Homework 2

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Q8.

(a)

```
summary(Auto)
```

```
##       mpg          cylinders      displacement     horsepower
##  Min.   : 9.00   Min.   :3.000   Min.   : 68.0   Min.   : 46.0
##  1st Qu.:17.00   1st Qu.:4.000   1st Qu.:105.0   1st Qu.: 75.0
##  Median :22.75   Median :4.000   Median :151.0   Median : 93.5
##  Mean   :23.45   Mean   :5.472   Mean   :194.4   Mean   :104.5
##  3rd Qu.:29.00   3rd Qu.:8.000   3rd Qu.:275.8   3rd Qu.:126.0
##  Max.   :46.60   Max.   :8.000   Max.   :455.0   Max.   :230.0
##
##      weight      acceleration        year           origin
##  Min.   :1613   Min.   : 8.00   Min.   :70.00   Min.   :1.000
##  1st Qu.:2225   1st Qu.:13.78   1st Qu.:73.00   1st Qu.:1.000
##  Median :2804   Median :15.50   Median :76.00   Median :1.000
##  Mean   :2978   Mean   :15.54   Mean   :75.98   Mean   :1.577
##  3rd Qu.:3615   3rd Qu.:17.02   3rd Qu.:79.00   3rd Qu.:2.000
##  Max.   :5140   Max.   :24.80   Max.   :82.00   Max.   :3.000
##
##                 name
##  amc matador       :  5
##  ford pinto        :  5
##  toyota corolla    :  5
##  amc gremlin       :  4
##  amc hornet        :  4
##  chevrolet chevette:  4
##  (Other)           :365
```

```
lm.fit =lm(mpg~horsepower ,data=Auto)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = mpg ~ horsepower, data = Auto)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5710  -3.2592  -0.3435   2.7630  16.9240
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 39.935861   0.717499   55.66   <2e-16 ***
## horsepower  -0.157845   0.006446  -24.49   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 4.906 on 390 degrees of freedom
## Multiple R-squared:  0.6059, Adjusted R-squared:  0.6049
## F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

i. Yes, p-value of the F-statistic is close to 0.

ii. Strong relationship.The mean of mpg is 23.45. Because of the RSE is 4.906 , it shows the percentage error of close to 20%. So A huge percentage of the variance in mpg is explained by horsepower.

iii. The relationship between mpg and horsepower is “Negative”

iv.

```
# the predicted mpg associated with a horsepower of 98
predict(lm.fit, data.frame(horsepower=c(98)))
```

```
##        1
## 24.46708
```

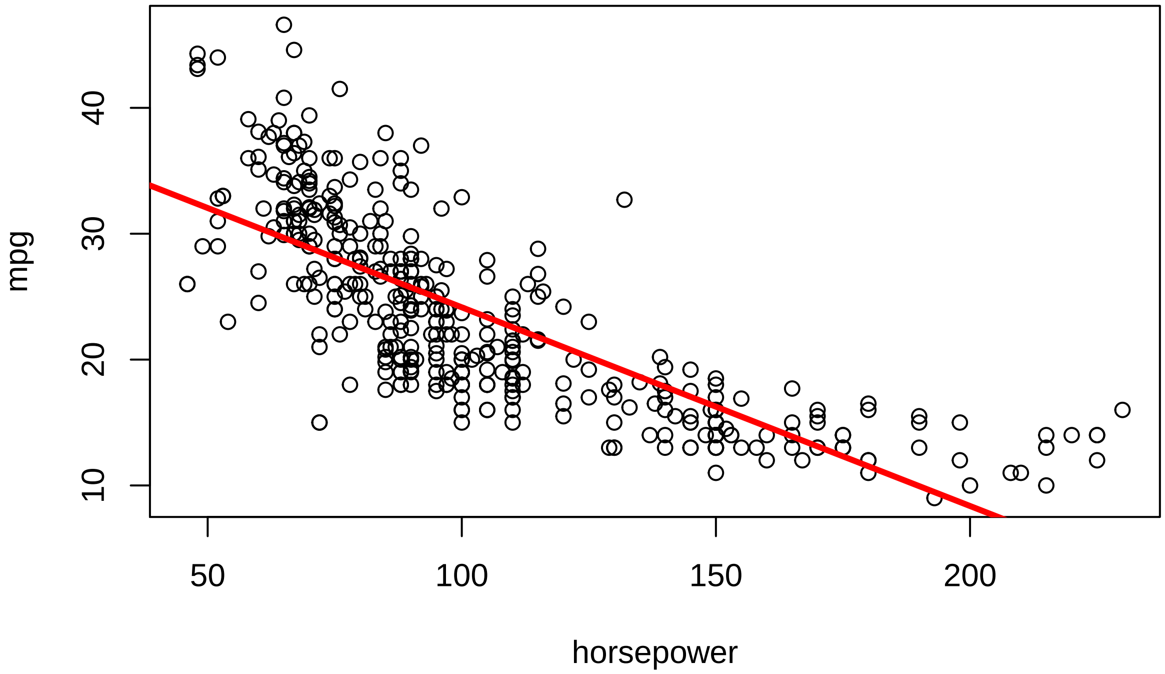
```
# the associated 95% confidence interval
predict(lm.fit, data.frame(horsepower=c(98)), interval = "confidence")
```

```
##        fit      lwr      upr
## 1 24.46708 23.97308 24.96108
```

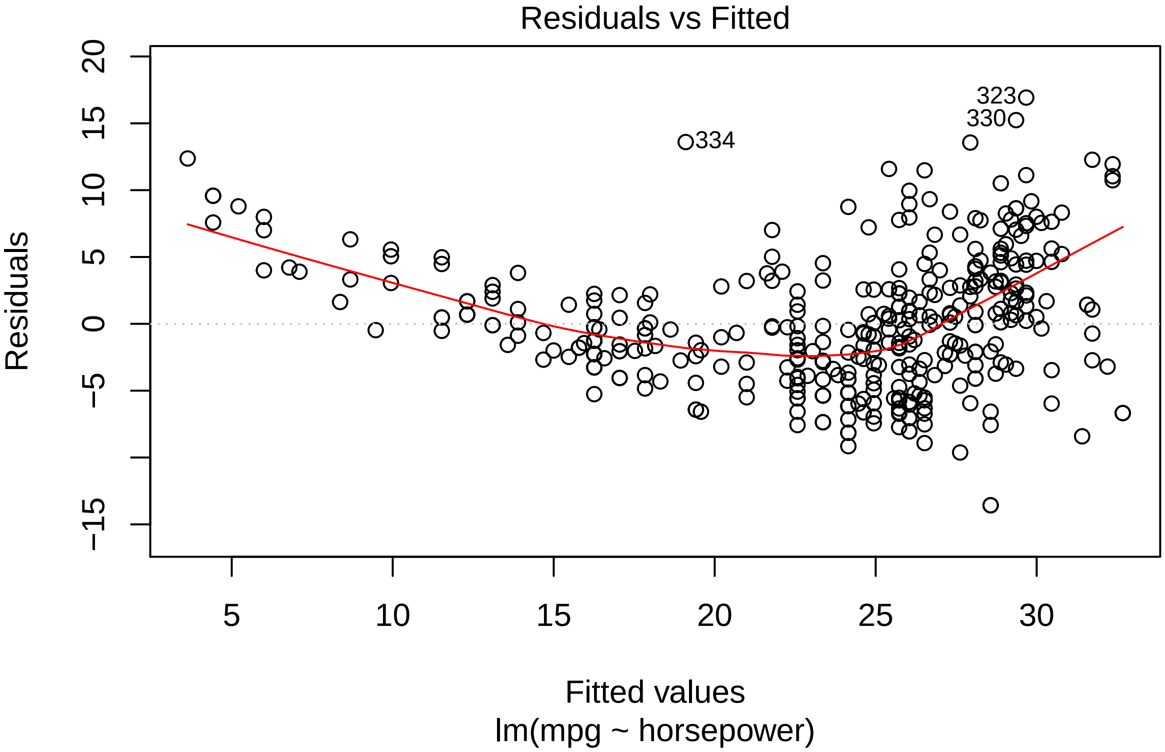
```
# the associated 95% prediction interval
predict(lm.fit, data.frame(horsepower=c(98)), interval = "prediction")
```

