Goals of this recitation

- Partial fraction decomposition
- Introduction to Laplace Transform

Contents

1. Pattern matching

1.1. **Introduction.** Pattern matching is recognising when two things need to be equal. It is based off of two things (in my opinion).

- The terms we have.
- The operations we have.
- **How** those operations are applied to the pieces.

For example, from the equation below

$$ax + b = 5 + 2x$$

Would you be able to figure out a and b? If so, what lets you know which is which? Is it

possible to carry out the same action in the situation below.

$$ax + b = \cos(x) + e^x$$

Why/ why not?

Try do the following exercises. Let x be a variable and all other letters be constants. Figure out the value of the constants

(1)

$$ax + b = 2 + x$$

(2)

$$7x + b = ax - 2$$

(3)

$$3\cos(x) + e^{bt} = r\cos(x) + e^{9t}$$

(4)

$$ax^2 + bx + c = 4x^2 + 8$$

(5)

$$ax^2 + 5x + c = 3x^2 + rx + 8$$

(6)

$$ax + b = x + 3 + x$$

(7)

$$ax^2 + bx + c = x(5x + 2) + 3$$

1.2. **Reformatting equations for pattern matching.** Lets go over the following example and ask ourselves why we took the steps that we took.

$$ax^2 + bx + c = x(5x + 2) + 3$$

What benefit does doing the operations you applied give you in this scenario?

Is there another way of doing the same action?

Things to bear in mind when giving notes.

Find the constants in the following equations.

(1)

$$c + x(ax + b) = x^2 + 4x + 7$$

(2)

$$\frac{3x^2 + bx + 4}{x^2} = \frac{rx^2 + 9x + c}{x^2}$$

(3)

$$\frac{ax + b}{x^2} = \frac{3}{x^2} + \frac{2}{x}$$

(4)

$$\frac{ax + b}{x^2} = \frac{4}{x} + \frac{7}{x^2}$$

Before we move onto partial fraction decomposition, lets look at an example where we are going to talk about some issues we have.

$$\frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$$

We are going to focus on just adjusting the right hand side. Can we do pattern matching straight away? If not what are the issues preventing us from using pattern matching?

Do we have a method of resolving this issue? If so what is it? Try apply that method.

Does this give another issue that prevents pattern matching? If so what is it and can you think of a way around it?

Try implement that

1.3. **Partial Fraction Decomposition.** Let's go through this problem together

$$\frac{17x - 53}{x^2 - 2x - 15}$$

If you have time, now you try

$$\frac{x-1}{x^2+8x+15}$$

1.4. **Pattern matching with tables.** Below is a table

$f(t)$	$F(s) = \mathcal{L}[f(t)]$		Formula
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$	A
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$	B
$f(t) = t^n$	$F(s) = \frac{n!}{s^{(n+1)}}$	$s > 0$	C
$f(t) = \sin(at)$	$F(s) = \frac{a}{s^2 + a^2}$	$s > 0$	D
$f(t) = \cos(at)$	$F(s) = \frac{s}{s^2 + a^2}$	$s > 0$	E
$f(t) = \sinh(at)$	$F(s) = \frac{a}{s^2 - a^2}$	$s > \lvert a\rvert$	F
$f(t) = \cosh(at)$	$F(s) = \frac{s}{s^2 - a^2}$	$s > \lvert a\rvert$	G
$f(t) = t^n e^{at}$	$F(s) = \frac{n!}{(s-a)^{(n+1)}}$	$s > a$	H
$f(t) = e^{at}\sin(bt)$	$F(s) = \frac{b}{(s-a)^2 + b^2}$	$s > a$	I
$f(t) = e^{at}\cos(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 + b^2}$	$s > a$	J
$f(t) = e^{at}\sinh(bt)$	$F(s) = \frac{b}{(s-a)^2 - b^2}$	$s - a > \lvert b\rvert$	K
$f(t) = e^{at}\cosh(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 - b^2}$	$s - a > \lvert b\rvert$	L

That people sometimes use to solve problems. One first question is how?

They try to recognise where on the table their input is by seeing which operation more closely align to the operations they have. For instance, if I gave you the following equation for $f(t)$ which formula letter do you think it would apply to and why?

$$f(t) = t^5$$

If we were to follow that row, what would $L(f(t))$ be?

$$L(f(t))$$

Try solve these problems in the same fashion.

(1) Let $f(t) = e^{4t}$. Then

$$L(f(t)) =$$

(2) Let $f(t) = \cos(6t)$. Then

$$L(f(t)) =$$

(3) Let $f(t) = \sin(3t)$. Then

$$L(f(t)) =$$

(4) Let $f(t) = t^3 e^{6t}$. Then

$$L(f(t)) =$$

(5) Suppose $L(f(t)) = \frac{3}{s^2-3^2}$

$$f(t) =$$

(6) Suppose $L(f(t)) = \frac{s}{s^2+7^2}$

$$f(t) =$$

2. Common strategy to tackle hard problems

2.1. Building Blocks: Looking at first few operations are built.

2.2. Logarithms: Why are they useful?

2.3. **Overview:** Previously solved problems

2.4. **Calc 1: What is the purpose of u substitution?**

$$\int \sin(e^x)e^x dx$$

3. Laplace Transform

3.1. **What is the point of it?** Here we are going to talk about the previous strategy but instead of counting operations building into each other we will discuss it with precalculus and calculus.

3.2. **Formal definition.** For a function $f(t)$, its laplace transform is

$$L(f(t))(s) = \int_{s=0}^{\infty} f(t)e^{-st}dt$$

3.3. **Nice properties that the Laplace transform has.** The laplace transform has two properties that when combined form a property called linearity. Let c be a constant, then

$$\begin{aligned} L(cf(t)) &= \int_{s=0}^{\infty} cf(t)e^{-st}dt \\ &= c\int_{s=0}^{\infty} f(t)e^{-st}dt \\ &= cL(f(t)) \end{aligned}$$

$$
\begin{aligned}
L(f(t)+g(t)) &= \int_{s=0}^{\infty}(f(t)+g(t))e^{-st}dt \\
&= \int_{s=0}^{\infty}(f(t)e^{-st}+g(t)e^{st})dt \\
&= \int_{s=0}^{\infty} f(t)e^{-st}dt + \int_{s=0}^{\infty} g(t)e^{-st}dt \\
&= L(f(t)) + L(g(t))
\end{aligned}
$$

3.4. **Ways it is used.** The direct definition is not often used directly, instead we break it apart into pieces to translate it and then solve it using the table we used earlier.

$f(t)$	$F(s) = \mathcal{L}[f(t)]$		Formula
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$	A
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$	B
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