正运动学解

连杆上的齐次变换矩阵公式如下：

In[2]:=
```
homoT =
  ( Cos[theta]                -Sin[theta]                0              a
    Sin[theta] * Cos[alpha]   Cos[theta] * Cos[alpha]    -Sin[alpha]    -Sin[alpha] * d
    Sin[theta] * Sin[alpha]   Cos[theta] * Sin[alpha]    Cos[alpha]     Cos[alpha] * d
    0                         0                          0              1 );
T[α_, aa_, dd_, θ_] := homoT /. {theta → θ, alpha → α, a → aa, d → dd};
```

In[5]:=
```
T[0, 4, 0, 60 Degree] // N
```
度　数值运算

Out[5]=

$$\begin{pmatrix} 0.5 & -0.866025 & 0. & 4. \\ 0.866025 & 0.5 & 0. & 0. \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

已知参数如下：

In[7]:=
```
l1 = 4; l2 = 3; l3 = 2;
```

每一个连杆上的其次变换矩阵分别为：

In[10]:=
```
T0to1 = T[0, 0, 0, theta1]
```

Out[10]=

$$\begin{pmatrix} \cos(\text{theta1}) & -\sin(\text{theta1}) & 0 & 0 \\ \sin(\text{theta1}) & \cos(\text{theta1}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[11]:=
```
T1to2 = T[0, l1, 0, theta2]
```

Out[11]=

$$\begin{pmatrix} \cos(\text{theta2}) & -\sin(\text{theta2}) & 0 & 4 \\ \sin(\text{theta2}) & \cos(\text{theta2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[12]:=
```
T2to3 = T[0, l2, 0, theta3]
```

Out[12]=

$$\begin{pmatrix} \cos(\text{theta3}) & -\sin(\text{theta3}) & 0 & 3 \\ \sin(\text{theta3}) & \cos(\text{theta3}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

末端的常量矩阵为

In[13]:=
```
T3toH = T[0, l3, 0, 0]
```

Out[13]=

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

因此，0到3的正运动学解为：

In[15]:=
```
T0to3 = (T0to1 . T1to2 . T2to3) // Simplify
                                   化简
```

Out[15]=
$$\begin{pmatrix} \cos(\text{theta1}+\text{theta2}+\text{theta3}) & -\sin(\text{theta1}+\text{theta2}+\text{theta3}) & 0 & 3\cos(\text{theta1}+\text{theta2})+4\cos(\text{theta1}) \\ \sin(\text{theta1}+\text{theta2}+\text{theta3}) & \cos(\text{theta1}+\text{theta2}+\text{theta3}) & 0 & 3\sin(\text{theta1}+\text{theta2})+4\sin(\text{theta1}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

0到H的正运动学解为：

In[16]:=
```
T0toH = (T0to1 . T1to2 . T2to3.T3toH) // Simplify
                                         化简
```

Out[16]=
$$\begin{pmatrix} \cos(\text{theta1}+\text{theta2}+\text{theta3}) & -\sin(\text{theta1}+\text{theta2}+\text{theta3}) & 0 & 2\cos(\text{theta1}+\text{theta2}+\text{theta3})+3\cos(\text{thet} \\ \sin(\text{theta1}+\text{theta2}+\text{theta3}) & \cos(\text{theta1}+\text{theta2}+\text{theta3}) & 0 & 2\sin(\text{theta1}+\text{theta2}+\text{theta3})+3\sin(\text{thet} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

若是θ1=0°, θ2=0°, θ3=0°

In[23]:=
```
T0to3 /. {theta1 → 0 Degree, theta2 → 0 Degree, theta3 → 0 Degree}
                     度                   度                   度
T0toH /. {theta1 → 0 Degree, theta2 → 0 Degree, theta3 → 0 Degree}
                     度                   度                   度
```

Out[23]=
$$\begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[24]=
$$\begin{pmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

若是θ1=10°, θ2=20°, θ3=30°

In[25]:=
```
T0to3 /. {theta1 → 10 Degree, theta2 → 20 Degree, theta3 → 30 Degree} // N
                      度                    度                    度       数
T0toH /. {theta1 → 10 Degree, theta2 → 20 Degree, theta3 → 30 Degree} // N
                      度                    度                    度       数
```

Out[25]=
$$\begin{pmatrix} 0.5 & -0.866025 & 0. & 6.53731 \\ 0.866025 & 0.5 & 0. & 2.19459 \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

Out[26]=
$$\begin{pmatrix} 0.5 & -0.866025 & 0. & 7.53731 \\ 0.866025 & 0.5 & 0. & 3.92664 \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

若是θ1=90°, θ2=90°, θ3=90 °

In[27]:=
```
T0to3 /. {theta1 → 90 Degree, theta2 → 90 Degree, theta3 → 90 Degree}
                   度                   度                    度
T0toH /. {theta1 → 90 Degree, theta2 → 90 Degree, theta3 → 90 Degree}
                   度                   度                    度
```

Out[27]=

$$\begin{pmatrix} 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[28]=

$$\begin{pmatrix} 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$