Goals of this recitation

- Some mathematical notation.
- Basic Mathematical Models.
- Direction Fields

Contents

1. Differential Equations

1.1. **Starting with notation:** There is plenty of notation in mathematics and today we are going to go over a couple of notational issues that happen during the course. One major convention you will have to go beyond is using x or y. The question is when do we use those letters and when do we not?

If we have a variable property that we know, we tend to use the initial of that property to represent that property. For instance if we are examining time, what letter do you think we would tend to use?

(1) What letter would we use for mass?

(2) What letter would we use for function?

(3) What letter would we use for gravitational force?

Note that not all maths is done in English and sometimes how people refer to things changes over time even if the variable representation does not.

1.2. **Introducing definitions:** A differential equation is essentially just an equation with a derivative. Why are these useful? Whenever we want to examine how something changes, as derivatives come from seeing how things change. Let's now start trying to make derivatives from sentences we are trying to examine.

(1) Change in population of mice over time.

(2) Change in money over time.

(3) Change in volume over height.

Lastly try to make an equation based off the sentence below.

The change in population of mice over time is s times the population of mice.

Lastly note about function notation in this course:

2. Review of derivatives

2.1. **Online Resources:** Here is a useful resource about how to take a derivative[1] and useful facts about derivatives that are used in this course[2] that you can find in the references page.

2.2. **What does the derivative tell us about a function?**

- If the derivative of a function is _______________ then the function is increasing.
- If the derivative of a function is _______________ then the function is decreasing.

Why is this important? Sometimes when we are given a differential function, we want to sketch how something would move depending on its stating point. From our answers above we can see that we can determine some behaviour from knowing our derivative. We are going to do an example using the following facts about algebaric inequalities.

- If s is positive we can write that as the inequality _______________
- What does it mean in terms of sign for $f'(x)$ to be less than 0.

3. Drawing a direction field for an equation

3.1. Getting the geometric data from the algebra. We are going to examine the following equation

$$\frac{dP}{dt} = 10 - 2P$$

Let's find out when P is increasing algebraically.

Let's find out when P is decreasing algebraically.

Lets graph $\frac{dP}{dt}$ versus P and see what this means for the derivative graphically.

Let's draw the direction field for $\frac{dP}{dt}$

4. Solving some differential equations

4.1. **Reviewing the fundamental theorem of calculus.** One very important thing the fundamental theorem of calculus tells us the differentiation and integration reverse each other. We are going to use this fact to get some integration rules.

If $f(x) = x^5 + C$
Then $f'(x) =$

Create an integration rule

Let x be a variable and f stand for a function letter.

Create an integration rule

If $f(x) = \sin(x)$ + C
Then $f'(x) =$

Let x be a variable and f stand for a function letter.

Create an integration rule

If $f(x) = e^x$ + C
Then $f'(x) =$

Let k be a variable and m stand for a function letter.

Create an integration rule

If $f(x) = \ln(|x|)$+ C
Then $f'(x) = \frac{1}{x}$

Let u represent $u(t)$ and then differentiate the following function to create an integration rule.

$$\frac{d}{dt}\ln(|u + C|)$$

5. Classifying Differential equations

5.1. **What is an ODE?.** An ODE is an equation that contains a function and its derivatives.

Create an ODE from the following equations.

- $$x(t) = e^{2t}$$
  $$x'(t) = 2e^{2t}$$

- $$x(t) = e^{-.5t}$$
  $$x'(t) = -.5e^{-.5t}$$

- $$x(t) = 4e^{3t}$$
  $$x'(t) = 12e^{3t}$$

5.2. **The order.** The order of a differential equation is the order of the highest derivative that appears in the equation. What was the order of the ODE's you created?

When someone asks you what the order of an ODE is, what are they asking you?

References

[1] Video about differentiation rules: https://www.youtube.com/watch?v=FLAm7Hqm-58.
[2] video about how derivatives are used: https://www.youtube.com/watch?v=Dyl7jPlJXOM.

6. Bonus Work if time

This is where we will create the power rule for integration if I am finished a couple of minutes early

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