Hochschule Esslingen
University of Applied Sciences

01_Introduction and Fundamentals

Lecture: Intelligent Data Analytics

Content

7. Problems

Problem 4:

Consider a white noise time series w_t, which is smoothed by building a moving average which averages the current value of w_t and its immediate neighbours in the past and future. That is let v_t be a new time series defined by:

$$v_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1})$$

a) Assume that $w_t \sim N(0,1)$ and plot a realisation of w_t for 500 data points by using the **R** commands

```
>set.seed(150)

>x=rnorm(500)

>plot.ts(x)
```

b) Calculate v_t and plot the result under the plot of w_t. Compare and discuss your results! In which way is the plotted time series w_t different from v_t?

Problem 4:

c) Is the time series v_t stationary? Calculate the mean, the autocovariance function (ACVF) and the autocorrelation function (ACF) for v_t.

d) Plot the autocorrelation function (ACF) of v_t in **R** for the lags $h = 4;\ 3;\ 2;\ 1; 0$. Use the **R** function `acf()`

Problem 5:

Consider a signal-plus-noise model of the general form $x_t = s_t + w_t$, where w_t is a Gaussian white noise with $\sigma_w = 1$ (s.t. $w_t \sim N(0,1)$ and w_t is iid). Simulate and plot $n = 200$ observations from each of the following two models

a) $x_t = s_t + w_t$, where

$$s_t = \begin{cases} 0, & t = 1, \dots, 100 \\ 10exp\left\{-\frac{(t-100)}{20}\right\} cos(2\pi t/4), & t = 101, \dots, 200 \end{cases}$$

b) $x_t = s_t + w_t$, where

$$s_t = \begin{cases} 0, & t = 1, \dots, 100 \\ 10exp\left\{-\frac{(t-100)}{200}\right\} cos(2\pi t/4), & t = 101, \dots, 200 \end{cases}$$

c) Compare the general appearance of the series in a) and b) with the earthquake series and the explosion series shown on slight 10.

Problem 6:

Consider two different MA(1) models:

(1) $x_t = w_t + \frac{1}{5} w_{t-1}, \qquad w_t \sim iid \ \mathrm{N}(0,25)$

(2) $y_t = v_t + 5 v_{t-1}, \qquad v_t \sim iid \ \mathrm{N}(0,1)$

a) Simulate two time series and make a plot of both of them.

b) Make a plot of the ACF function of both processes.

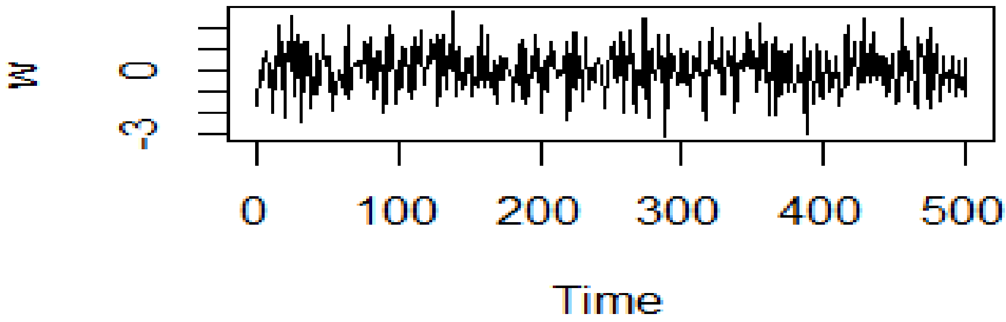
c) Calculate the mean and the autocovariance function for all lags $\mathrm{h} \geq 0$.

d) What do you discover? What is the difference between the two models?

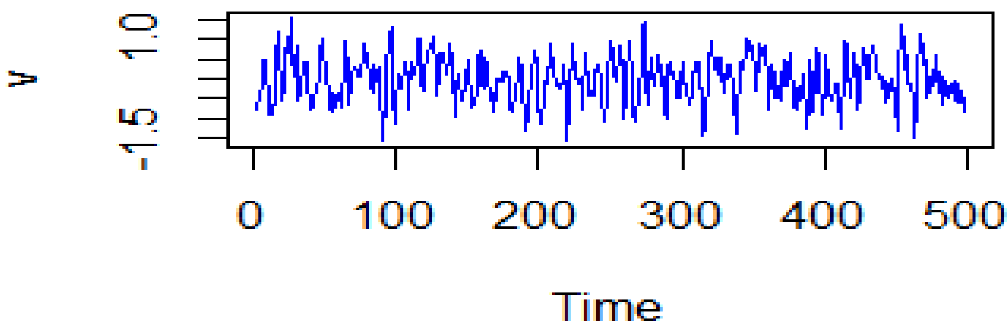
Solution to Problem 4:

b) #Solution to Problem 1

```
set.seed(150)
w=rnorm(100)
w=ts(w)
par(mfrow=c(2,1))
plot.ts(w)
v=1/3*(lag(w,1)+w+lag(w,-1))
plot.ts(v, col="blue")
```



The time series v_t is smoother than w_t. Large peaks (positive and negative) become smaller.



