EM算法求解GMM模型参数

1
```
# 导入各个module
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import os

sns.set_theme(style="darkgrid")

current_path = os.getcwd()
output_path = os.path.join(current_path, 'output')
```

数据预处理与可视化

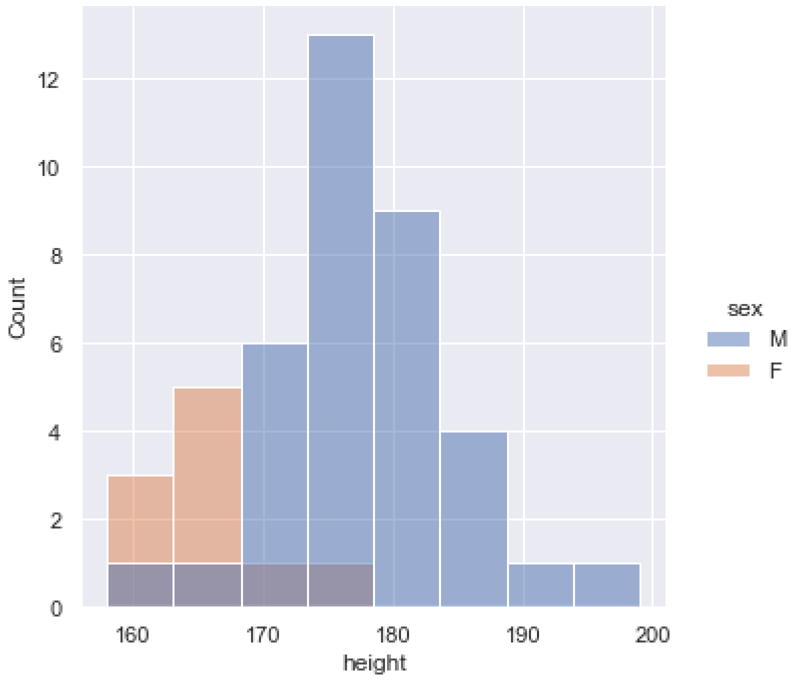
导入数据

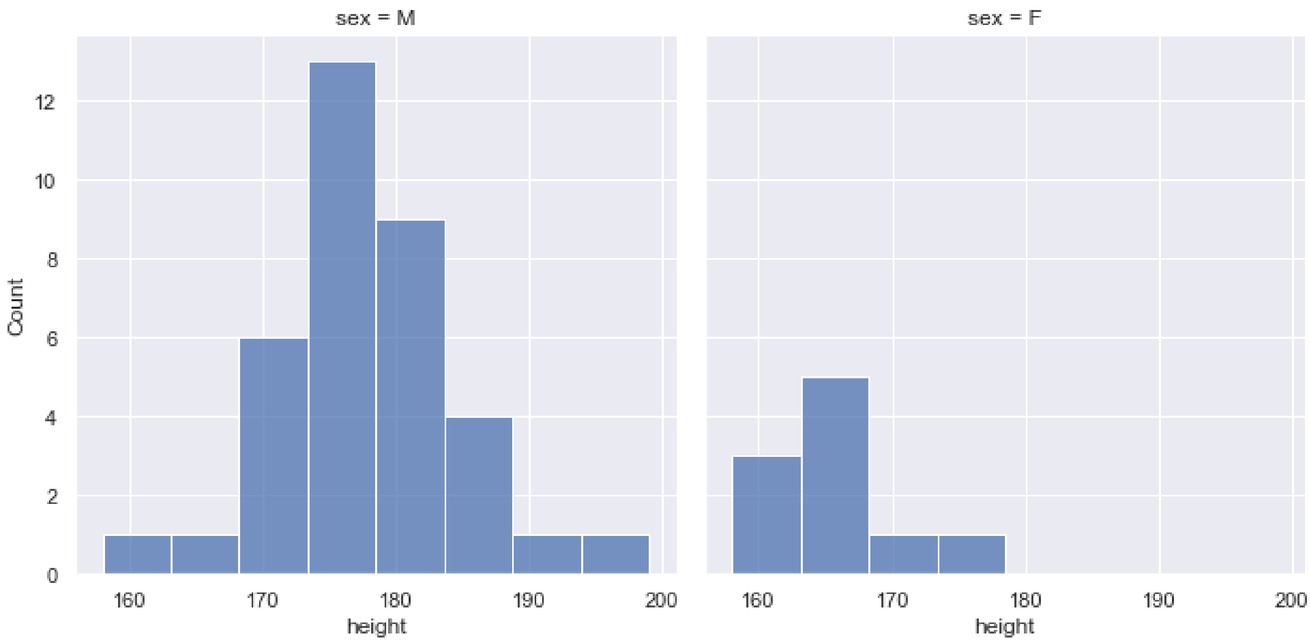
2
```
df = pd.read_excel('hw1_data.xlsx', header=None)
df.columns = ['sex', 'height']
male_df = df[df['sex']=='M']
female_df = df[df['sex']=='F']
```

