Goals of this recitation

- Phase Lines
- Characteristic equation
- General Solutions

Contents

1. Phase Lines

1.1. **Intuition.**

1.1.1. *Derivative Refresher.*

- If a function is increasing, then its derivative is ______________.

- If a function is decreasing, then its derivative is ______________.

1.1.2. *Stability.* How to view stability:

1.2. **Example problem.** Figure out when y is at equilibrium and whether those equilibrium points are stable or not by looking at a graph of $\frac{dy}{dt}$ versus y

$$\frac{dy}{dt} = y(1 - y^2)$$

1.3. **Abstraction.** What is abstraction?

1.4. **Applying abstraction.** Let's examine what we used in the last section.

2. Linear Equations

2.1. **Homogeneous.** Before we mentioned that equations were linear if the coefficient of each power of the derivative depends on the independent variable.

When we have a linear equation equal to zero it is called homogeneous. An example below is

$$\frac{d^2y}{dt^2} + e^t\frac{dy}{dt} + \cos(t)y = 0$$

We have solved some equations before so let's examine them together to see what we can expand upon.

$$\frac{dy}{dt} - y = 0$$

$$\frac{d^2y}{dt^2} - y = 0$$

Let's do a problem together.

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 0$$

Solve the following equations

(1)

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0$$

(2)

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 0$$

2.2. **Inhomogenous.** There are also inhomogeneous equations, i.e. equations where

Inhomogeneous equations are when the equation is equal to some function of the independant variable. For instance

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = 2\sin(t)$$

These can come in many different forms so a question we ask is, what is the strategy to figure this out?

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = e^t \cos(2t)$$

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