Goals of this recitation

- Eigenvalues.
- Eigenvectors.
- Real unique eigenvalues in 2 or 3 dimensions.
- What to do with repeated eigenvalues.
- Complex eigenvalues in 2 or 3 dimensions.

Contents

1. Determinants

1.1. **2 Dimensions.** For a matrix A of the following form

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\det(A) = ad - bc$. Find the determinant in the following cases.

(1)

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

(2)

$$A = \begin{pmatrix} 3 & 6 \\ 7 & 9 \end{pmatrix}$$

1.2. **3 Dimensions.** For a matrix A of the following form

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix}$$

Then we get that

$$\det(A) = a \cdot \det \begin{pmatrix} e & f \\ i & j \end{pmatrix} - b \cdot \det \begin{pmatrix} d & f \\ h & j \end{pmatrix} + c \cdot \det \begin{pmatrix} d & e \\ h & i \end{pmatrix}$$

Lets do an example

$$A = \begin{pmatrix} 1 & -1 & -1 \\ 3 & -1 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

Now you try!

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 0 & 3 \end{pmatrix}$$

2. Real Eigenvalues

2.1. **Example without repeated roots.** We are going to learn about how to deal with eigenvectors and eigenvalues for situations with 3 dimensions. We are going to do an example with a matrix to find the eigenvalues of that matrix and we will do it together. Find the eigenvalues of the following matrix.

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

Now find the eigenvectors of the previous matrix

2.2. **Example with repeated roots without generalised eigenvalues.** We are going to find the eigenvalues of the following vector

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

a

2.3. **Example with repeated roots with generalised eigenvalues.** We are going to find the eigenvalues of the following vector

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

a

3. Complex eigenvalues

3.1. **Complex conjugation.** In complex analysis we say that the complex conjugate of a complex number $\overline{a+ib} = a - ib$. Find the complex conjugates in the following cases.

(1)

$$\overline{3+2i} =$$

(2)

$$\overline{5-7i} =$$

(3)

$$\overline{3+i+i} =$$

3.2. **Forming eigenvectors with complex eigenvalues.** When finding our eigenvectors, we will get eigenvectors with complex values, to figure out what the eigenvectors are we need to get them into a certain form like below.

$$u(t) + iv(t)$$

Then our eigenvectors are $u(t)$ and $v(t)$. Lets do an example below.

$$\begin{pmatrix} 4 \\ \sin(t) \\ i\cos(t) \end{pmatrix} \left(ie^t + ln(t)\right)$$

Lets do an example with complex eigenvalues

$$x' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} x$$

a

Dept. of Mathematics, Colorado State University, Fort Collins, CO, USA
Email address: brian.collery@colostate.edu