可视化

男女生身高的频数直方图

3
```
hist_fig1 = sns.displot(df, x='height', hue='sex')
hist_fig2 = sns.displot(df, x='height', col='sex')

hist_fig_name = 'hist_fig.pdf'
output_fig_path = os.path.join(output_path, hist_fig_name)
hist_fig1.savefig(output_fig_path, dpi=400)
```

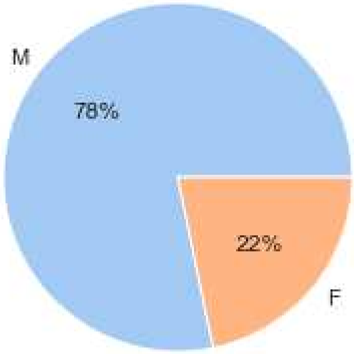




使用饼图显示男女生比例

4

```
sex_cnt = df['sex'].value_counts()
#define Seaborn color palette to use
colors = sns.color_palette('pastel')[0:5]
#create pie chart
plt.pie(sex_cnt.values, labels = list(sex_cnt.index), colors = colors, autopct='%.0f%%')

pie_fig_name = 'pie_fig.pdf'
output_fig_path = os.path.join(output_path, pie_fig_name)
plt.savefig(output_fig_path, dpi=400, bbox_inches='tight')

plt.show()
```



计算

1. 取参数的初始值开始迭代
2. E步：根据当前模型参数，计算分模型k对观测数据y_j的响应度$\hat{\gamma}_{jk}$

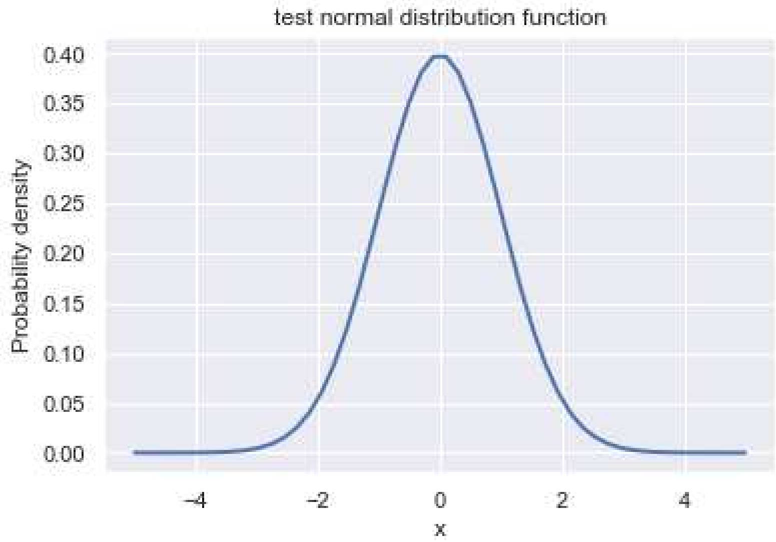
$$\hat{\gamma}_{jk} = \frac{\alpha_k \phi(y_j|\theta_k)}{\sum_{k=1}^{K} \alpha_k \phi(y_j|\theta_k)}$$