```
##        fit     lwr      upr
## 1 24.46708 14.8094 34.12476
```

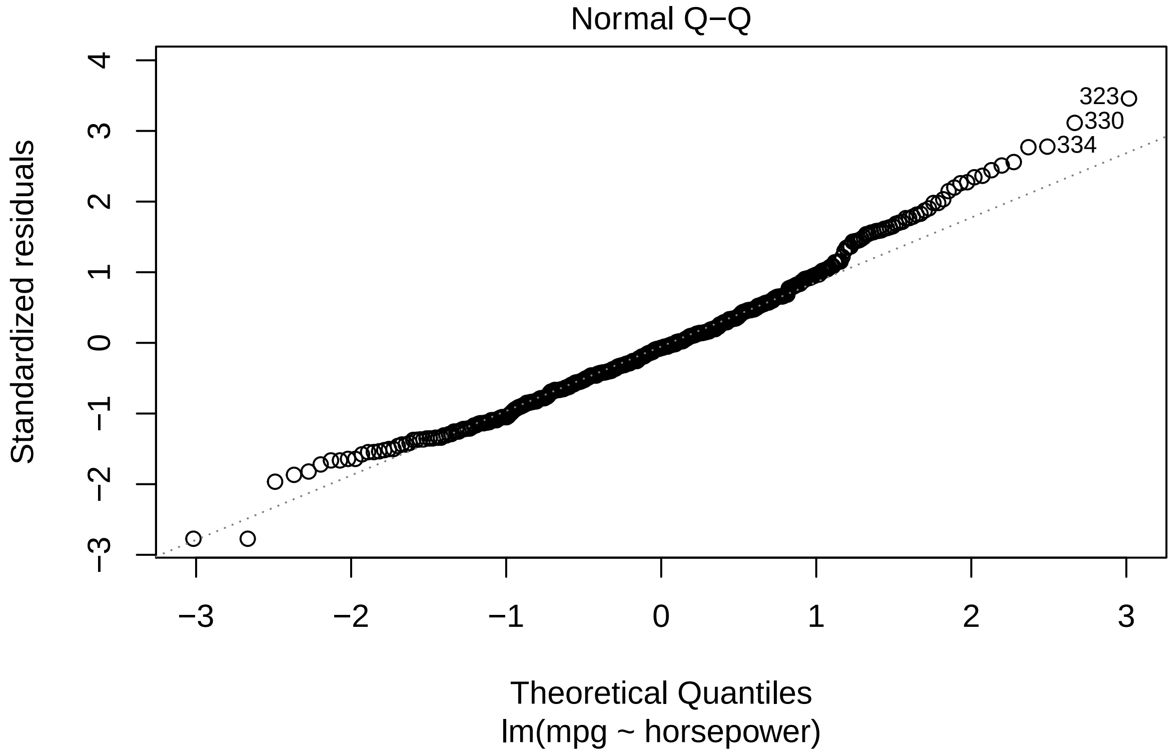
(b)



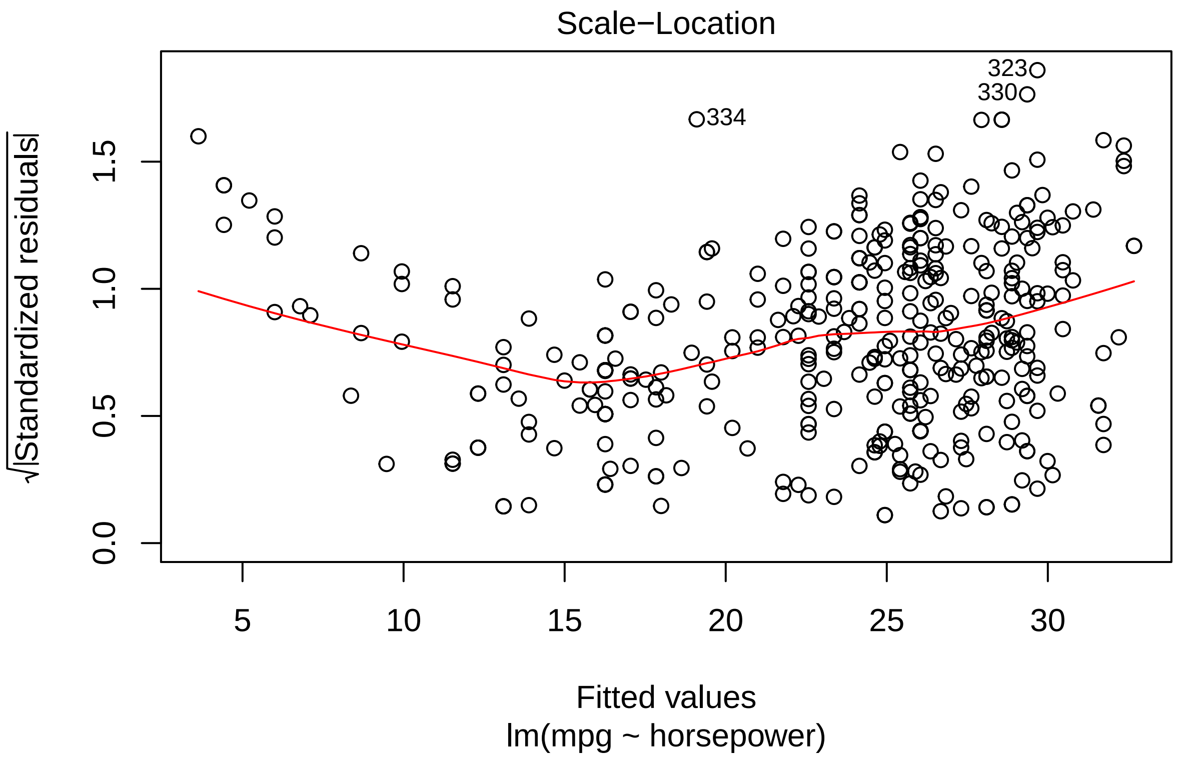
(c)

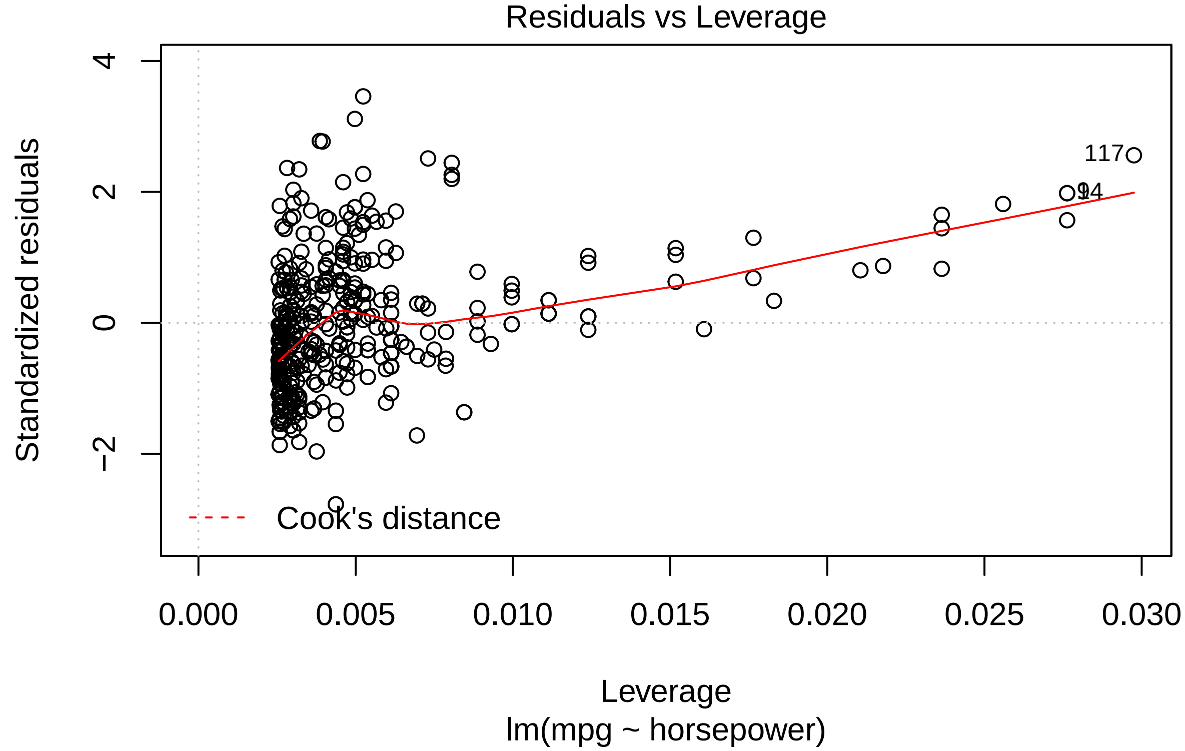


Residuals vs Fitted
Residuals
20
15
10
5
0
−5
−15
323
330
334
5
10
15
20
25
30
Fitted values
lm(mpg ~ horsepower)



Normal Q−Q
Standardized residuals
4
3
2
1
0
−1
−2
−3
323
330
334
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(mpg ~ horsepower)

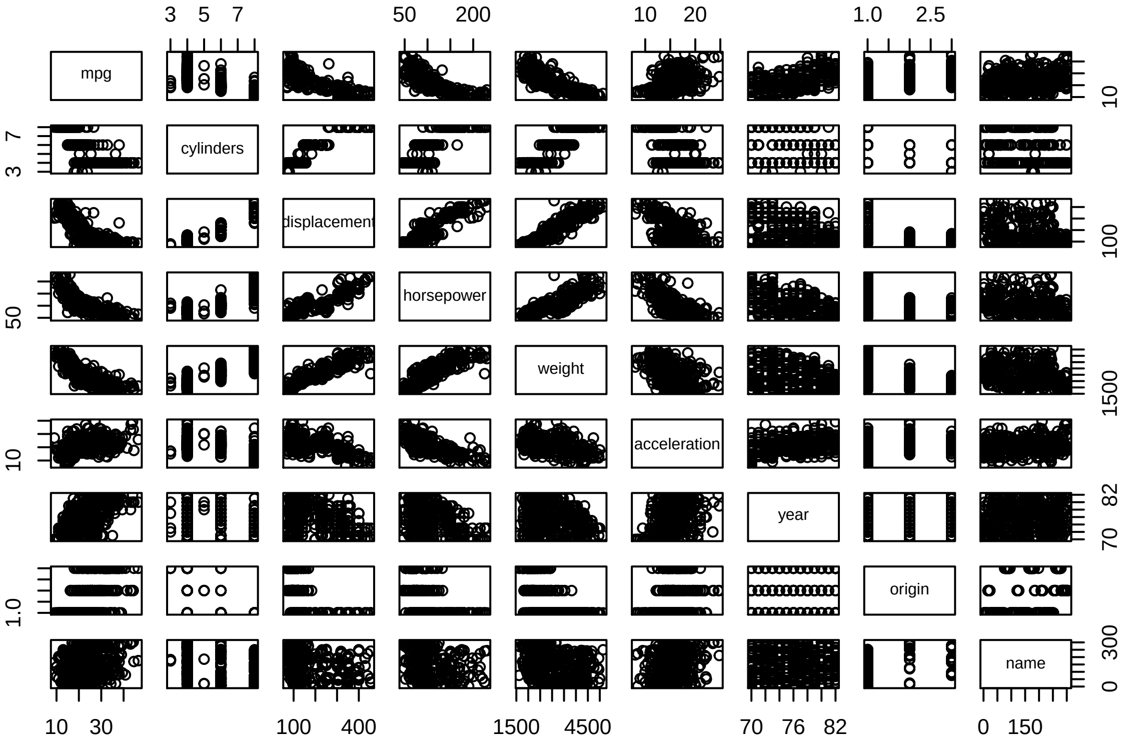

Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
5
10
15
20
25
30
334
323
330
Fitted values
lm(mpg ~ horsepower)


Residuals vs Leverage
Standardized residuals
−2
0
2
4
0.000
0.005
0.010
0.015
0.020
0.025
0.030
117
94
Cook's distance
Leverage
lm(mpg ~ horsepower)

Q9.

(a)

