References and Further Reading

The material in Section 3.1 is from Gonzalez [1986]. Additional reading for the material in Section 3.2 may be found in Schowengerdt [1983], Poyton [1996], and Russ [1999]. See also the paper by Tsujii et al. [1998] regarding the optimization of image displays. Early references on histogram processing are Hummel [1974], Gonzalez and Fittes [1977], and Woods and Gonzalez [1981]. Stark [2000] gives some interesting generalizations of histogram equalization for adaptive contrast enhancement. Other approaches for contrast enhancement are exemplified by Centeno and Haertel [1997] and Cheng and Xu [2000]. For enhancement based on an ideal image model, see Highnam and Brady [1997]. For extensions of the local histogram equalization method, see Caselles et al. [1999], and Zhu et al. [1999]. See Narendra and Fitch [1981] on the use and implementation of local statistics for image enhancement. Kim et al. [1997] present an interesting approach combining the gradient with local statistics for image enhancement.

Image subtraction (Section 3.4.1) is a generic image processing tool widely used for change detection. As noted in that section, one of the principal applications of digital image subtraction is in mask mode radiography, where patient motion is a problem because motion smears the results. The problem of motion during image subtraction has received significant attention over the years, as exemplified in the survey article by Meijering et al. [1999]. The method of noise reduction by image averaging (Section 3.4.2) was first proposed by Kohler and Howell [1963]. See Peebles [1993] regarding the expected value of the mean and variance of a sum of random variables.

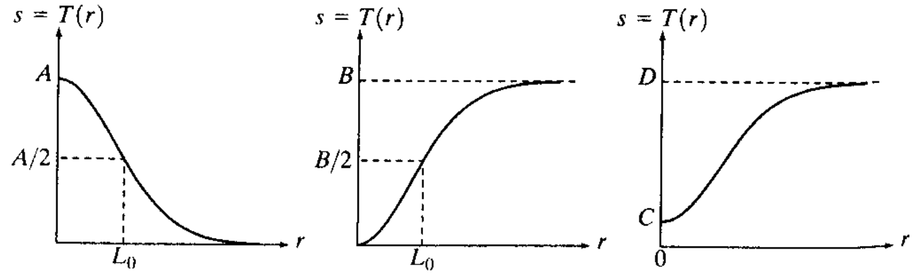
For additional reading on linear spatial filters and their implementation, see Umbaugh [1998], Jain [1989], and Rosenfeld and Kak [1982]. Rank-order filters are discussed in these references as well. Wilburn [1998] discusses generalizations of rank-order filters. The book by Pitas and Venetsanopoulos [1990] also deals with median and other nonlinear spatial filters. A special issue of *IEEE Transactions in Image Processing* [1996] is dedicated to the topic of nonlinear image processing. The material on high-boost filtering is from Schowengerdt [1983]. We will encounter again many of the spatial filters introduced in this chapter in discussions dealing with image restoration (Chapter 5) and edge detection (Chapter 10).

Problems

See inside front cover

Detailed solutions to the problems marked with a star can be found in the book web site. The site also contains suggested projects based on the material in this chapter.

3.1 Exponentials of the form $e^{-\alpha r^2}$, with α a positive constant, are useful for constructing smooth gray-level transformation functions. Start with this basic function and construct transformation functions having the general shapes shown in the following figures. The constants shown are *input* parameters, and your proposed transformations must include them in their specification. (For simplicity in your answers, L_0 is not a required parameter in the third curve.)



(a) (b) (c)

3.2 ★ (a) Give a continuous function for implementing the contrast stretching transformation shown in Fig. 3.2(a). In addition to m, your function must include a parameter, E, for controlling the slope of the function as it transitions from low to high gray-level values. Your function should be normalized so that its minimum and maximum values are 0 and 1, respectively.

(b) Sketch a family of transformations as a function of parameter E, for a fixed value $m = L/2$, where L is the number of gray levels in the image.

(c) What is the smallest value of E that will make your function *effectively* perform as the function in Fig. 3.2(b)? In other words, your function does not have to be identical to Fig. 3.2(b). It just has to yield the same result of producing a binary image. Assume that you are working with 8-bit images, and let $m = 128$. Also, let C be the smallest positive number representable in the computer you are using.