3. M步：计算新一轮迭代的模型参数
4. 重复上面两步直到收敛

正态分布函数

5

```
def normal_dist(x, mu, sigma):
    """
    pdf of normal distribution
    :param x:
    :param mu: mean
    :param sigma: variance
    :return:
    """
    a = 1.0 / sigma / np.sqrt(2*np.pi)
    exp = -1.0/2.0*((x-mu)/(sigma))**2
    exp = np.exp(exp)
    return a*exp

test_x = np.linspace(-5, 5)
test_y = normal_dist(test_x, 0.0, 1.0)

# plot
fig, ax = plt.subplots()
ax.plot(test_x, test_y, linewidth=2.0)
ax.set_title('test normal distribution function')
ax.set_xlabel('x')
ax.set_ylabel('Probability density')
plt.show()
```



Membership probabilities

6
```
def membership_prob(x,
                    mu:list,
                    sigma:list,
                    tau:list,
                    idx):
    """
    For the i th data, compute the probability that it belongs to
    the j th model, based on current parameters
    :param x: value of the i th data
    :param mu: list of mean values of all models
    :param sigma: list of variant values of all models
    :param tau: list of weights
    :param idx: index of the model
    :return:
    """
    if len(mu)==len(sigma) and len(sigma)==len(tau):
        pass
    else:
        raise ValueError("The length of the input values is not identical")
    if idx >= len(mu):
        raise IndexError("The index is larger than the number of models")

    denominator = 0
    for j in range(len(mu)):
        denominator += tau[j]*normal_dist(x, mu[j], sigma[j])
    numerator = tau[idx] * normal_dist(x, mu[idx], sigma[idx])

    return numerator/denominator
```

New parameters

7
```
def new_tau(gamma:np.ndarray):
    return np.mean(gamma)

def new_mu(x:np.ndarray, gamma:np.ndarray):
    """
    Calculate the mean of j th model, using all the data
    :param x:
    :param gamma:
    :return:
    """
    if x.size == gamma.size:
        numerator = x * gamma # do multiply element wise
```

```
        numerator = np.sum(numerator)
        denominator = np.sum(gamma)
    else:
        raise ValueError("The length of x and gamma should be the same when calculating new mu")
    return numerator/denominator

def new_sigma(x:np.ndarray, gamma:np.ndarray, mu):
    """
    Calculate the variant of j th model, using all the data
    :param x:
    :param gamma:
    :param mu:
    :return:
    """
    if x.size == gamma.size:
        numerator = x - mu
        numerator = numerator**2
        numerator = numerator * gamma
        numerator = np.sum(numerator)
        denominator = np.sum(gamma)
    else:
        raise ValueError("The length of x and gamma should be the same when calculating new sigma")

    return np.sqrt(numerator/denominator)
```

EM算法计算

66

```
def run(max_step, mu1, mu2, sigma1, sigma2, tau1):
    tau2 = 1 - tau1
    stat_label = ['mu1', 'mu2', 'sig1', 'sig2', 'tau1', 'tau2']
    stat_data = [mu1, mu2, sigma1, sigma2, tau1, tau2]
    stat_cnt = [0]*6
    gamma1 = np.zeros(shape=height_data.values.shape)
    gamma2 = np.zeros(shape=height_data.values.shape)
    for i in range(max_step):
        # E STEP
        for j in range(len(height_data.values)):
            gamma1[j] = membership_prob(height_data[j], [mu1, mu2], [sigma1, sigma2], [tau1, tau2], 0
            gamma2[j] = membership_prob(height_data[j], [mu1, mu2], [sigma1, sigma2], [tau1, tau2], 1
        # M step
        mu1 = new_mu(height_data.values, gamma1)
        mu2 = new_mu(height_data.values, gamma2)
        sigma1 = new_sigma(height_data.values, gamma1, mu1)
        sigma2 = new_sigma(height_data.values, gamma2, mu2)
        tau1 = new_tau(gamma1)
        tau2 = new_tau(gamma2)
        # record the parameter
        stat_label = stat_label+['mu1', 'mu2', 'sig1', 'sig2', 'tau1', 'tau2']
        stat_data = stat_data + [mu1, mu2, sigma1, sigma2, tau1, tau2]
        stat_cnt = stat_cnt + [i+1]*6
    stat = pd.DataFrame({'step':stat_cnt,'label':stat_label, 'data':stat_data})

    print("EM algorithm for GMM model -- Rsult:")
    print("mu1=",mu1,"mu2=",mu2)
    print("sigma1=",sigma1, "sigma2=",sigma2)
    print("tau1=",tau1, "tau2=",tau2)

    test = pd.DataFrame({"$\mu_1$":[mu1],"$\mu_2$":[mu2], "$\sigma_1$":[sigma1], "$\sigma_2$":[sigma2
    print(test)
    return mu1, mu2, sigma1, sigma2, tau1, tau2, stat
```

