Goals of this recitation

- Draw sketches to help understand the mathematical properties of situations.
- Split up a situation based off of different behaviour.
- Put the pieces together to form a first order differential equation.
- Exactness

Contents

1. First order Modeling

1.1. Derivative Refresher.

- If a function is increasing, then its derivative is ______________.
- If a function is decreasing, then its derivative is ______________.

Let's use these ideas to build intuition on how to make first order differential equations.

- If an action will increase the value of the function, what will its effect on the derivative be?
- If an action will decrease the value of the function, what will its effect on the derivative be?

1.2. Strategy for modeling first order differential equations.

In maths, when we have difficult problems, an often used strategy is to break up the pieces into simpler parts based off different behaviour/operations. What do you think the strategy for solving first order differential equations based off your answers above?

1.3. **Creating a first order differential equation.** Let's do an example together.

A tank initially contains 120 L of pure water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

A tank initially contains 200 L of pure water. A mixture containing a concentration of 3 g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of salt in the tank at any time t.

2. Exactness

2.1. Intuition for variables that reappear through integration. Let's do some examples of differentiation and look at what "disappears"(get's sent to 0).

(1) Let $f(x) = x^2 + 7$
- Differentiate this function with respect to x.
- What "disappeared"?
- If we did integration with respect to x in the one variable case, what would we not be able to refind?

(2) Let $f(x, y) = 3x + e^y + xy^2 + 7$
- Differentiate this function with respect to x.
- What "disappeared"?
- If we did integration with respect to x in the two variable case, what would we not be able to refind?

(3) Let $f(x, y, z) = zx + ze^y + xy^2 + 7$
- Differentiate this function with respect to x.
- What "disappeared"?
- If we did integration with respect to x in the three variable case, what would we not be able to refind?

2.2. **Potential Functions.** Note about Calc 3 Potential functions

2.3. **Note about Generalising. KEY MATHEMATICAL IDEA:**

2.4. **Test for exactness.** The following equation is exact if

$$M(x)dx + N(y)dy = 0$$

$$M_y(x, y)dx = N_x(x, y)$$

2.5. **Exactness example.**

$$(y\cos(x) + 2xe^y) + (\sin(x) + x^2e^y - 1)y' = 0$$

Practice

Show that the following equation is exact and then find the solution

$$(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)y' = 0$$

References

Dept. of Mathematics, Colorado State University, Fort Collins, CO, USA
Email address: brian.collery@colostate.edu