Goals of this recitation

- Wronskian
- Variation of parameters
- Section 3.8

Contents

1. Wronskian

1.1. **Determinants.** This is something that appears in linear algebra. It has many uses but one of the most important things it shows is whether a function (when applicable) is reversible or one to one. Let's do some examples after looking at the formula.

If you have a matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\det(A) = ad - bc$. Note that a common convention for finding the determinant of a matrix is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Let's look at the example. Find the determinant of the matrix

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

Find the determinant of the following matrices

(1)

$$\begin{pmatrix} 6 & 1 \\ 1 & 2 \end{pmatrix}$$

(2)

$$\begin{pmatrix} 1 & -1 \\ -4 & 1 \end{pmatrix}$$

(3)

$$\begin{pmatrix} e^t & -e^t \\ -2e^t & 6e^t \end{pmatrix}$$

(4)

$$\begin{pmatrix} t^2 & 3t \\ 2t & 3t \end{pmatrix}$$

1.2. **The Wronskian of a homogeneous differential equation.** If a differential equation has solutions y_1 and y_2 form a Wronskian which is

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

If the Wronskian of a homogeneous differential equation is non zero at a point t_0 we can form a solution $y = c_1y_1 + c_2y_2$. We call this a fundamental set of solutions. Let's form the Wronskian of a few questions and see if they form a fundamental set of solutions. Lets do an example.

$$y'' + 4y' - 5y = 0$$

Now you try do an example

$$y'' - 2y' - 8y = 0$$

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