```r
pairs(Auto) # scatterplot matrix
```



(b)

```r
cor(Auto[,1:8]) # correlations between the variables without names
```

```
##                     mpg  cylinders displacement horsepower     weight
## mpg           1.0000000 -0.7776175   -0.8051269 -0.7784268 -0.8322442
## cylinders    -0.7776175  1.0000000    0.9508233  0.8429834  0.8975273
## displacement -0.8051269  0.9508233    1.0000000  0.8972570  0.9329944
## horsepower   -0.7784268  0.8429834    0.8972570  1.0000000  0.8645377
## weight       -0.8322442  0.8975273    0.9329944  0.8645377  1.0000000
## acceleration  0.4233285 -0.5046834   -0.5438005 -0.6891955 -0.4168392
## year          0.5805410 -0.3456474   -0.3698552 -0.4163615 -0.3091199
## origin        0.5652088 -0.5689316   -0.6145351 -0.4551715 -0.5850054
##              acceleration       year     origin
## mpg             0.4233285  0.5805410  0.5652088
## cylinders      -0.5046834 -0.3456474 -0.5689316
## displacement   -0.5438005 -0.3698552 -0.6145351
## horsepower     -0.6891955 -0.4163615 -0.4551715
## weight         -0.4168392 -0.3091199 -0.5850054
## acceleration    1.0000000  0.2903161  0.2127458
## year            0.2903161  1.0000000  0.1815277
## origin          0.2127458  0.1815277  1.0000000
```

(c)

