Goals of this recitation

- Converting second order equations to first order equations using matrices.
- Interesting matrix facts.

Contents

1. Second order ODE's

Not finished

1.1. Converting equations into matrices.

Before we had equations such as

$$x_1' = 3x_1 + 2x_2$$
$$x_2' = 1x_1 + 4x_2$$

If you are to convert this into a matrix, how would you do that?

Matrices can be a useful way of organising and studying data. However, sometimes data does not come in the exact format we would like to put into a matrix. The solution people came up with was to find ways of reformatting data so that we could use techniques that work on matrices on this data. The rest of this section is focused on how they did this.

One thing to remember as it is the guiding principle of a lot of the motivation behind assumptions or strategies we use is the answer to the following questions.

If we have one equation, how many variables can we solve for?

If we have a situation that involves more variables than equations, what potential effects would we want our assumptions to have?

(1)

(2)

(3)

Lets look at the following equation and discuss issues with solving it.

$$u'' + 2u' - 3u = 0$$

We do have ways of solving these equations but lets take another perspective on this situation.

How many equations are given to us?

How many variables are given to us?

Lets walk through our strategies that we talked about in the first page.

Strategy:

Strategy:

Let's walk through our example together to see how it is done.

$$u'' + 2u' - 3u = 0$$

What connections between terms do you see?

Let's create equations that highlight some of these connections and then try to solve our system.

Now you try.

For this system, just try set up the system of equations for this. I don't think you have went into the methods

$$u'' + 0.25u' + 4u = 2\cos(3t)$$

1.2. **Sketching vector fields from matrix equations.** If we are given an equation such as

$$x' = Ax$$

What is the method to sketch a vector field from this information?

Lets do an example together.

$$x' = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} x$$

2. Interesting Matrix facts

2.1. Useful properties about Matrices.

Scalar multiplication

Let A be a matrix, λ be a scalar and v be a vector. Then

$$A\lambda v = \lambda A v$$

Identity operation

Suppose that A is an invertible matrix, i.e. $A^{-1}A = I$. Then

$$v = Iv$$
$$v = A^{-1}Av$$

2.2. Facts about eigenvalues.

2.3. Connection between eigenvalues and determinant.

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