69

```
def draw_result(mu1, mu2, sigma1, sigma2, filename):
    x_M = np.linspace(155, 200)
    x_F = np.linspace(150, 200)
    pdf_M = normal_dist(x_M, mu1, sigma1)
```

```
    pdf_F = normal_dist(x_F, mu2, sigma2)
    # fig = plt.figure()
    fig, ax = plt.subplots()
    sns.histplot(ax=ax, data=df, x='height', hue='sex', stat="density", common_norm=False)
    sns.lineplot(ax=ax, x=x_M, y=pdf_M)
    sns.lineplot(ax=ax, x=x_F,y=pdf_F)
    ax.set_title("Result")
    filename = os.path.join(output_path, filename)
    fig.savefig(filename, dpi=400)

def draw_history(stat, filename):
    fig, axes = plt.subplots(2, 3, figsize=(15, 8), sharex=True)
    fig.suptitle('History of params during iteration')
    sns.lineplot(ax=axes[0,0], data=stat.query("label=='mu1'"), x="step", y="data", marker='o')
    axes[0,0].set_title('$\mu_1$')
    sns.lineplot(ax=axes[1,0], data=stat.query("label=='mu2'"), x="step", y="data", marker='o')
    axes[1,0].set_title('$\mu_2$')
    sns.lineplot(ax=axes[0,1], data=stat.query("label=='sig1'"), x="step", y="data", marker='o')
    axes[0,1].set_title('$\sigma_1$')
    sns.lineplot(ax=axes[1,1], data=stat.query("label=='sig2'"), x="step", y="data", marker='o')
    axes[1,1].set_title('$\sigma_2$')
    sns.lineplot(ax=axes[0,2], data=stat.query("label=='tau1'"), x="step", y="data", marker='o')
    axes[0,2].set_title('$p_1$')
    sns.lineplot(ax=axes[1,2], data=stat.query("label=='tau2'"), x="step", y="data", marker='o')
    axes[1,2].set_title('$p_2$')

    filename = os.path.join(output_path, filename)
    fig.savefig(filename, dpi=400)
```

加载数据

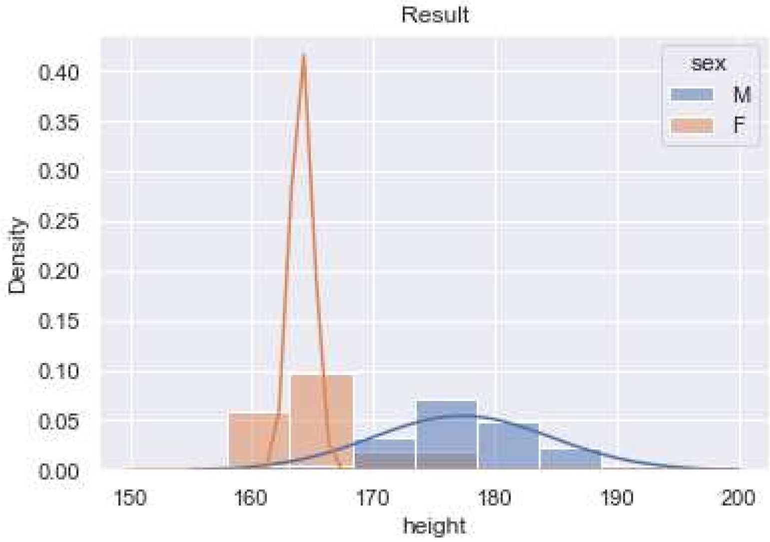
```
10 df = pd.read_csv('hw1_data.csv', header=None)
df.columns = ['sex', 'height']
height_data = df['height']
```