```
lm.fit =lm(mpg~.-name,data=Auto)
summary(lm.fit)
```

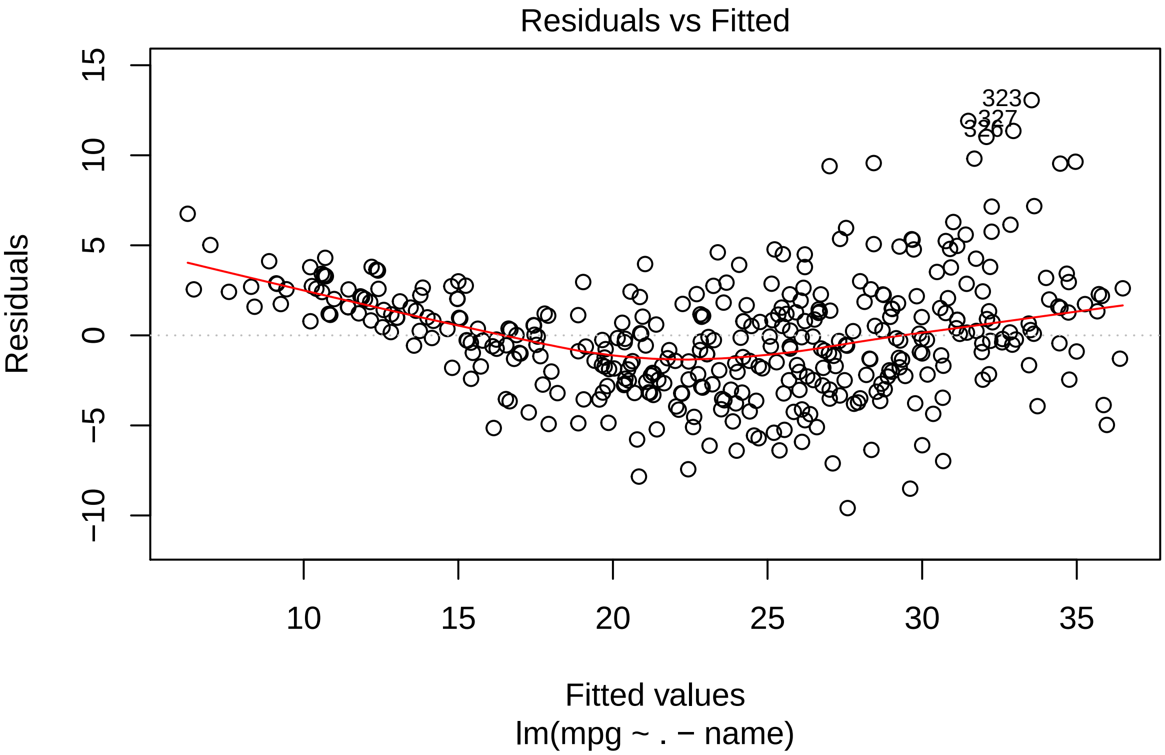
```
##
## Call:
## lm(formula = mpg ~ . - name, data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5903 -2.1565 -0.1169  1.8690 13.0604
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.218435   4.644294  -3.707  0.00024 ***
## cylinders     -0.493376   0.323282  -1.526  0.12780
## displacement   0.019896   0.007515   2.647  0.00844 **
## horsepower    -0.016951   0.013787  -1.230  0.21963
## weight        -0.006474   0.000652  -9.929  < 2e-16 ***
## acceleration   0.080576   0.098845   0.815  0.41548
## year           0.750773   0.050973  14.729  < 2e-16 ***
## origin         1.426141   0.278136   5.127 4.67e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.328 on 384 degrees of freedom
## Multiple R-squared:  0.8215, Adjusted R-squared:  0.8182
## F-statistic: 252.4 on 7 and 384 DF,  p-value: < 2.2e-16
```

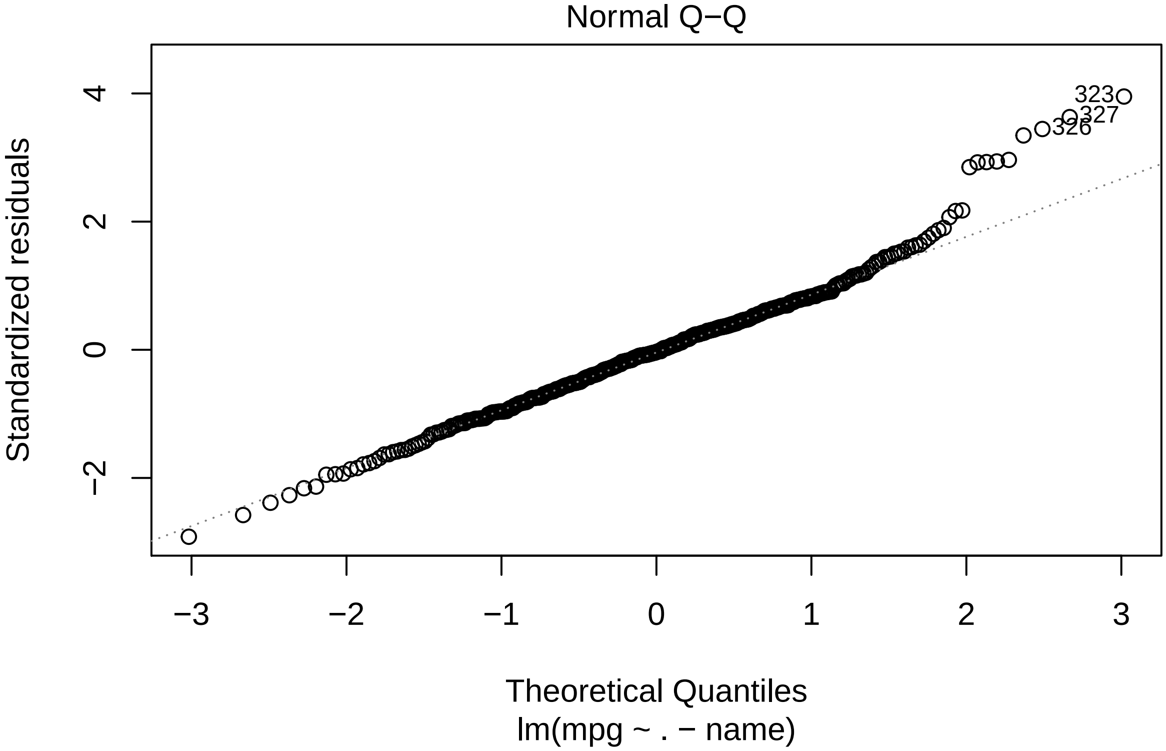
i. Yes

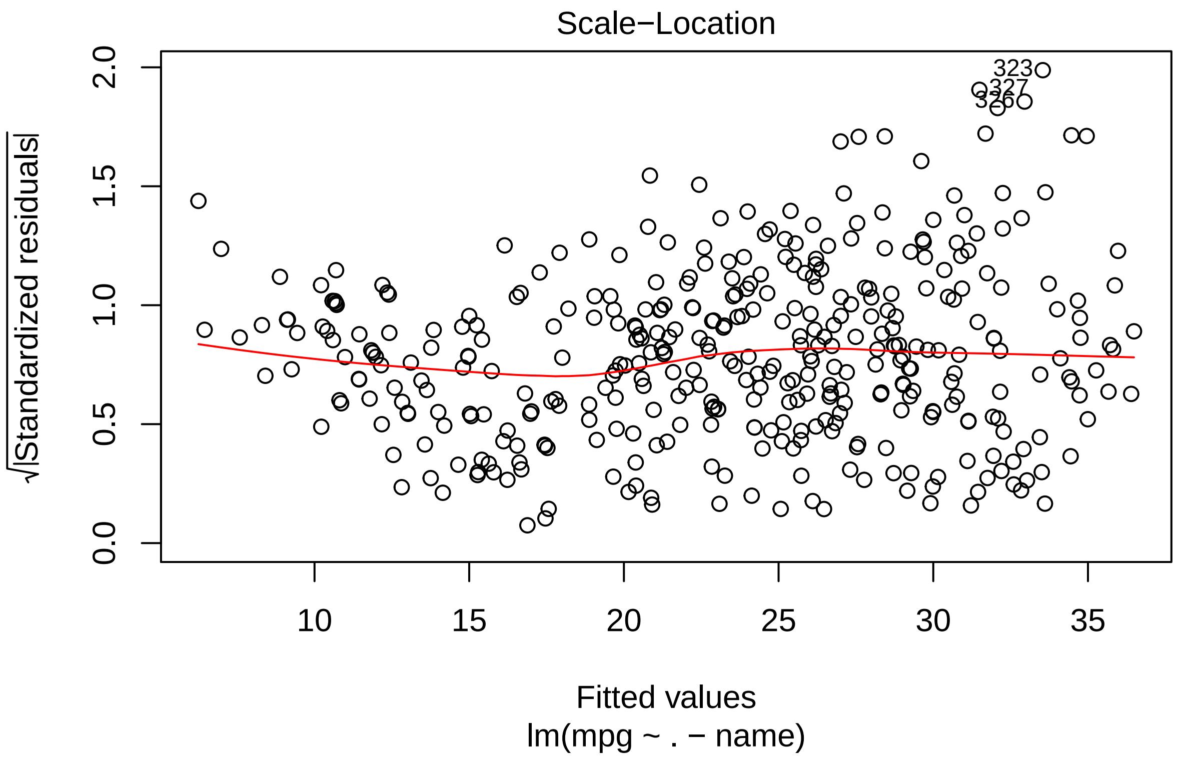
ii. displacement, weight, year and origin

iii. Regression coefficient of year: 0.75 The lower the “year” (older), the higher the “mpg”

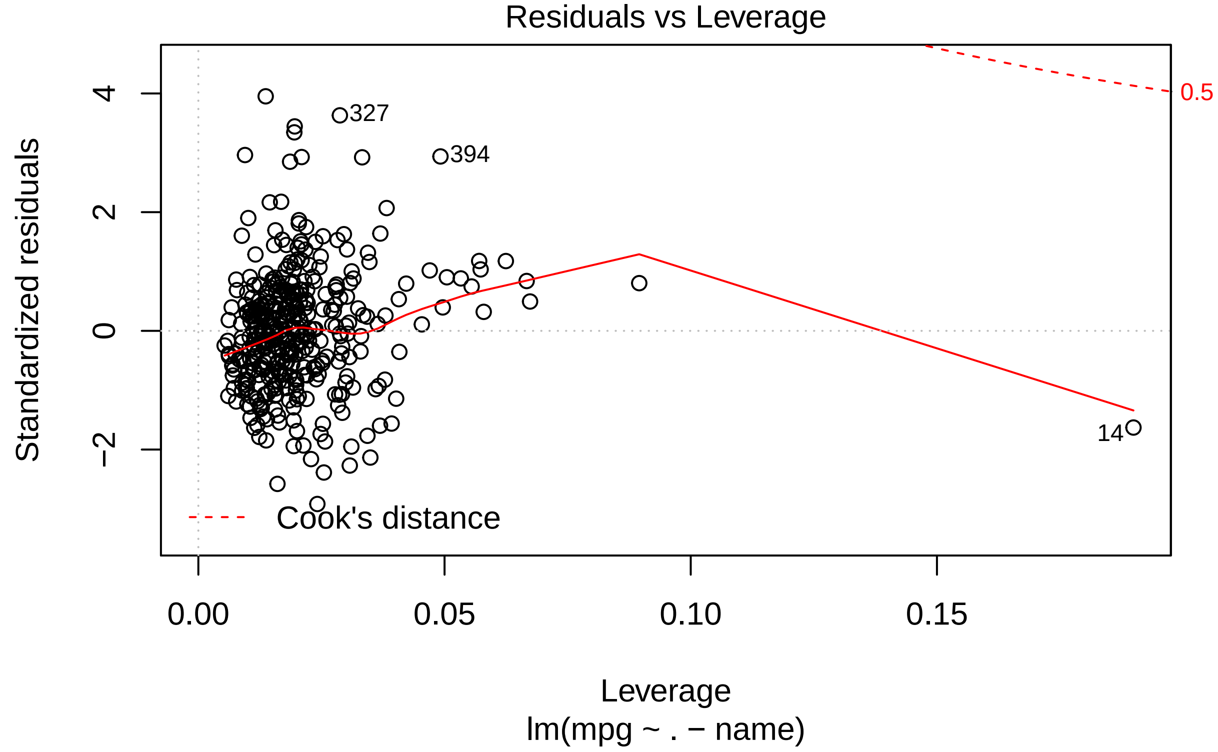
(d)


Residuals vs Fitted
Residuals
15
10
5
0
−5
−10
10
15
20
25
30
35
Fitted values
lm(mpg ~ . − name)
323
327
326

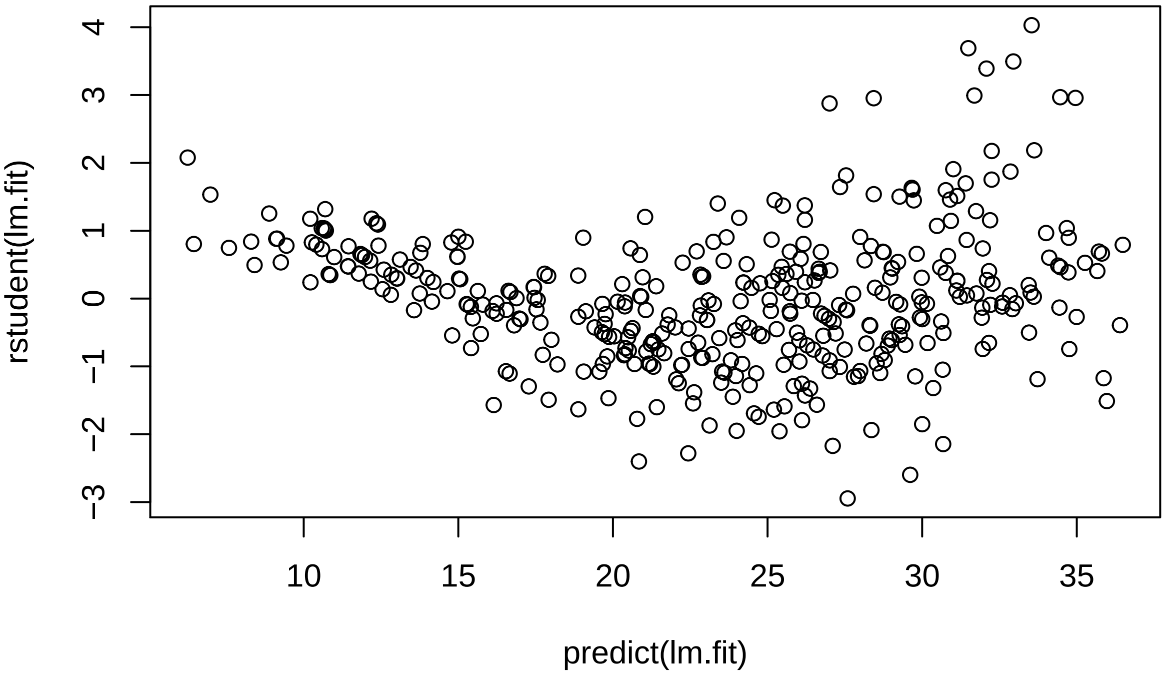

Normal Q−Q
Standardized residuals
4
2
0
−2
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(mpg ~ . − name)
323
327
326



Scale−Location



Residuals vs Leverage



- The fit seems not linear. There is a curve.
- There are some values in “rstudent” > 3

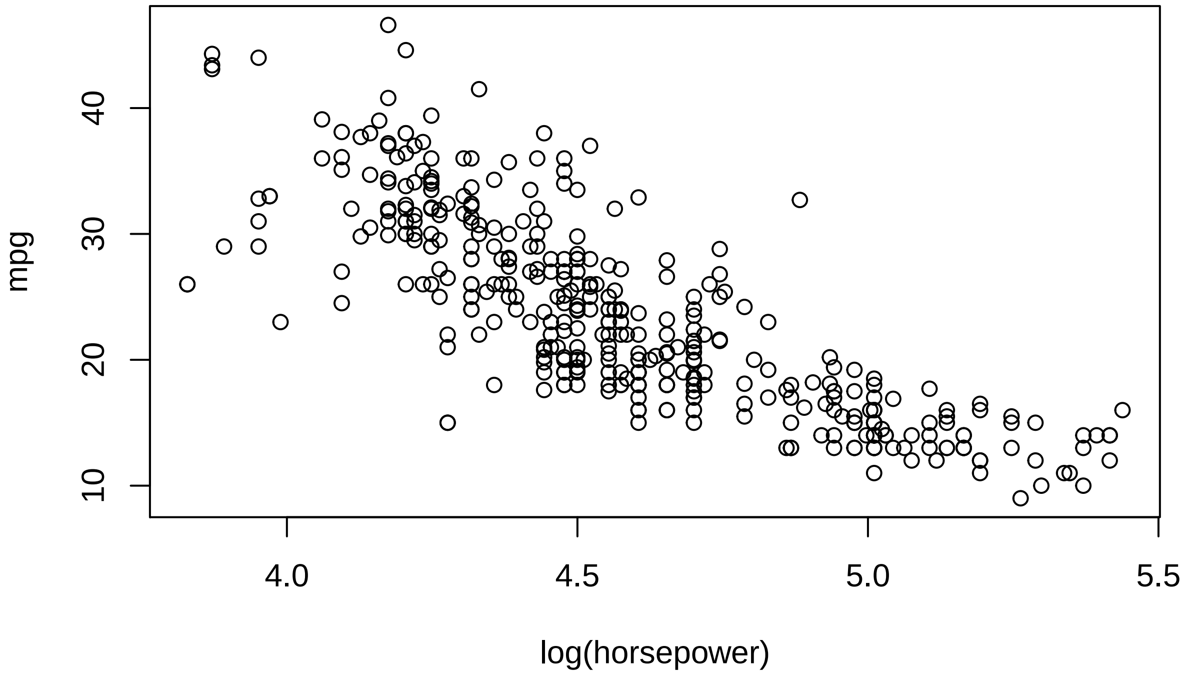
(e)

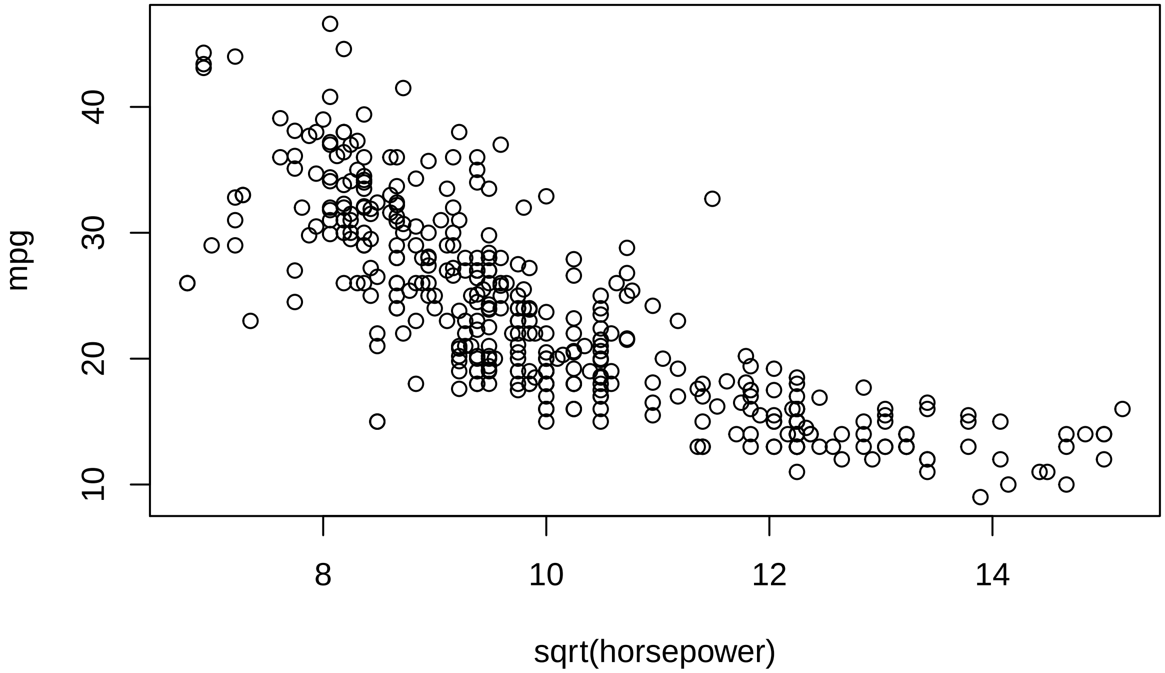
```
Auto2 = Auto[,1:8]
lm2.fit = lm(mpg~.*., data = Auto2)
summary(lm2.fit)
```

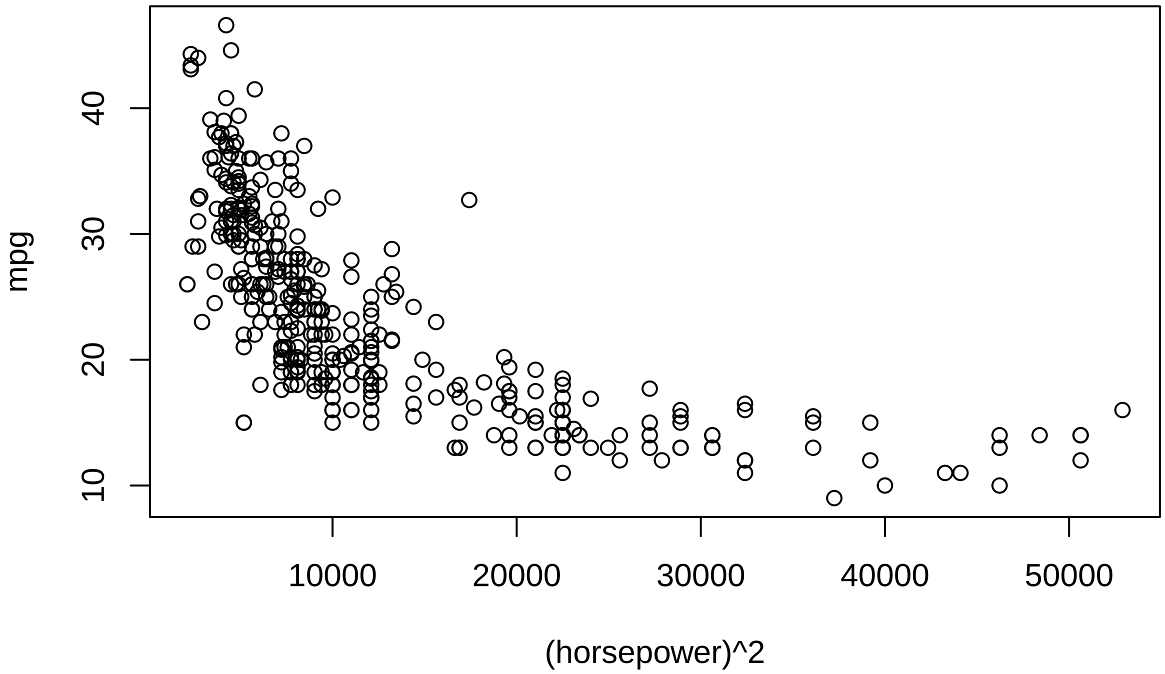
```
##
## Call:
## lm(formula = mpg ~ . * ., data = Auto2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.6303 -1.4481  0.0596  1.2739 11.1386
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                3.548e+01  5.314e+01   0.668  0.50475
