Goals of this recitation

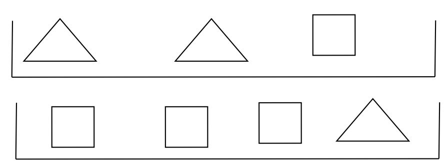
- Position to vectors.
- Derivatives with matrices.
- Sketching the path of vectors.

Contents

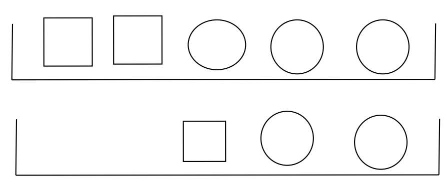
1. Intuition about Matrices

1.1. **Drawing matrices.** Refresher on things.

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}\begin{pmatrix}\triangle\\ \square\end{pmatrix}$$



$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}\begin{pmatrix}\bigcirc\\ \square\end{pmatrix}$$



$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix}\bigcirc\\ \square\end{pmatrix}$$

1.2. **Drawing Matrix multiplication.** : Let's draw this to see what is happening.

$$\begin{pmatrix} 1 & & 2 \\ & & \\ 2 & & 3 \end{pmatrix} \begin{pmatrix} 2 & & 1 \\ & & \\ 1 & & 3 \end{pmatrix} \begin{pmatrix} \bigcirc \\ \\ \square \end{pmatrix} = \begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \bigcirc \\ \\ \square \end{pmatrix}$$

Try draw this yourself! Think about the process of gathering things together.

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} \bigcirc \\ \triangle \end{pmatrix} = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \begin{pmatrix} \bigcirc \\ \triangle \end{pmatrix}$$

1.3. **Process for matrix multiplication.** Lets do some matrix multiplication with some examples.

$$\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 3 \\ 9 & 2 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 7 & 1 \\ 9 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} =$$

2. Derivatives

2.1. A perspective on one dimensional cases. Let's talk about what we need (and don't need!) for in a derivative in one dimension.

What we need:

What we don't need:

2.2. How we use the derivative in one dimension. Let's sketch a graph below and talk a little about how we try use derivatives in one dimension.

Let's write down this perspective on how we use the derivative in one dimension and reflect upon this for later in this recitation.

3. Derivatives in multiple dimensions

3.1. Issues when extending this to two dimensions. What do you think will become an issue when we start talking about derivatives in multiple dimensions.

If this is true, what will our solution to this have to contain that one dimensional derivatives would not?

If we are to generalise our method of using the derivative from one dimensional derivative, what would we expect to happen.

3.2. **Using matrices to talk about derivatives.** Let's take these differential equations and turn them into matrix equations and then we will use them with points (1,0) and (0,1).

$$x' = 2x$$
$$y' = 4y$$

Let's take these differential equations and turn them into matrix equations and then we will use them with points (1,1) and (1,-1).

$$x' = -3x - y$$
$$y' = -x - 3y$$

Let's talk about another example, suppose that when $(x, y) = (1, 1)$, $(x', y') = (-.5, .25)$. Sketch this situation and then move onto the last part of this exercise.

Go through the previous examples and imagine the origin was the equilibrium point. Which of these do you think would be stable/unstable?

What makes it easier to talk about stability/ not easy to talk about stability.

4. Talking about Eigenvectors and Eigenvalues

When is it easy to talk about stability?

What does that mean mathematically?

Let's make a definition about when we would like to examine to determine stability.

Dept. of Mathematics, Colorado State University, Fort Collins, CO, USA
Email address: brian.collery@colostate.edu