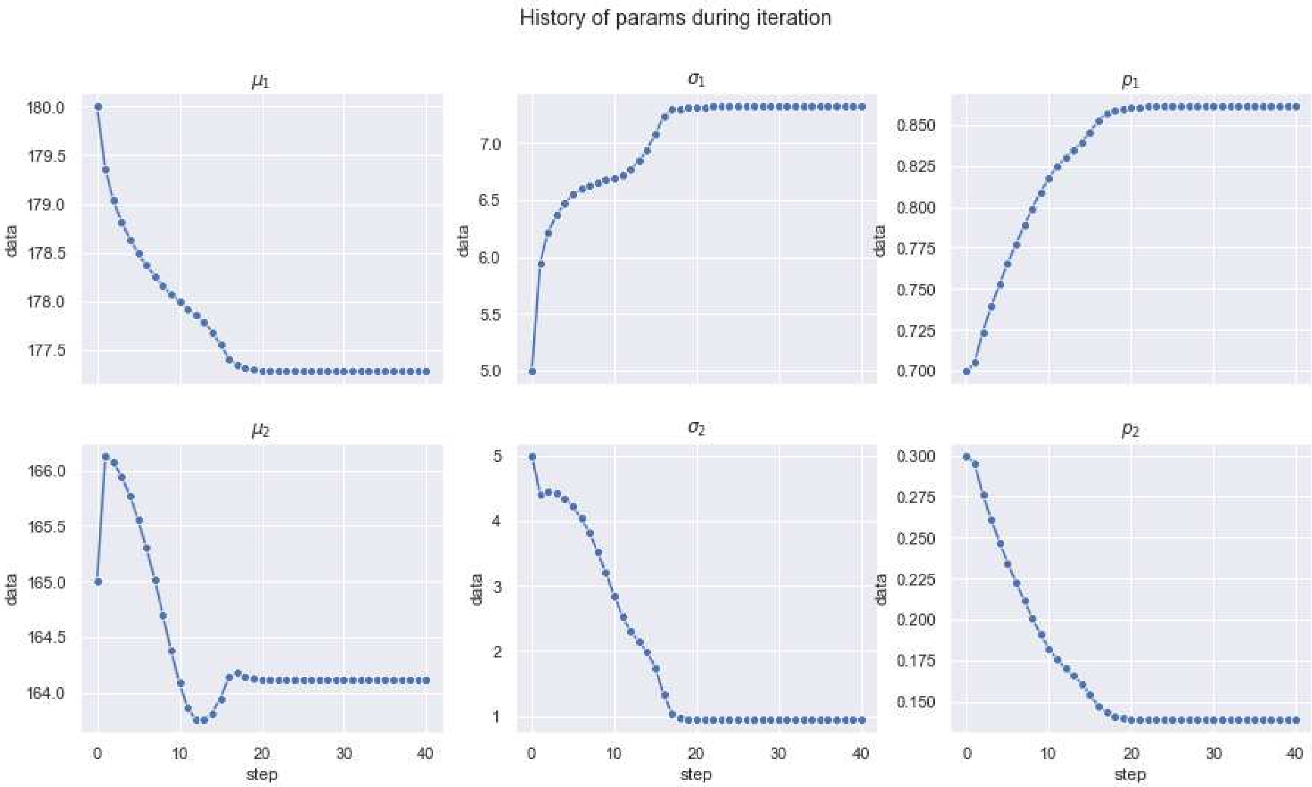
计算与结果可视化

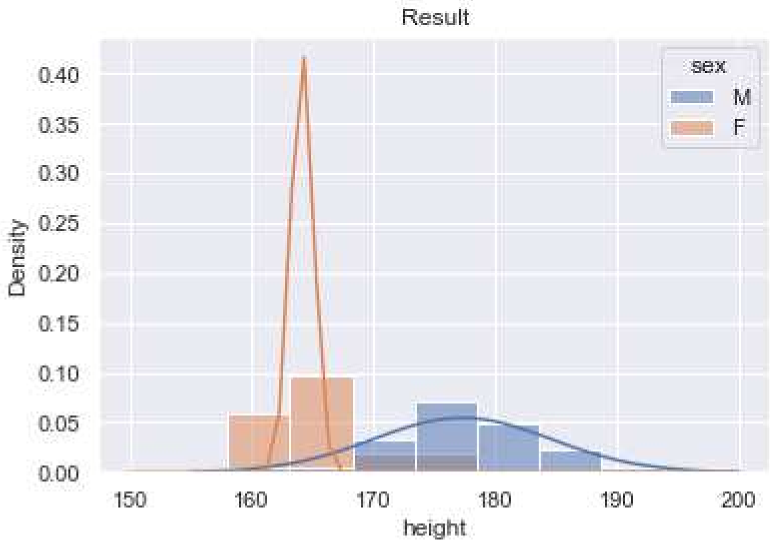
```
70 mu1, mu2, sigma1, sigma2, tau1, tau2, stat = run(max_step=40,
        mu1=180, mu2=165, sigma1=5, sigma2=5, tau1=0.7)

draw_result(mu1, mu2, sigma1, sigma2, "result1.pdf")
# 各个参数在计算过程中的变化
draw_history(stat, "history1.pdf")


mu1, mu2, sigma1, sigma2, tau1, tau2, stat = run(max_step=40,
        mu1=170, mu2=160, sigma1=5, sigma2=5, tau1=0.5)