## cylinders                  6.989e+00  8.248e+00   0.847  0.39738
## displacement              -4.785e-01  1.894e-01  -2.527  0.01192 *
## horsepower                 5.034e-01  3.470e-01   1.451  0.14769
## weight                     4.133e-03  1.759e-02   0.235  0.81442
## acceleration              -5.859e+00  2.174e+00  -2.696  0.00735 **
## year                       6.974e-01  6.097e-01   1.144  0.25340
## origin                    -2.090e+01  7.097e+00  -2.944  0.00345 **
## cylinders:displacement    -3.383e-03  6.455e-03  -0.524  0.60051
## cylinders:horsepower       1.161e-02  2.420e-02   0.480  0.63157
## cylinders:weight           3.575e-04  8.955e-04   0.399  0.69000
## cylinders:acceleration     2.779e-01  1.664e-01   1.670  0.09584 .
## cylinders:year            -1.741e-01  9.714e-02  -1.793  0.07389 .
## cylinders:origin           4.022e-01  4.926e-01   0.816  0.41482
## displacement:horsepower   -8.491e-05  2.885e-04  -0.294  0.76867
## displacement:weight        2.472e-05  1.470e-05   1.682  0.09342 .
## displacement:acceleration -3.479e-03  3.342e-03  -1.041  0.29853
## displacement:year          5.934e-03  2.391e-03   2.482  0.01352 *
## displacement:origin        2.398e-02  1.947e-02   1.232  0.21875
## horsepower:weight         -1.968e-05  2.924e-05  -0.673  0.50124
## horsepower:acceleration   -7.213e-03  3.719e-03  -1.939  0.05325 .
## horsepower:year           -5.838e-03  3.938e-03  -1.482  0.13916
## horsepower:origin          2.233e-03  2.930e-02   0.076  0.93931
## weight:acceleration        2.346e-04  2.289e-04   1.025  0.30596
## weight:year               -2.245e-04  2.127e-04  -1.056  0.29182
## weight:origin             -5.789e-04  1.591e-03  -0.364  0.71623
## acceleration:year          5.562e-02  2.558e-02   2.174  0.03033 *
## acceleration:origin        4.583e-01  1.567e-01   2.926  0.00365 **
## year:origin                1.393e-01  7.399e-02   1.882  0.06062 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.695 on 363 degrees of freedom
## Multiple R-squared:  0.8893, Adjusted R-squared:  0.8808
## F-statistic: 104.2 on 28 and 363 DF,  p-value: < 2.2e-16
```

From the p-values, e.g. acceleration:origin is statistically signifcant

(f)


mpg
10
20
30
40
4.0
4.5
5.0
5.5
log(horsepower)


mpg
10
20
30
40
8
10
12
14
sqrt(horsepower)



It shows that “log” is a better fit than the original one.

Q10.

(a)

