Goals of this recitation

- Jacobian Matrix
- Laplace Transforms
- ODE knowledge

Contents

1. Jacobian Matrix and its eigenvalues

Example:

Find the Jacobian of the following system.

$$x'(t) = 5x^2 + 4xy$$
$$y'(t) = 7xy^2$$

Find the Jacobian at the point (2,3)

Find the Jacobian of the following systems at certain points and then find the eigenvectors.

(1) Find the Jacobian of the following system at the point (1,1) and then find its eigenvalues.

$$x'(t) = 3xy + y^3$$
$$y'(t) = x^2 - 6y$$

(2) Find the Jacobian of the following system at the point (0,1) and then find its eigenvalues.

$$x'(t) = e^x + xy$$
$$y'(t) = y^4$$

2. Laplace Transform

$f(t)$	$F(s) = \mathcal{L}[f(t)]$		Formula
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$	A
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$	B
$f(t) = t^n$	$F(s) = \frac{n!}{s^{(n+1)}}$	$s > 0$	C
$f(t) = \sin(at)$	$F(s) = \frac{a}{s^2 + a^2}$	$s > 0$	D
$f(t) = \cos(at)$	$F(s) = \frac{s}{s^2 + a^2}$	$s > 0$	E
$f(t) = \sinh(at)$	$F(s) = \frac{a}{s^2 - a^2}$	$s > \lvert a \rvert$	F
$f(t) = \cosh(at)$	$F(s) = \frac{s}{s^2 - a^2}$	$s > \lvert a \rvert$	G
$f(t) = t^n e^{at}$	$F(s) = \frac{n!}{(s-a)^{(n+1)}}$	$s > a$	H
$f(t) = e^{at}\sin(bt)$	$F(s) = \frac{b}{(s-a)^2 + b^2}$	$s > a$	I
$f(t) = e^{at}\cos(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 + b^2}$	$s > a$	J
$f(t) = e^{at}\sinh(bt)$	$F(s) = \frac{b}{(s-a)^2 - b^2}$	$s - a > \lvert b \rvert$	K
$f(t) = e^{at}\cosh(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 - b^2}$	$s - a > \lvert b \rvert$	L

Solve the following equations using the Laplace Transform.

(1) $y'' + 3y' + 2y = 0$, $y(0) = 1$ and $y'(0) = 0$

(2) $y'' - 2y' + 4y = 0$, $y(0) = 2$ and $y'(0) = 0$

(3)
$$y'' - y' - 6y = 0 \qquad y(0) = 2, \quad y'(0) = -1$$

(4) $y'' - 2y' + 2y = 0$, $y(0) = 0$ and $y'(0) = 1$

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