draw_result(mu1, mu2, sigma1, sigma2, "result2.pdf")
# 各个参数在计算过程中的变化
draw_history(stat, "history2.pdf")

mu1, mu2, sigma1, sigma2, tau1, tau2, stat = run(max_step=40,
        mu1=170, mu2=160, sigma1=10, sigma2=10, tau1=0.5)

draw_result(mu1, mu2, sigma1, sigma2, "result3.pdf")
# 各个参数在计算过程中的变化
draw_history(stat, "history3.pdf")
```

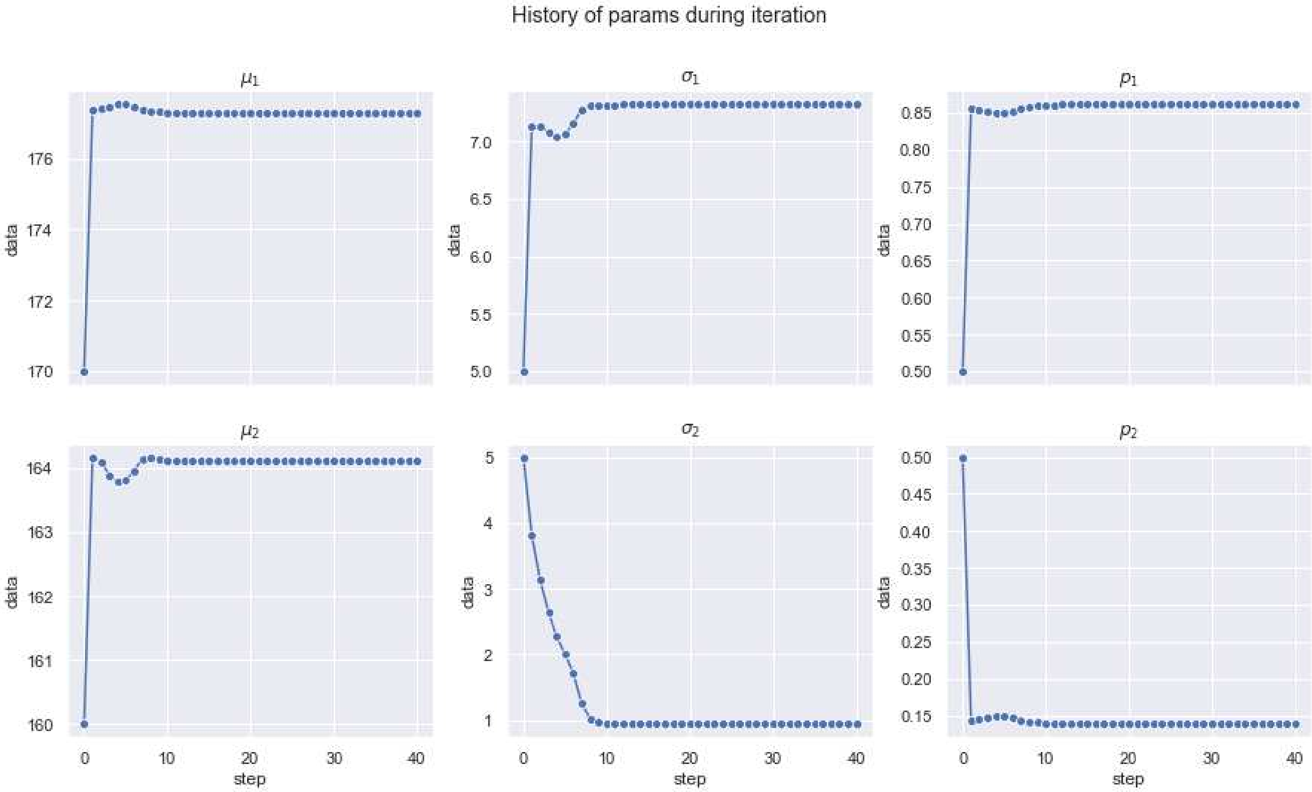
```
EM algorithm for GMM model -- Rsult:
mu1= 177.28427949725432 mu2= 164.11589378938336
```