```
lm.fit =lm(Sales~Price + Urban + US, data = Carseats)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Sales ~ Price + Urban + US, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9206 -1.6220 -0.0564  1.5786  7.0581
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 13.043469   0.651012  20.036  < 2e-16 ***
## Price       -0.054459   0.005242 -10.389  < 2e-16 ***
## UrbanYes    -0.021916   0.271650  -0.081    0.936
## USYes        1.200573   0.259042   4.635 4.86e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.472 on 396 degrees of freedom
## Multiple R-squared:  0.2393, Adjusted R-squared:  0.2335
## F-statistic: 41.52 on 3 and 396 DF,  p-value: < 2.2e-16
```

(b)

Price

- According to the linear regression, there is a relationship. he higher price, the lower sales.

urbanYes

- According to the linear regression, there isn't a relationship between the location of the store and the number of sales

USYes

- According to the linear regression, there is a relationship. If the store is in the US, the sales will increase.

(c)

Sales = 13.04 + -0.05 Price + -0.02 UrbanYes + 1.20 USYes

(d)

Price and USYes

(e)

```r
#uses the predictors for which there is evidence of association with the outcome
lm2.fit = lm(Sales~Price + US, data = Carseats)
summary(lm2.fit)
```

```
##
## Call:
## lm(formula = Sales ~ Price + US, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9269 -1.6286 -0.0574  1.5766  7.0515
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 13.03079    0.63098  20.652  < 2e-16 ***
## Price       -0.05448    0.00523 -10.416  < 2e-16 ***
## USYes        1.19964    0.25846   4.641 4.71e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.469 on 397 degrees of freedom
## Multiple R-squared:  0.2393, Adjusted R-squared:  0.2354
## F-statistic: 62.43 on 2 and 397 DF,  p-value: < 2.2e-16
```

(f)

```
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Sales ~ Price + Urban + US, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9206 -1.6220 -0.0564  1.5786  7.0581
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 13.043469   0.651012  20.036  < 2e-16 ***
## Price       -0.054459   0.005242 -10.389  < 2e-16 ***
## UrbanYes    -0.021916   0.271650  -0.081    0.936
## USYes        1.200573   0.259042   4.635 4.86e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.472 on 396 degrees of freedom
## Multiple R-squared:  0.2393, Adjusted R-squared:  0.2335
## F-statistic: 41.52 on 3 and 396 DF,  p-value: < 2.2e-16
```

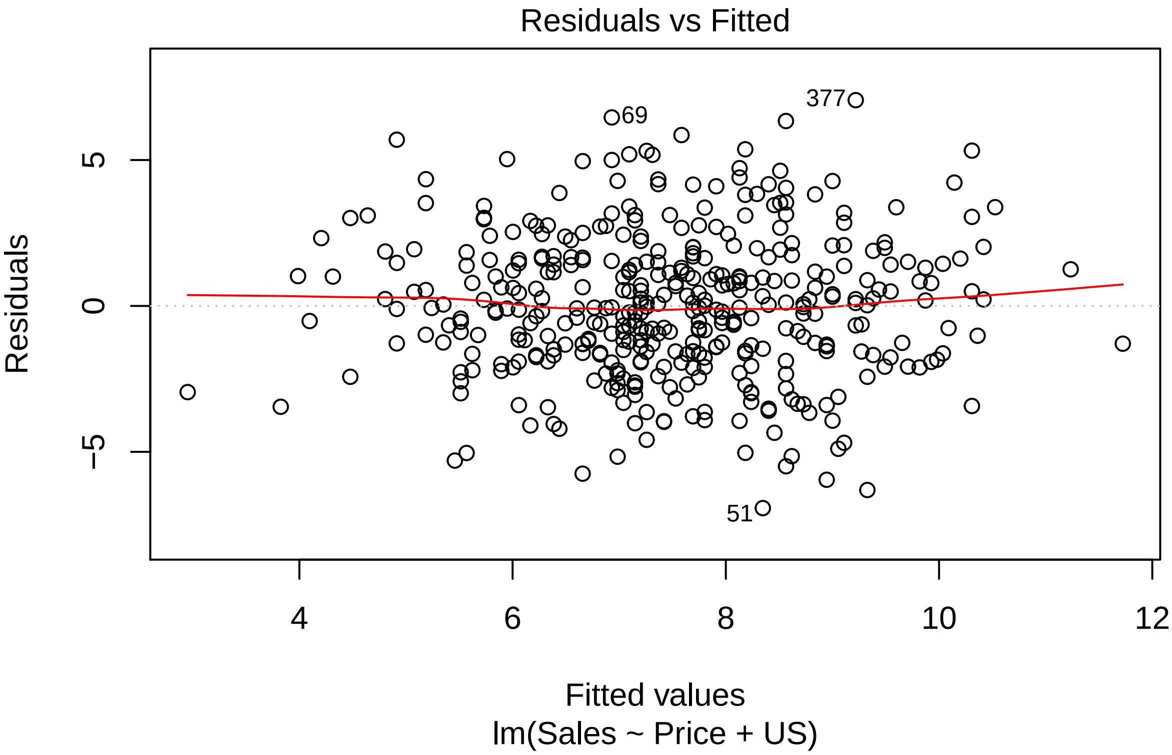
Comparing to lm2.fit, the two models are similarly fit.

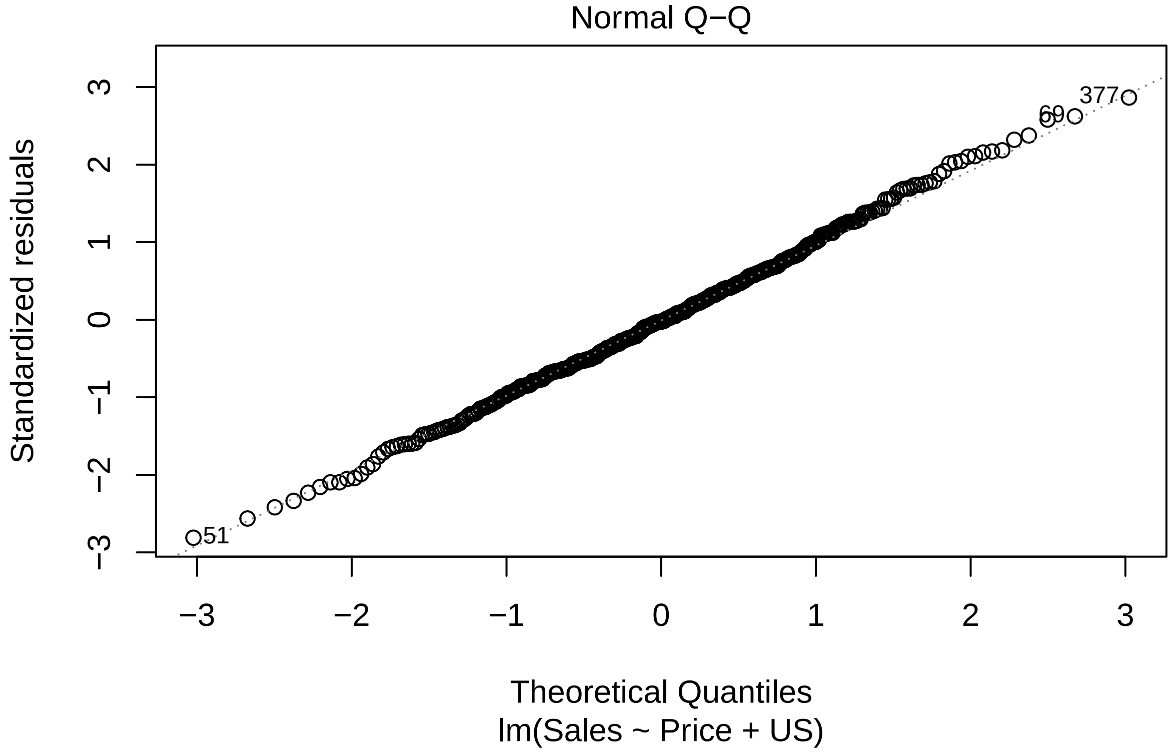
(g)

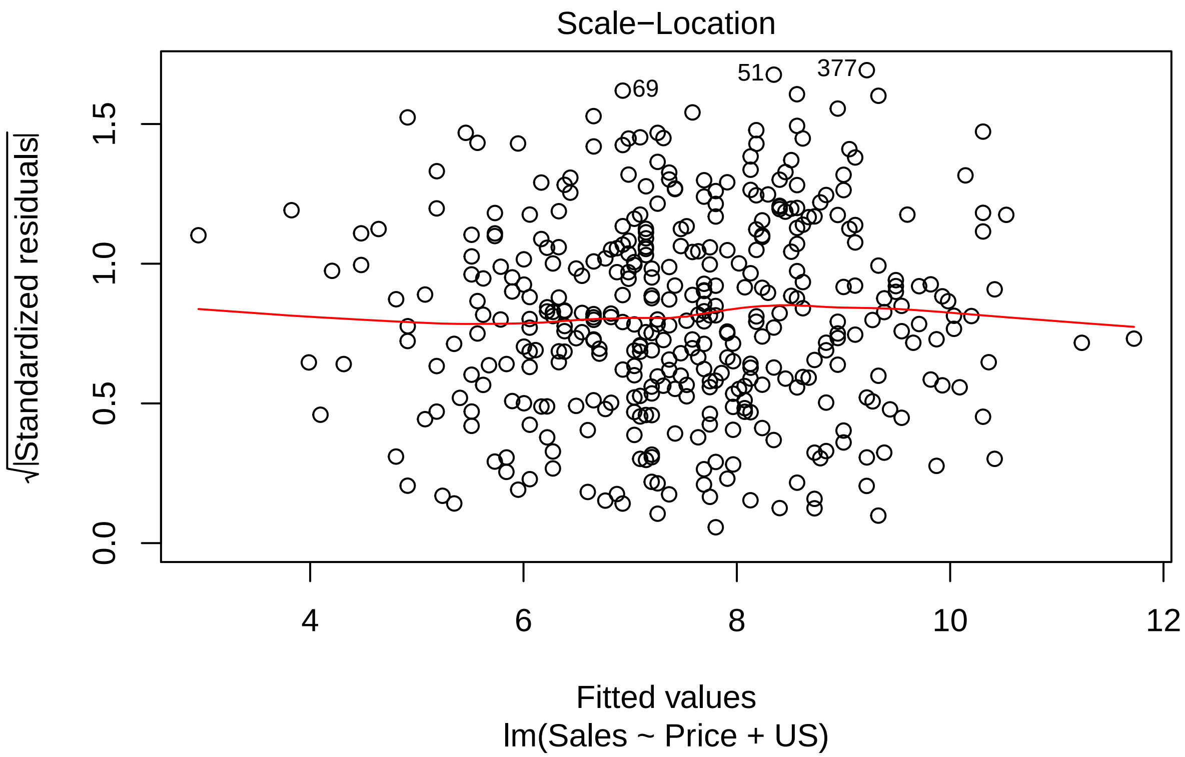
```
# Confidence Intervals:
confint(lm2.fit)
```

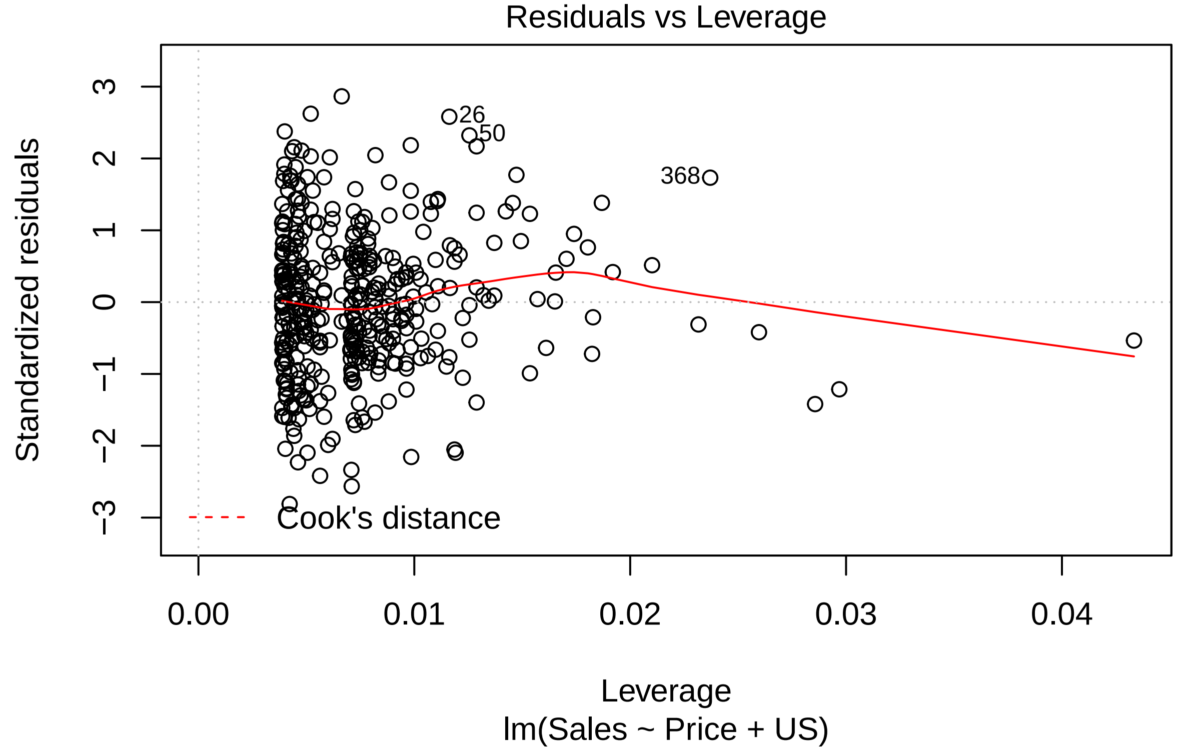
```
##                   2.5 %      97.5 %
## (Intercept) 11.79032020 14.27126531
## Price       -0.06475984 -0.04419543
## USYes        0.69151957  1.70776632
```

(h)

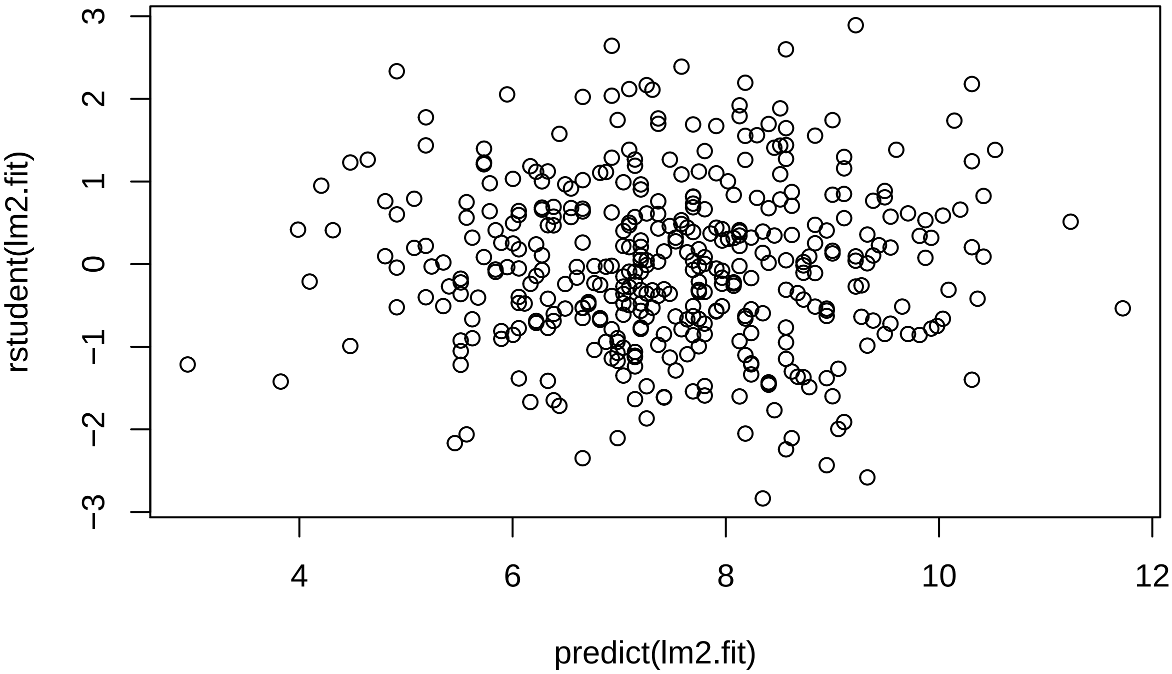

Residuals vs Fitted
Residuals
5
0
−5
4
6
8
10
12
Fitted values
lm(Sales ~ Price + US)
69
377
51


Normal Q−Q
Standardized residuals
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(Sales ~ Price + US)
377
69
51





As the rstudents values, there is no outliers are suggested.



The graph shows that there are a few high leverage points.

Q13.

