Goals of this recitation

- Understand where integration factors come from.
- Separability.
- Modeling first order equations.

Contents

1. Connections between wanted operations and used operations

Let's go through a few scenarios with algebra to figure out some natural processes we do in mathematics. **The goal is not to figure out the answer but rather how we got there.**

(1) Suppose we wanted an unknown value that when we add 5 to it we get 8.
   (a) Write out and solve this equation. Do all the steps.

   (b) What operations were originally mentioned in the question?

   (c) What operations did you use to solve the question?

(2) Suppose we wanted an unknown value that when multiplied by 2 we get 18.

(a) Write out and solve this equation. Do all the steps.

(b) What operations were originally mentioned in the question?

(c) What operations did you use to solve the question?

(3) Reflecting upon the process of how you did the last two questions. Answer the following questions.

(a) Was there any connection between the operations mentioned in the question and the operations used to solve the question?

(b) Suppose that you were in the situation where you wanted the derivative of a function to be something and you did not know what the original function was. Would there be any operation that you might expect to use in order to solve for your original function?

(c) Suppose we had the following equation

$$\frac{d}{dx}f(x) = g(x)$$

Write out the operation you would use to solve for $f(x)$ notationally.

2. Examining some differential equations and seeing what we need

2.1. **Building intuition.** In this subsection, we are going to try to solve a few different problems to try see how to develop strategies to solve some differential equations.

Let's examine the following ODE

$$\frac{dy}{dx} + y = 0$$

The sum rule

If $f(x) = g(x) + h(x)$
Then $\dfrac{df}{dx} = \dfrac{dg}{dx} + \dfrac{dh}{dx}$

The product Rule
If $f(x) = g(x)h(x)$
Then $\dfrac{df}{dx} = g(x)\frac{dh}{dx} + \frac{dg}{dx}h(x)$

Chain Rule

If $f(x) = g(h(x))$
Then $\dfrac{df}{dx} = \dfrac{dg}{dh} \cdot \dfrac{dh}{dx}$

Lets examine our ODE and see what properties it has.

Operations:

Any difference in behaviour of the terms?

If you had to pick the result of a differentiation rule that was the closest in behaviour to this, which one would you pick and why?

Let's do a few attempts to solve this problem that are wrong to see what the answer should be.

What was the behaviour of the functions that caused an issue in our attempts?

So if we want to pick a function that works, what should this function do?

What function are we talking about?

Let's solve this equation together now.

2.2. **Stretching it further.** Let's look at the following ODE

$$\frac{dy}{dx} + 2y = 0$$

and let's try use the strategy that we used before.

2.3. **Exercises.**

$$y' + \frac{1}{2}y = 2t \quad y(0) = -1$$

3. Separability

3.1. **What is it?** For a differential equation to be separable, we need to be able to write the equation as follows.

$$N(x)dx + M(y)dy = 0$$

3.2. **Why we care?** Can solve separable equations in a specific way that is usually not allowed. Let's do an example.

$$\frac{dy}{dx} = \frac{x - e^x}{y + e^y}$$

4. Modeling first order equations

What does a derivative measure about a function?

So if we wish to write an equation that is a first order equation, there are two things we need to figure out.

(1)
(2)

Lets do a question together that does this.
Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

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