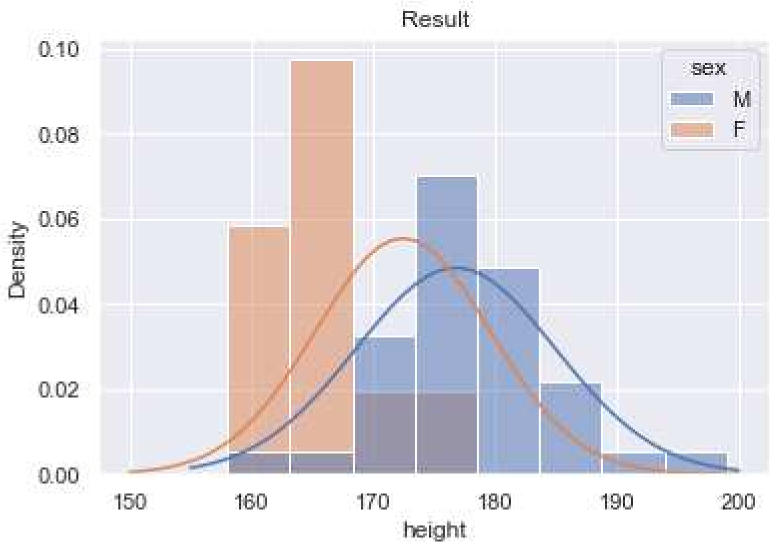
```
sigma1= 7.319264254664316 sigma2= 0.9428005047498634
tau1= 0.8612010766793227 tau2= 0.1387989233206773
      $\mu_1$     $\mu_2$  $\sigma_1$  $\sigma_2$     $p_1$     $p_2$
0  177.284279  164.115894    7.319264    0.942801  0.861201  0.138799
EM algorithm for GMM model -- Rsult:
mu1= 177.28427949660465 mu2= 164.11589378931535
sigma1= 7.319264255059287 sigma2= 0.9428005047027699
tau1= 0.8612010767225255 tau2= 0.13879892327747453
      $\mu_1$     $\mu_2$  $\sigma_1$  $\sigma_2$     $p_1$     $p_2$
0  177.284279  164.115894    7.319264    0.942801  0.861201  0.138799
EM algorithm for GMM model -- Rsult:
mu1= 176.74955961795646 mu2= 172.3986702972358
sigma1= 8.23287562075184 sigma2= 7.197744928504974
tau1= 0.7028106707591764 tau2= 0.2971893292408235
     $\mu_1$    $\mu_2$  $\sigma_1$  $\sigma_2$     $p_1$     $p_2$
0  176.74956  172.39867    8.232876    7.197745  0.702811  0.297189
```