```
set.seed(1)
```

set.seed(1) prior to starting part (a): according to the question

(a)

```
x = rnorm(100, 0, 1)
```

(b)

```
eps = rnorm(100, 0, 0.25)
```

(c)

```
y = -1 + 0.5*x + eps
length(y)
```

```
## [1] 100
```

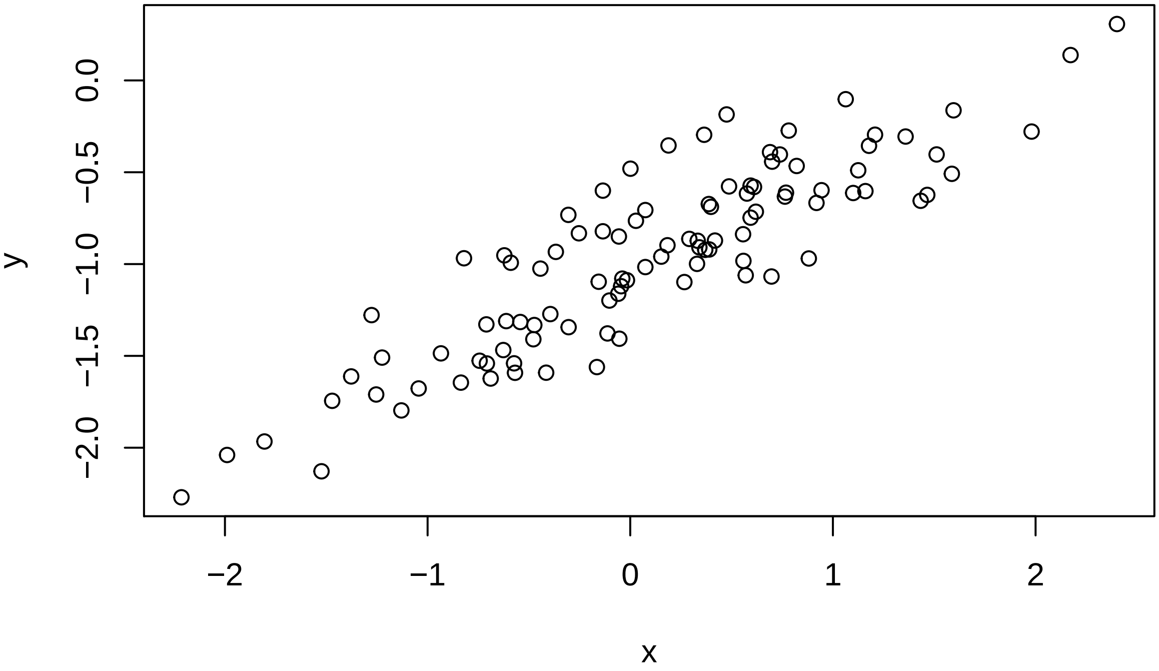
```
summary(y)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -2.2700 -1.3294 -0.9215 -0.9550 -0.6021  0.3071
```

length of vector y = 100 ; beta0 = -1 ; beta1 = 0.5

(d)

