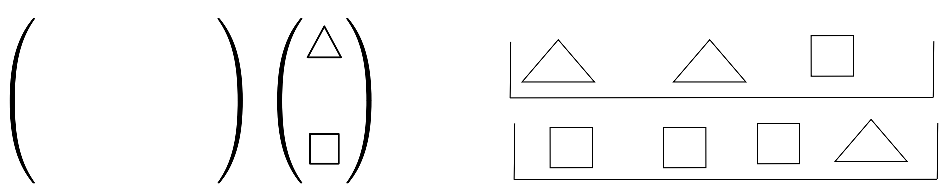
Goals of this recitation

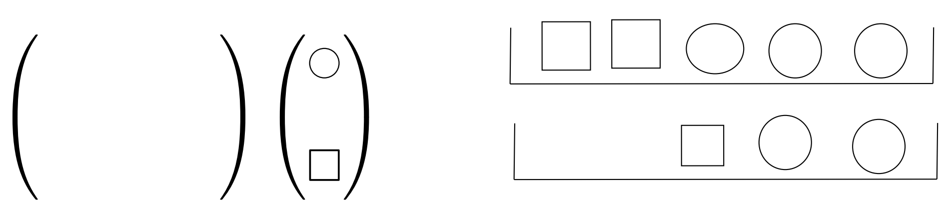
- Creating Matrices from equations.
- Creating operations with matrices.
- Intuition on vectors.
- Using matrices on vectors.
- Fix the last section to add in how matrices can take in vectors and create new vectors.

Contents

1. Creating Matrices from equations

1.1. Creating intuition on matrices. Split page in two





1.2. **Extending this to expressions.** Lets do this with some equations!

$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$	$\begin{matrix} 3x + 4y \\ 2x + y \end{matrix}$
$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$	$\begin{matrix} -2x + 6y \\ 5y \end{matrix}$
$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$	$\begin{matrix} -6x + 7y \\ x + 8y \end{matrix}$

Convert the following set of equations into a matrix.

$$\begin{matrix} 2x & +6y & +7z \\ -4x & +3y & +5z \\ 6x & +7y & -2z \\ x & +5y & -4z \end{matrix}$$

Write the matrix below as a set of equations to the right of it.

$$\begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

1.3. **Extending this to equations.** Lets make the expressions equations

$$\begin{pmatrix} & & \\ & & \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$$\begin{aligned} x + 2y &= 1 \\ 4x + 8y &= 2 \end{aligned}$$

$$\begin{pmatrix} & & \\ & & \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$$\begin{aligned} -2x + 6y &= -2 \\ x + y &= 7 \end{aligned}$$

$$\begin{pmatrix} & & \\ & & \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

$$\begin{aligned} -4x + 9y &= 4 \\ 2x + 2y &= 3 \end{aligned}$$

For the following equation, treat x and x' and y and y' as separate variables and try to create a matrix equation out of the following equations.

$$\begin{aligned} x' &= 3x + 1 \\ y' &= x + 2y \end{aligned}$$

2. Creating operations with matrices

2.1. Matrix Notation. We are going to be building operations with matrices in this section so we need to start creating notation to talk about notation. Often times a matrix is denotes with a capital letter and we use lower case with subscripts to represent a particular entry in the matrix. Lets talk through an example. Let M be described as below.

$$M = \begin{pmatrix} 3 & 4 \\ 1 & -3 \end{pmatrix}$$

Then if we wanted to talk about a particular entry in this matrix, we would use the notation m_{rc} where r represent the row and c stands for the column. In a lot of textbooks they use i and j but for now I am going to use these letters as they make more sense to me but you should be aware of this if you use other resoruces.

What do you think m_{12} stands for?

What is $m_{12} =$ Try answer the following questions.

$$A = \begin{pmatrix} 3 & 1 \\ 9 & 7 \end{pmatrix}$$

What is $a_{21} =$

What is $a_{11} =$

$$B = \begin{pmatrix} 6 & -12 \\ 10 & 27 \end{pmatrix}$$

What is $b_{12} =$

What is $b_{22} =$

$$R = \begin{pmatrix} 4 & -5 & 7 \\ 1 & 0 & 3 \\ 32 & 91 & -15 \end{pmatrix}$$

What is $r_{33} =$

What is $r_{13} =$

What is $r_{32} =$

2.2. **Intuition behind matrix addition.** We have used matrices to store data with order, now we want to create operations on said data in order to be more functional. Lets look at certain circumstances and see if we can create operations from them.

$$\begin{pmatrix} 1 & 3 \\ 9 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2 & -3 \\ 1 & 21 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 10 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 6 & 7 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 & 5 \\ 8 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Now suppose that $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$. What do you think we will get for $A + B$?

$$A + B =$$

3. Vectors

3.1. **Intuition on vectors.** We will be trying to solve the following questions in relation to the the directions given.

(1) Suppose you went 2 steps east and 1 step north. Then you went a further 3 steps east and 10 steps north. Could you describe how far you have traveled in terms of steps east and north?

(2) Suppose you went 4 steps east and 2 steps north. Then you went a further 1 steps east and 5 steps north. Could you describe how far you have traveled in terms of steps east and north?

(3) Suppose you went -1 steps east and 12 steps north. Then you went a further 2 steps east and 3 steps north. Could you describe how far you have traveled in terms of steps east and north?

(4)
$$(3 \text{ East} + 4 \text{ North}) + (2 \text{ East} + 7 \text{ North}) =$$
Suppose if I represented east by i and north by j, what would you say the answer to the question below would be?
$$(2i + j) + (4i + 6j) =$$
Suppose we were to go further with more general notation.
$$(ai + bj) + (ci + dj) =$$

Note to Brian in case I forget during class: Draw and discuss i and j as directions on a graph.

3.2. **Using notation.**

(1)
$$4\overrightarrow{i} + 3\overrightarrow{j} + 6\overrightarrow{i} - \overrightarrow{j} =$$

(2)
$$2\overrightarrow{i} + 5\overrightarrow{j} + \overrightarrow{i} - 3\overrightarrow{j} =$$

4. Matrices in relation to vectors

4.1. **Examing a useful situation.** A case where we care about Matrices is where we put in vectors and get out vectors of the same dimension (we can have alternate dimensions but this is the situation I care about).

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} i_{in} \\ j_{in} \end{pmatrix} = \begin{pmatrix} i_{out} \\ j_{out} \end{pmatrix}$$

Convert the question above into a set of equations and then let's talk about what each entry in the matrix does.

What purpose does a_{11} have in this situation?

What purpose does a_{21} have in this situation?

4.2. **Creating Matrix functions.** Lets talk about strategy based off of what we did at the end of the last page. To do this we will focus on what interacts with each other and what does not and see if we can break down a strategy to do what we want.(Note that there are multiple equivalent viable strategies I am just going to use mine)

Let's use this case to do an example. Let's talk about rotating vectors counter clockwise.

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