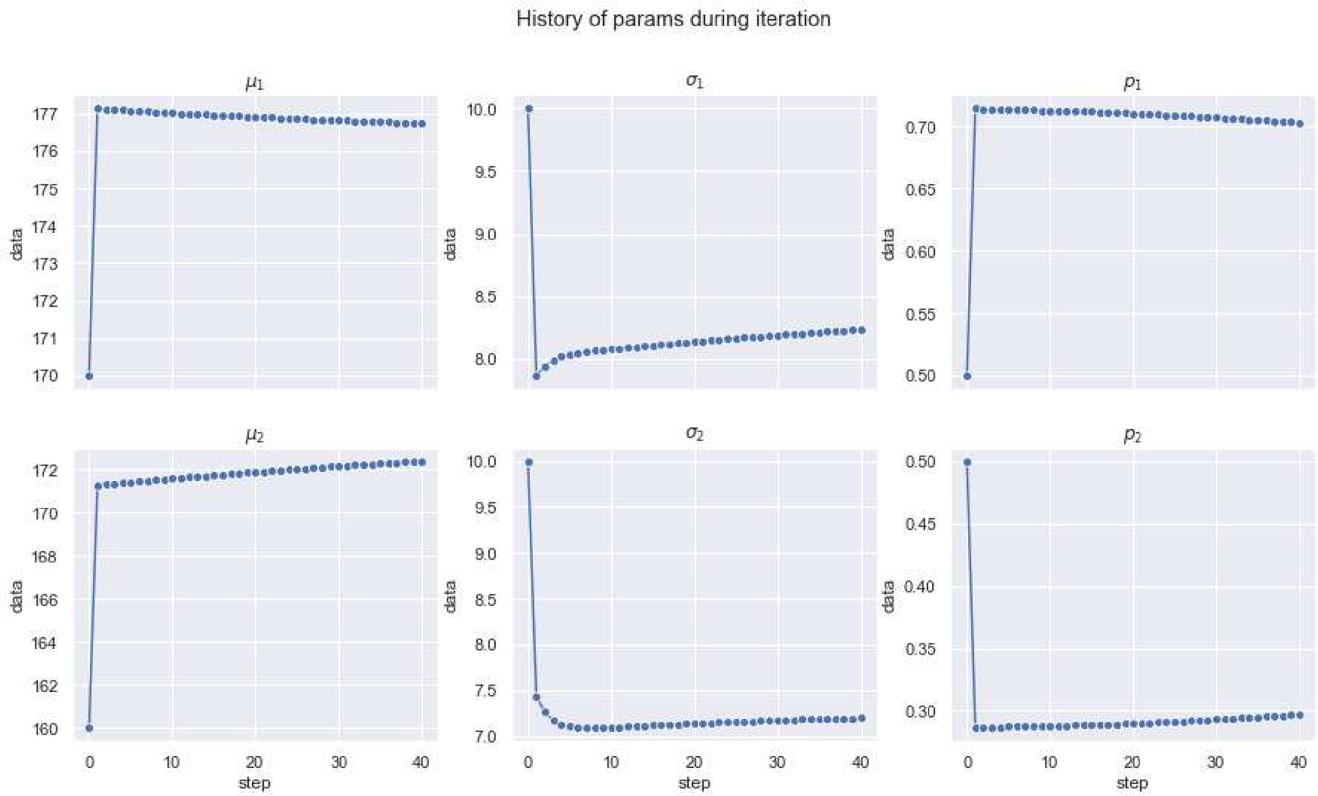





Result
sex
M
F
Density
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
150
160
170
180
190
200
height


History of params during iteration
μ_1
σ_1
p_1
μ_2
σ_2
p_2
data
step
0
10
20
30
40


Result
sex
M
F
Density
0.00
0.02
0.04
0.06
0.08
0.10
150
160
170
180
190
200
height


History of params during iteration
μ_1
σ_1
p_1
μ_2
σ_2
p_2
data
step
177
176
175
174
173
172
171
170
10.0
9.5
9.0
8.5
8.0
0.70
0.65
0.60
0.55
0.50
172
170
168
166
164
162
160
10.0
9.5
9.0
8.5
8.0
7.5
7.0
0.50
0.45
0.40
0.35
0.30
0
10
20
30
40