Goals of this recitation

- First order modeling problems.
- Solving non homogeneous linear systems.

Contents

1. First order Modeling

1.1. Derivative Refresher.

- If a function is increasing, then its derivative is ________________.
- If a function is decreasing, then its derivative is ________________.

Let's use these ideas to build intuition on how to make first order differential equations.

- If an action will increase the value of the function, what will its effect on the derivative be?
- If an action will decrease the value of the function, what will its effect on the derivative be?

1.2. **Strategy for modeling first order differential equations.** In maths, when we have difficult problems, an often used strategy is to break up the pieces into simpler parts based off different behaviour/operations. What do you think the strategy for solving first order differential equations based off your answers above?

1.3. **Creating a first order differential equation.** Let's do an example together.

A tank initially contains 120 L of pure water. A mixture containing a concentration of 4 g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of 4 for the amount of salt in the tank at any time t. Find the limiting amount of salt in the tank as $t \to \infty$

A tank initially contains 200 L of pure water. A mixture containing a concentration of 3 g/L of salt enters the tank at a rate of 5 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of salt in the tank at any time t. Find the limiting amount of salt in the tank as $t \to \infty$

2. Solving non homogeneous linear systems

2.1. U substitution.

Why do we use u sub?

When we use u sub what is the last step before the end of the problem?

2.2. Integration Factor.

2.2.1. *Building intuition.* In this subsection, we are going to try to solve a few different problems to try see how to develop strategies to solve some differential equations.

The sum rule

If $f(x) = g(x) + h(x)$
Then $\frac{df}{dx} = \frac{dg}{dx} + \frac{dh}{dx}$

The product Rule
If $f(x) = g(x)h(x)$
Then $\frac{df}{dx} = g(x)\frac{dh}{dx} + \frac{dg}{dx}h(x)$

Chain Rule

If $f(x) = g(h(x))$
Then $\frac{df}{dx} = \frac{dg}{dh} \cdot \frac{dh}{dx}$

Let's examine the following ODE

$$\frac{dy}{dx} + y = 0$$

Lets examine our ODE and see what properties it has.

Operations:

Any difference in behaviour of the terms?

If you had to pick the result of a differentiation rule that was the closest in behaviour to this, which one would you pick and why?

2.2.2. *Exercises.*

(1)

$$y' + 2y = 3e^t \qquad y(0) = 0$$

(2)

$$y' + 6y = 2$$

2.3. **Reading equations from Matrices.** To see why we use a change of variables strategy we are first going to examine equations without using it first and see what goes wrong. Figure out the equations from the following equations and let's examine what the issues are after we have figured them out.

(1)

$$x' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$$

(2)

$$x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x + \begin{bmatrix} e^{t} \\ t \end{bmatrix}$$

I am going to examine this equation and talk about what an ideal situation would be like.

$$x' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$$

2.4. **Non homogeneous linear systems.** Let's solve this equation using diagonalisation methods

$$x' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$$

Now you try.

$$x' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} x + \begin{bmatrix} e^t \\ t \end{bmatrix}$$

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