(\text{theta6})) - \sqrt{2}\sin(\text{theta4})\sin(\text{theta5})\sin(\text{theta6})) + \cos(\text{theta2})\,(\sin(\text{theta6}) - \sqrt{2}\sin(\text{theta5})\cos(\text{theta6}))) + \sin(\text{theta3})\,(\cos(\text{theta2})\,(-\sqrt{2}\sin(\text{theta4})\sin(\text{theta5})\sin(\text{theta6}) + \cos(\text{theta4})\,(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6}) + \sin(\text{theta6}))) + \cos(\text{theta5})\,(2\sin(\text{theta4})\cos(\text{theta6}) + \cos(\text{theta4})$$

$$\cos(\text{theta3})\,(\cos(\text{theta4})\,(-\sqrt{2}\sin(\text{theta5})\sin(\text{theta6}) + \cos(\text{theta5})\cos(\text{theta6}) + \cos(\text{theta6})) - \sin(\text{theta4})\,(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6}) + 2\cos(\text{theta5})\sin(\text{theta6}))) + \sin(\text{theta1})\cos(\text{theta2})\,(\sin(\text{theta3})\,(\cos(\text{theta4})\,(-\sqrt{2}\sin(\text{theta5})\sin(\text{theta6}) + \cos(\text{theta5})\cos(\text{theta6}) + \cos(\text{theta6})) - \sin(\text{theta4})\,(\sqrt{2}\sin(\text{theta5})\cos(\text{theta6}) + 2\cos(\text{theta5})\sin(\text{theta6}))) + \cos(\text{theta3})\,(\sqrt{2}\sin(\text{theta5})\sin(\text{theta6}) - \cos(\text{theta5})\cos(\text{theta6}) + \cos(\text{theta6}))) -$$

$$\left(\frac{1}{\sqrt{2}}\sin(\text{theta1})\sin(\text{theta3})\sin(\text{theta4})\sin(\text{theta5}) + \sin(\text{theta1})\sin(\text{theta3})\cos(\text{theta4})\sin^2\left(\frac{\text{theta5}}{2}\right) + \sin(\text{theta1})\cos(\text{theta3})\cos^2\left(\frac{\text{theta5}}{2}\right)\right) + \sin(\text{theta1})\sin(\text{theta2})\cos(\text{theta3})\cos(\text{theta4})\sin^2\left(\frac{\text{theta5}}{2}\right) + \frac{1}{\sqrt{2}}\sin(\text{theta1})\sin(\text{theta2})\cos(\text{theta3})\sin(\text{theta4})\sin(\text{theta5}) - \sin(\text{theta1})\sin(\text{theta2})\sin(\text{theta3})\cos^2\left(\frac{\text{theta5}}{2}\right) + \cos(\text{theta1})\sin(\text{theta4})\sin^2\left(\frac{\text{theta5}}{2}\right) - \frac{1}{\sqrt{2}}\cos(\text{theta1})\cos(\text{theta4})\sin(\text{theta5})$$

$$(\cos(\text{theta3})\,(\cos(\text{theta4})\,(139.35 - 40.65\cos(\text{theta5})) + 57.4878\sin(\text{theta4})\sin(\text{theta5}) - 300.) + \sin(\text{theta3})\,(-40.65\cos(\text{theta5}) - 1689.35) - 950.) + \cos(\text{theta2})\,(\sin(\text{theta3})\,(\cos(\text{theta4})\,(139.35 - 40.65\cos(\text{theta5})) + 57.4878\sin(\text{theta4})\sin(\text{theta5}) - 300.) + \cos(\text{theta3})\,(40.65\cos(\text{theta5}) + 1689.35)) + 250.) + \cos(\text{theta1})\,(\sin(\text{theta4})\,(139.35 - 40.65\cos(\text{theta5})) - 57.4878\cos(\text{theta4})\sin(\text{theta5}))$$

cos(theta4)
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cos(theta6) +
(cos(
theta5) − 1)
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theta6)))) +
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(sin(
theta4)
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sin(theta5)
cos(theta6) +
(cos(
theta5) + 1)
sin(
theta6)) +
cos(
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2 cos(theta5)
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theta6) +
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theta2)
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theta3)
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theta5) cos(
theta6) + sin(
theta6)) –
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theta6))) –

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(cos(theta4)
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theta6)) –
sin(theta4)
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cos(theta6) +
2 cos(theta5)
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theta6))) +
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cos(theta5)
cos(theta6) +
cos(
theta6))) +
cos(theta2)

$\frac{1}{2}$(cos(theta3)
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$\cos^2\left(\frac{theta5}{2}\right)$ –
cos(theta2)
($\sqrt{2}$
sin(theta4)
sin(theta5) +
2 cos(theta4)
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$\frac{theta5}{2}$))) +
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2 cos(theta4)
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40.65 sin(theta2)
sin(theta3)
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1689.35) +
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139.35 sin(
theta2)
sin(theta3)
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theta3)
(40.65 cos(theta5) +

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theta6))) +
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sin(theta5)
cos(theta6) +
(cos(
theta5) − 1)
sin(theta6))))

(cos(
theta3)
(sin(theta4)
(√2
sin(theta5)
cos(theta6) +
2 cos(theta5)
sin(
theta6)) +
cos(theta4)
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sin(theta6) −
(cos(
theta5) + 1)
cos(
theta6))) +
sin(theta3)
(√2
sin(theta5)
sin(theta6) −
cos(theta5)
cos(theta6) +
cos(
theta6))))

1689.35) −
300. sin(theta2)
sin(
theta3) + 795.5

0.	0.	0.	1.