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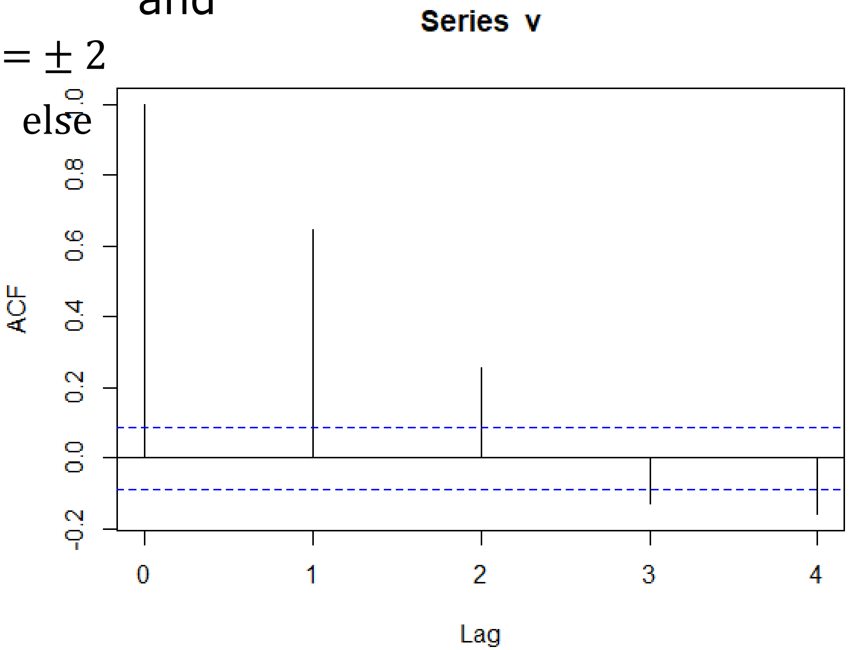
Solution to Problem 4:

c) Mean value: $\mu_v(t) = E(v_t) = \frac{1}{3}(E(w_{t-1}) + E(w_t) + E(w_{t+1})) = 0$

ACVF : $\gamma_v(h) = cov(v_{t+h}, v_t) = \begin{cases} \frac{1}{3}\sigma^2, & \text{if } h=0 \\ \frac{2}{9}\sigma^2, & \text{if } h=\pm 1 \\ \frac{1}{9}\sigma^2, & \text{if } h=\pm 2 \\ 0, & \text{else} \end{cases}$ and

ACF: $\rho_v(h) = \frac{\gamma_v(h)}{\gamma_v(0)} = \begin{cases} 1, & \text{if } h=0 \\ \frac{2}{3}, & \text{if } h=\pm 1 \\ \frac{1}{3}, & \text{if } h=\pm 2 \\ 0, & \text{else} \end{cases}$

