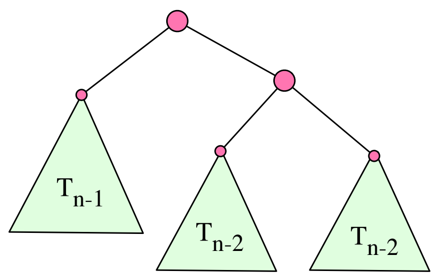
Chris Wong SID 860 923 521

CS 111 ASSIGNMENT 3

due 5/10/2011

Problem 1: We construct recursively binary trees $T_0, T_1, T_2, ...,$ as follows. Both T_0 and T_1 consist of a single node. For $n \geq 2$, to obtain T_n, we link one copy of T_{n-1} and two copies of T_{n-2}, as in the figure below:



Let b_n be the number of nodes in T_n. For example, $b_0 = 1$, $b_1 = 1$, $b_2 = 5$ and $b_3 = 9$. Give the formula for b_n. Show your work. The solution must consist of the following steps: (i) Set up a recurrence equation and give a brief justification. (ii) Give the associated homogeneous equation. (iii) Determine the characteristic equation and solve it. (iv) Give the general solution for the homogeneous equation. (v) Determine a particular solution for the non-homogeneous equation. (vi) Give the general solution for the non-homogeneous equation. (vii) Use the initial conditions to compute the final answer.

Solution 1:
i) First, lets count the number nodes and recurances in the tree. There are two nodes, one T_{n-1} recurrance, and two T_{n-2} recurannces. Now can derive the recurrance eqaution:
$b(n) = T_{n-1} + 2T_{n-2} + 2$

ii)Homogeneous equation: $Tn = T_{n-1} + 2T_{n-2}$

iii)Characteristic equation: $x^2 - x - 2 = 0$ then $(x-2)(x+1)$ So, roots = 2 and -1

iv)General soultion: b(n) = $C_1(2)^n + C_2(-1)^n + Y_p$(particular sol.)

v)Non-homogeneous equation:$Tn = T_{n-1} + 2T_{n-2} + 2 \Rightarrow A = A + 2A + 2 \Rightarrow A = -1$ So, particular solution = -1

vi)General solution = Characteristic solution + Particular solution $\Rightarrow C_1(2)^n + C_2(-1)^n - 1$

vii)Plugging in:
$b_0 = 1 \Rightarrow 1 = C_1 + C_2 - 1$
$b_1 = 1 \Rightarrow 1 = (2)C_1 + (-1)C_2 - 1$
$C_1 = \frac{4}{3}, C_2 = \frac{2}{3}$
Thus, the final answer is: $b(n) = \frac{4}{3}(2)^n + \frac{2}{3}(-1)^n - 1$

Problem 2: Let S_n be the number of tilings of the $n \times 3$ grid with 1×3 and 2×3 tiles. (Tiles can be rotated by 90 degrees.) Give a recurrence relation for S_n and justify its correctness.

Note: This recurrence will be of degree higher than 2, and you *do not* have to solve it. The reverse page shows all tilings of the 5×3 grid, showing that $S_5 = 23$. You can use this value to verify you recurrence.

Solution 2:
First, set n = 1. So, we will find the number of times a 1x3 and 2x3 fits into a 1x3 square. Only one 1x3 piece can fit into a 1x3. no 2x3 pieces can fit inside a 1x3. Those there exists only one possible combination for n=1

Next set n = 2. So, we will find the number of times a 1x3 and 2x3 fits into a 2x3 square without repitions from previous answers. Two 1x3 pieces can fit insde a 2x3, but this is a repition of n=1, so we will not count that combination. A single 2x3 piece will fit inside a 2x3, and those are the only possible combinations. Thus, there exists only one more combination for n=2

Third, we set n = 3. So we will find the number of times a 1x3 and 2x3 fits into a 3x3 square without repitions from pervious n's. Three horizontal 1x3's, one 1x3 followed by a 2x3 horizontal, and a 2x3 followed by a 1x3 horizontal are the possible combinations that exist. There are more, but those are just repeats of the pervious n's. So, there exsits 3 combinations for n = 3

There are no new combinations for n ¿ 3 because of repition. We can now derive the recurrence relation: $S_n = S_{n-1} + S_{n-2} + 3S_{n-3}$

Problem 3: No partner

Solution 3: No partner