```
## Warning in abline(lm.fit): only using the first two of 4 regression
## coefficients
```



Linear relationship

(e)

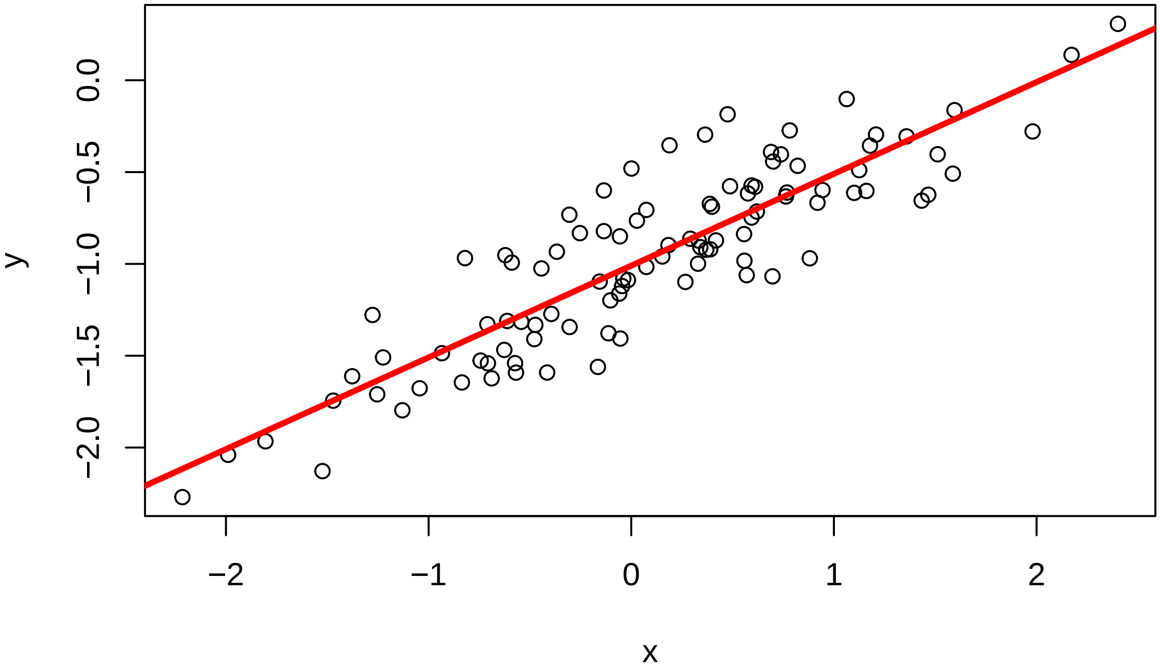
```
lm.fit = lm(y~x)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.46921 -0.15344 -0.03487  0.13485  0.58654
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.00942    0.02425  -41.63   <2e-16 ***
## x            0.49973    0.02693   18.56   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2407 on 98 degrees of freedom
## Multiple R-squared:  0.7784, Adjusted R-squared:  0.7762
## F-statistic: 344.3 on 1 and 98 DF,  p-value: < 2.2e-16
```

beta0 and 1 are similar to the original values.

(f)

```
plot(x,y)
abline(lm.fit)
abline (lm.fit, lwd =3, col ="red")
```



(g)

```
lm2.fit = lm(y~poly(x, 2))
summary(lm2.fit)
```

```
##
## Call:
## lm(formula = y ~ poly(x, 2))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.4913 -0.1563 -0.0322  0.1451  0.5675
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.95501    0.02395 -39.874   <2e-16 ***
## poly(x, 2)1  4.46612    0.23951  18.647   <2e-16 ***
## poly(x, 2)2 -0.33602    0.23951  -1.403    0.164
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.2395 on 97 degrees of freedom
## Multiple R-squared:  0.7828, Adjusted R-squared:  0.7784
## F-statistic: 174.8 on 2 and 97 DF,  p-value: < 2.2e-16
```

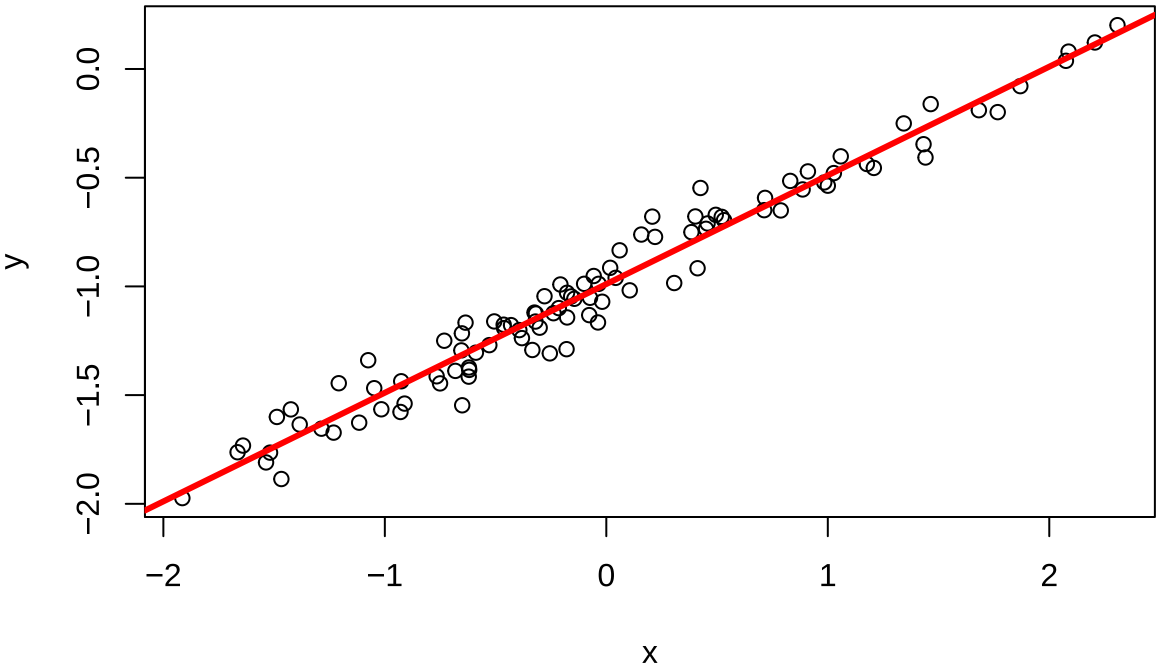
Regression coefficient of the model is insignificant.

(h)

```
set.seed(1)
eps = rnorm(100, 0, 0.1) # less noise
x = rnorm(100)
y = -1 + 0.5*x + eps
plot(x, y)
lm1.fit = lm(y~x)
summary(lm1.fit)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.232416 -0.060361  0.000536  0.058305  0.229316
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.989115   0.009035 -109.48   <2e-16 ***
## x            0.499907   0.009472   52.78   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09028 on 98 degrees of freedom
## Multiple R-squared:  0.966,  Adjusted R-squared:  0.9657
## F-statistic:  2785 on 1 and 98 DF,  p-value: < 2.2e-16
```

```
abline(lm1.fit, lwd =3, col ="red")
```



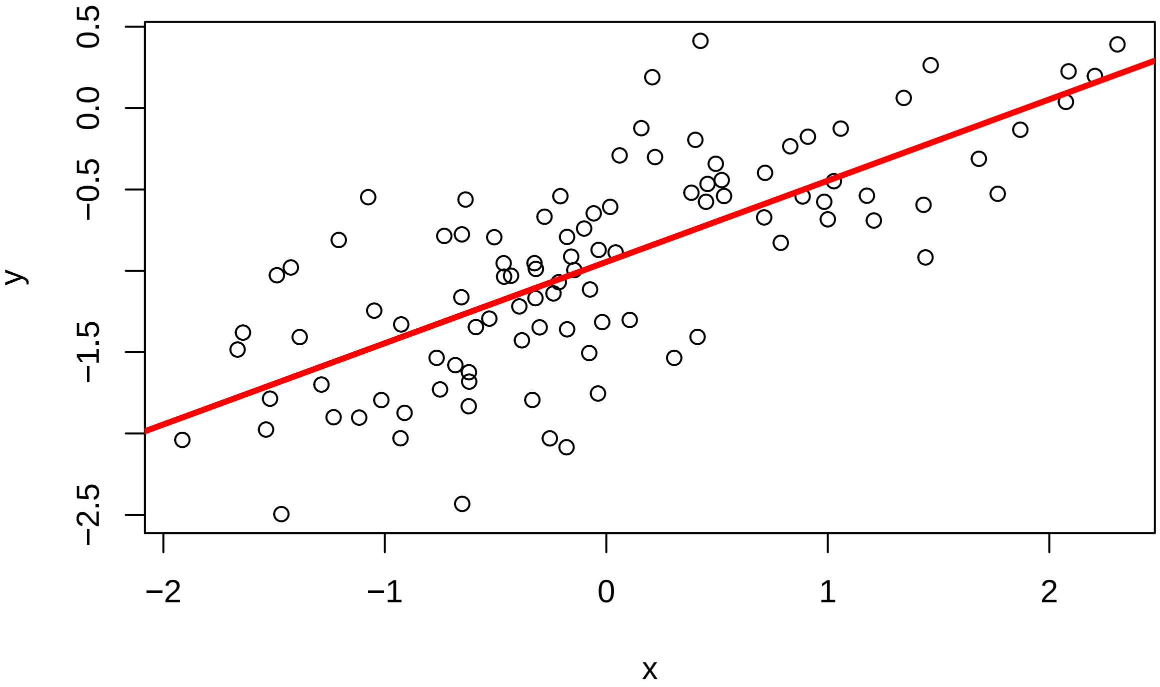
RSE dencreases

(i)

```
set.seed(1)
eps = rnorm(100, 0, 0.5)  # more noise
x = rnorm(100)
y = -1 + 0.5*x + eps
plot(x, y)
lm2.fit = lm(y~x)
summary(lm2.fit)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.16208 -0.30181  0.00268  0.29152  1.14658
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.94557    0.04517  -20.93   <2e-16 ***
## x            0.49953    0.04736   10.55   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4514 on 98 degrees of freedom
## Multiple R-squared:  0.5317, Adjusted R-squared:  0.5269
## F-statistic: 111.2 on 1 and 98 DF,  p-value: < 2.2e-16
```

```
abline(lm2.fit, lwd =3, col ="red")
```



RSE increases

(j)

```
confint(lm.fit)
```

```
##                  2.5 %     97.5 %
## (Intercept) -1.0575402 -0.9613061
## x            0.4462897  0.5531801
```

```
confint(lm1.fit)
```

```
##                  2.5 %     97.5 %
## (Intercept) -1.0070441 -0.9711855
## x            0.4811096  0.5187039
```

```
confint(lm2.fit)
```

```
##                  2.5 %     97.5 %
## (Intercept) -1.0352203 -0.8559276
## x            0.4055479  0.5935197
```

With different level of noises, Intervals are still around 0.5.