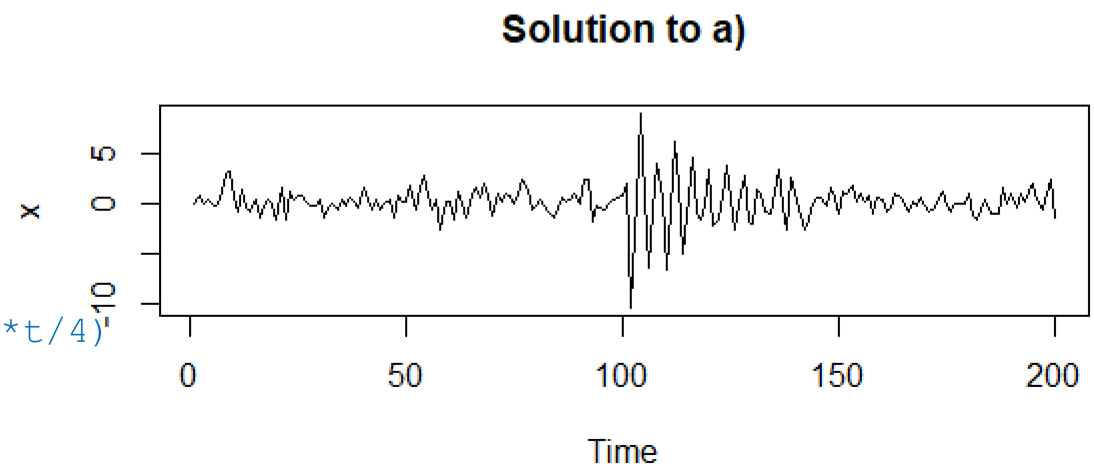


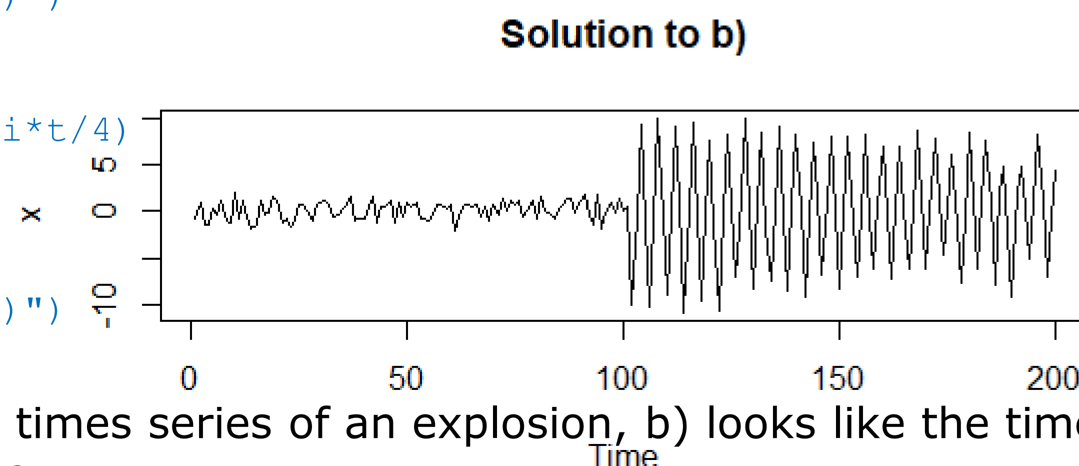
d) `acf(v, lag.max=4)`

Solution to Problem 5:

a)+b)

```
#Solution to Problem 2 a)
t=time(1:200)
s=10*exp(-(t-100)/20)*cos(2*pi*t/4)
s[1:100]=0
w=rnorm(200)
x=s+w
par(mfrow=c(2,1))
ts.plot(x, main="Solution to a)")
#Solution to Problem 2 b)
t=time(1:200)
s=10*exp(-(t-100)/200)*cos(2*pi*t/4)
s[1:100]=0
w=rnorm(200)
x=s+w
ts.plot(x, main="Solution to b)")
```

