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CS 111 ASSIGNMENT 4

due 5/24/2011

Problem 1: Give the asymptotic value (using the Θ-notation) for the number of words that will be printed by the algorithms below. Your solution needs to consist of an appropriate recurrence equation and its solution, with a brief justification. (See the suggested format at the bottom of the assignment).

(a) **Algorithm** Jazz (n : **integer**)
    if $n = 1$
        print("jazz")
    else
        for $i \leftarrow 1$ **to** $2n$ **do** print("jazz")
        for $j \leftarrow 1$ **to** 4 **do** Jazz($\lceil n/4 \rceil$)

(b) **Algorithm** Salsa (n : **integer**)
    if $n = 1$
        print("salsa")
    else
        for $j \leftarrow 1$ **to** 8 **do** Salsa($\lfloor n/2 \rfloor$)
        for $i \leftarrow 1$ **to** n^2 **do** print("salsa")

(c) **Algorithm** Mariachi (n : **integer**)
    if $n = 1$
        print("mariachi")
    else
        Mariachi($\lceil n/4 \rceil$)
        Mariachi($\lceil n/4 \rceil$)
        Mariachi($\lceil n/4 \rceil$)
        for $i \leftarrow 1$ **to** $3n$ **do** print("mariachi")

(d) **Algorithm** Reggae (n : **integer**)
    if $n = 1$
        print("reggae")
    else
        Reggae($\lceil n/3 \rceil$)
        Reggae($\lfloor n/3 \rfloor$)
        for $i \leftarrow 1$ **to** 7 **do** print("reggae")

(e) **Algorithm** Rhumba (n : **integer**)
    if $n = 1$
        print("rhumba")
    else
        for $j \leftarrow 1$ **to** 16 **do** Rhumba($\lfloor n/4 \rfloor$)
        for $i \leftarrow 1$ **to** $2n^3$ **do** print("rhumba")

Solution 1:

a)here are 4 recursive calls, each with parameter $\lceil n/4 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$T(n) \quad = \quad 4T(n/4) + 2n.$$

We apply the Master Theorem with $a = 4$, $b = 4$, $d = 1$. Here, we have $a = b^d$, so the solution is $\Theta(nlog(n))$.

b)here are 8 recursive calls, each with parameter $\lceil n/2 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$T(n) \quad = \quad 8T(n/2) + n^2.$$

We apply the Master Theorem with $a = 8$, $b = 2$, $d = 2$. Here, we have $a > b^d$, so the solution is $\Theta(n^3)$.

c)here are 3 recursive calls, each with parameter $\lceil n/4 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$T(n) \quad = \quad 3T(n/4) + 3n.$$

We apply the Master Theorem with $a = 3$, $b = 4$, $d = 1$. Here, we have $a < b^d$, so the solution is $\Theta(n)$.

d)here are 2 recursive calls, each with parameter $\lceil n/3 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$T(n) \quad = \quad 2T(n/3) + 7.$$

We apply the Master Theorem with $a = 2$, $b = 3$, $d = 0$. Here, we have $a > b^d$, so the solution is $\Theta(n^{log_3 2})$.

e)here are 16 recursive calls, each with parameter $\lceil n/4 \rceil$. Since we are looking for an asymptotic solution, we can ignore rounding. Then the number of letters printed can be expressed by the recurrence:

$$T(n) \quad = \quad 16T(n/4) + 2n^3.$$

We apply the Master Theorem with $a = 16$, $b = 4$, $d = 3$. Here, we have $a < b^d$, so the solution is $\Theta(n^3)$.

Problem 2: Determine (using the inclusion-exclusion principle) the number of integer solutions of the equation:

$$x + y + z \quad = \quad 17,$$

under the constraints

$$\begin{array}{ccccc} 0 & \leq & x & \leq & 4 \\ 0 & \leq & y & \leq & 6 \\ 0 & \leq & z & \leq & 9 \end{array}$$

Show your work.

Solution 2:

s = solutions
⇒ s(x≤4 ∩ y6 ∩ z≤9)
⇒ s - (x≥5 ∪ y≥7 ∪ z≥10)
⇒ s -s(x≥5) -s(y≥7) -s(z≥10) +s(x≥5 ∪ y≥7) +s(x≥5 ∪ z≥10) +s(y≥5 ∪ z≥10) +(x≥5 ∪ y≥7 ∪ z≥10)
⇒ $\binom{19}{2}$ -$\binom{14}{2}$ -$\binom{12}{2}$ -$\binom{9}{2}$ +$\binom{7}{2}$ +$\binom{4}{2}$ +$\binom{2}{2}$
⇒ 171 -91 -66 -36 +21 +6 +1 = 6
Number of integer soultions of the equation: 6

Problem 3: No partner

Solution 3: No partner
