Goals of this recitation

- Understand pattern matching.
- Partial fraction decomposition.
- Matrix multiplication.
- Eigenvectors.

Contents

1. Adding fractions

1.1. **Introduction.** Let's do a couple of examples of trying to add fractions together and focus on when it is easy and when it is not.

Change numbers of equations and the pictures

(1)

$$\frac{3}{4} + \frac{7}{4} =$$

(2)

$$\frac{2}{7} + \frac{9}{7} =$$

(3)

$$\frac{2}{3} + \frac{3}{5} =$$

1.2. **Starting to add complications.** Looking at the last problem in the last section, what is different to the previous two problems?

What approach did you take?

What did that approach do? What is that purpose of it?

(1)

$$\frac{4}{3} + \frac{1}{2} =$$

(2)

$$\frac{4}{6} + \frac{3}{4} =$$

(3)

$$\frac{2}{x} + \frac{5}{3} =$$

(4)

$$\frac{2}{x} + \frac{3}{\sin(x)} =$$

1.3. **Fractions with functions.** With adding fractions, our motivation and approach while doing it is the same regardless. So if we have fractions with functions in the denominator, what approach do you take and why?

Lets carry out that approach here.

(1)

$$\frac{3}{x} + \frac{7}{e^x} =$$

(2)

$$\frac{x}{\ln(x)} + \frac{x^2}{\cos(x)} =$$

(3)

$$\frac{e^x}{x} + \frac{\ln(x)}{x^2} =$$

2. Pattern matching

2.1. **Introduction.** Pattern matching is recognising when two things need to be equal. It is based off of two things (in my opinion).

- The terms we have.
- The operations we have.
- **How** those operations are applied to the pieces.

For example, from the equation below

$$ax + b = 3 + 7x$$

Would you be able to figure out a and b? If so, what lets you know which is which? Is it possible to carry out the same action in the situation below.

$$ax + b = \sin(x) + \cos(x)$$

Why/ why not?

Try do the following exercises. Let x be a variable and all other letters be constants. Figure out the value of the constants

(1)

$$ax + b = 6 + 9x$$

(2)

$$3x + b = ax + 17$$

(3)

$$7\sin(x) + e^{bt} = r\sin(x) + e^{4t}$$

(4)

$$ax^2 + bx + c = 3x^2 + x$$

(5)

$$ax^2 + 5x + c = 3x^2 + rx + 8$$

(6)

$$ax + b = x + x + 5$$

(7)

$$ax^2 + bx + c = x(4x + 9) + 8$$

2.2. **Reformatting equations for pattern matching.** Lets go over the following example and ask ourselves why we took the steps that we took.

$$ax^2 + bx + c = x(4x + 9) + 8$$

What benefit does doing the operations you applied give you in this scenario?

Is there another way of doing the same action?

Things to bear in mind when giving notes.

Find the constants in the following equations.

(1)

$$c + x(ax + b) = x^2 + 4x + 7$$

(2)

$$\frac{3x^2 + bx + 4}{x^2} = \frac{rx^2 + 9x + c}{x^2}$$

(3)

$$\frac{ax + b}{x^2} = \frac{3}{x^2} + \frac{2}{x}$$

(4)

$$\frac{ax + b}{x^2} = \frac{4}{x} + \frac{7}{x^2}$$

Before we move onto partial fraction decomposition, lets look at an example where we are going to talk about some issues we have.

$$\frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$$

We are going to focus on just adjusting the right hand side. Can we do pattern matching straight away? If not what are the issues preventing us from using pattern matching?

Do we have a method of resolving this issue? If so what is it? Try apply that method.

Does this give another issue that prevents pattern matching? If so what is it and can you think of a way around it?

Try implement that

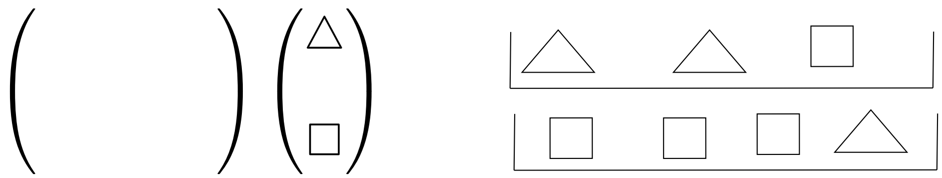
2.3. **Partial Fraction Decomposition.** Let's go through this problem together

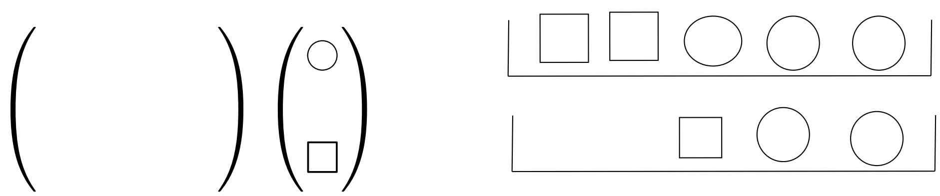
$$\frac{17x - 53}{x^2 - 2x - 15}$$

2.4. **Quiz problem.**

3. Matrices

3.1. Creating Matrix Equations. Let's do it with visual examples





Lets do this with some equations!

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$$x + 6y = 1$$

$$3x + 7y = 3$$

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$$-x + 2y = 9$$

$$7y = -‘1$$

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$$-3x = 0$$

$$1x + 4y = 2$$

Convert the following set of equations into a matrix.

$$\begin{array}{llll} x & +\,3y & +9z & = 1 \\ -\,4x & +\,3y & +2z & = 2 \\ 8x & +\,3y & -1z & = 6 \\ 5x & +\,15y & -24z & = 1 \end{array}$$

Write the matrix below as a set of equations to the right of it.

$$\begin{pmatrix} 2 & 4 \\ 9 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

3.2. **Process for matrix multiplication.** Lets do some matrix multiplication with some examples.

$$\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 3 \\ 9 & 2 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 7 & 1 \\ 9 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} =$$

3.3. **Differential Equations with Matrices.** Let's take these differential equations and turn them into matrix equations and then we will use them with points (1,0) and (0,1).

$$x' = 2x$$
$$y' = 4y$$

Let's take these differential equations and turn them into matrix equations and then we will use them with points (1,1) and (1,-1).

$$x' = 3x - y$$
$$y' = -x - 3y$$

Let's talk about another example, suppose that when $(x, y) = (1, 1)$, $(x', y') = (-.5, .25)$. Sketch this situation and then move onto the last part of this exercise.

Go through the previous examples and imagine the origin was the equilibrium point. Which of these do you think would be stable/unstable?

What makes it easier to talk about stability/ not easy to talk about stability.

3.4. **Eigenvalues.** Lets find the eigenvalues of the matrix below.

$$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$$

Now you try!

$$\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$$

3.5. **Eigenvectors.** Lets figure out the eigenvectors of

$$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$$

Now you try!

$$\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$$

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