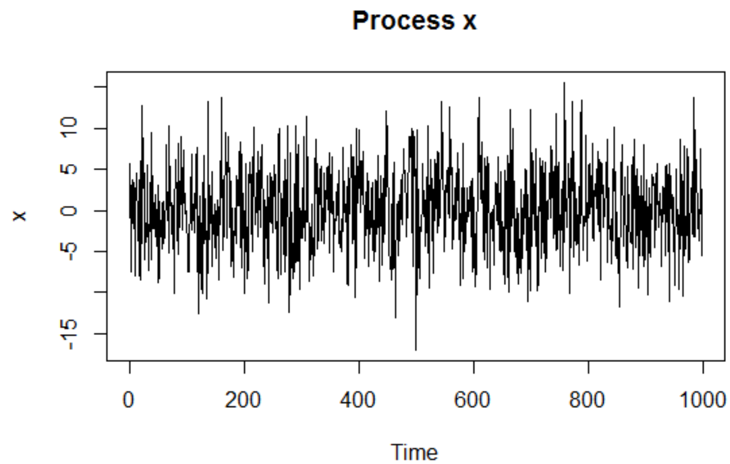


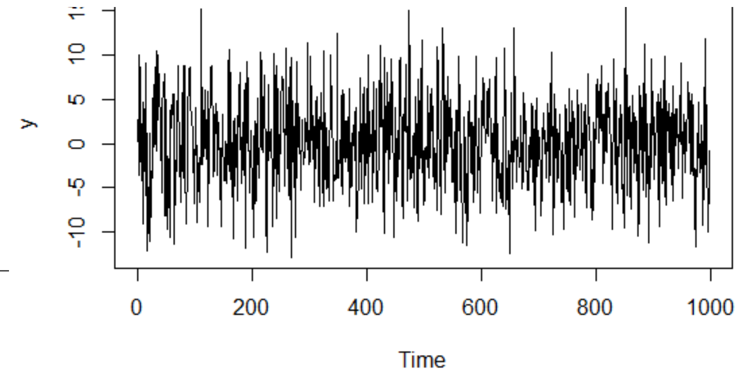


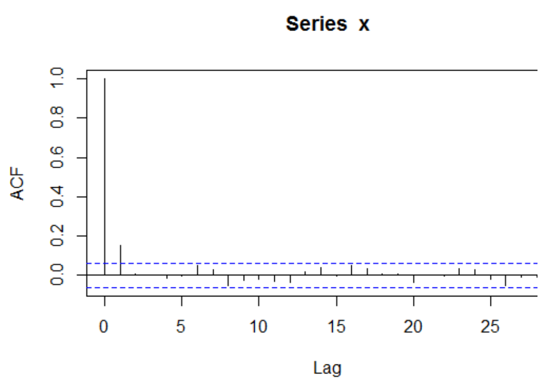
c) While a) looks like the times series of an explosion, b) looks like the time series of an earthquake

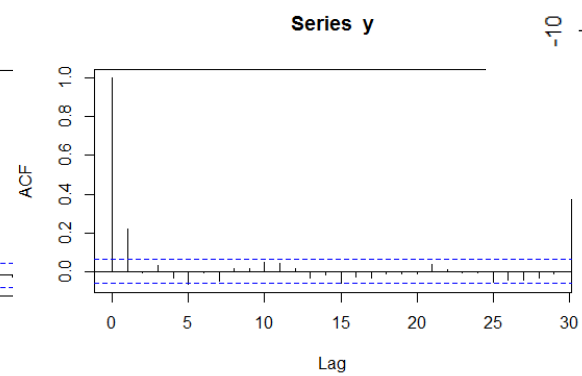
Solution to Problem 6:

```
#Solution to Problem 6 a)
w=rnorm(1000,sd=5)
w=ts(w)
x=w+0.2*lag(w)
plot(x, main="Process x")
v=rnorm(1000)
v=ts(v)
y=v+5*lag(v)
plot(y, main="Process y")
#Solution to Problem 6 b)
```









Solution to Problem 6:

c) $E(x_t) = E(w_t) + 0.2 \cdot E(w_{t-1}) = 0$ und $E(y_t) = E(v_t) + 5 \cdot E(v_{t-1}) = 0$

$$cov(x_t, x_{t-h}) = \gamma_x(h) =$$

$$cov(w_t + 0.2 \cdot w_{t-1}, w_{t-h} + 0.2 \cdot w_{t-h-1})$$

$$= cov(w_t, w_{t-h}) + 0.2cov(w_t, w_{t-h-1}) + 0.2cov(w_{t-1}, w_{t-h}) + 0.04cov(w_{t-1}, w_{t-h-1})$$

$$= \begin{cases} 25(1 + 0.04) & \text{für } h = 0 \\ 25 \cdot 0.2 & \text{für } h = \pm 1 \\ 0 & \text{sonst} \end{cases} \Longrightarrow \rho_x(h) = \begin{cases} 1 & für\ h = 0 \\ \frac{25 \cdot 0.2}{25 \cdot (1+0.04)} = \frac{5}{26} & für\ h = \pm 1 \\ 0 & sonst \end{cases}$$

$$cov(y_t, y_{t-h}) = \gamma_y(h) =$$

$$cov(v_t + 5 \cdot v_{t-1}, v_{t-h} + 5 \cdot v_{t-h-1})$$

$$= cov(v_t, v_{t-h}) + 5cov(v_t, v_{t-h-1}) + 5cov(v_{t-1}, v_{t-h}) + 25cov(v_{t-1}, v_{t-h-1})$$

$$= \begin{cases} 25 + 1 & \text{für } h = 0 \\ 5 & \text{für } h = \pm 1 \\ 0 & \text{sonst} \end{cases} \Longrightarrow \rho_x(h) = \begin{cases} 1 & für\ h = 0 \\ \frac{5}{26} & für\ h = \pm 1 \\ 0 & sonst \end{cases}$$

Solution to Problem 6:

d)

The two modells are not distinguishable by looking at their plots, their mean value or their ACF.

The noise is different, but since only the whole time series is observed, not the noise, we cannot distingues between them.