3.3 Propose a set of gray-level-slicing transformations capable of producing all the individual bit planes of an 8-bit monochrome image. (For example, a transformation function with the property $T(r) = 0$ for r in the range $[0, 127]$, and $T(r) = 255$ for r in the range $[128, 255]$ produces an image of the 7th bit plane in an 8-bit image.)

3.4 ★ (a) What effect would setting to zero the lower-order bit planes have on the histogram of an image in general?

(b) What would be the effect on the histogram if we set to zero the higher-order bit planes instead?

★3.5 Explain why the discrete histogram equalization technique does not, in general, yield a flat histogram.

3.6 Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization will produce exactly the same result as the first pass.

3.7 In some applications it is useful to model the histogram of input images as Gaussian probability density functions of the form

$$p_r(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(r-m)^2}{2\sigma^2}}$$

where m and σ are the mean and standard deviation of the Gaussian PDF. The approach is to let m and σ be measures of average gray level and contrast of a given image. What is the transformation function you would use for histogram equalization?

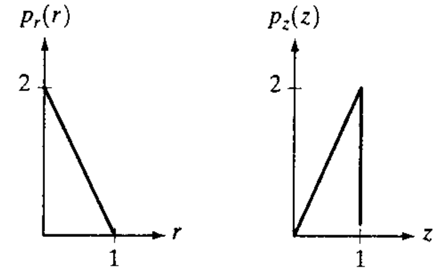
★3.8 Assuming continuous values, show by example that it is possible to have a case in which the transformation function given in Eq. (3.3-4) satisfies Conditions (a) and (b) in Section 3.3.1, but its inverse may fail to be single valued.

3.9 (a) Show that the discrete transformation function given in Eq. (3.3-8) for histogram equalization satisfies conditions (a) and (b) in Section 3.3.1.

(b) Show by example that this does not hold in general for the inverse discrete transformation function given in Eq. (3.3-9).

★ (c) Show that the inverse discrete transformation in Eq. (3.3-9) satisfies Conditions (a) and (b) in Section 3.3.1 if none of the gray levels r_k, $k = 0, 1, \ldots, L - 1$, are missing.

3.10 An image has the gray level PDF $p_r(r)$ shown in the following diagram. It is desired to transform the gray levels of this image so that they will have the specified $p_z(z)$ shown. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.



★**3.11** Propose a method for updating the local histogram for use in the local enhancement technique discussed in Section 3.3.3.

3.12 Two images, $f(x, y)$ and $g(x, y)$, have histograms h_f and h_g. Give the conditions under which you can determine the histograms of

(a) $f(x, y) + g(x, y)$

(b) $f(x, y) - g(x, y)$

(c) $f(x, y) \times g(x, y)$

(d) $f(x, y) \div g(x, y)$

in terms of h_f and h_g. Explain how to obtain the histogram in each case.

3.13 Consider two 8-bit images whose gray levels span the full range from 0 to 255.

(a) Discuss the limiting effect of repeatedly subtracting image (b) from image (a).

(b) Would reversing the order of the images yield a different result?

★**3.14** Image subtraction is used often in industrial applications for detecting missing components in product assembly. The approach is to store a "golden" image that corresponds to a correct assembly; this image is then subtracted from incoming images of the same product. Ideally, the differences would be zero if the new products are assembled correctly. Difference images for products with missing components would be nonzero in the area where they differ from the golden image. What conditions do you think have to be met in practice for this method to work?

3.15 Prove the validity of Eqs. (3.4-4) and (3.4-5).

3.16 In an industrial application, X-ray imaging is to be used to inspect the inside of certain composite castings. The objective is to look for voids in the castings, which typically appear as small blobs in the image. However, due to properties in of the casting material and X-ray energy used, high noise content often makes inspection difficult, so the decision is made to use image averaging to reduce the noise and thus improve visible contrast. In computing the average, it is important to keep the number of images as small as possible to reduce the time the parts have to remain stationary during imaging. After numerous experiments, it is concluded that decreasing the noise variance by a factor of 10 is sufficient. If the imaging device can produce 30 frames/s, how long would the castings have to remain stationary during imaging to achieve the desired decrease in variance? Assume that the noise is uncorrelated and has zero mean.

3.17 The implementation of linear spatial filters requires moving the center of a mask throughout an image and, at each location, computing the sum of products of the mask coefficients with the corresponding pixels at that location (see Section 3.5). In the case of lowpass filtering, all coefficients are 1, allowing use of a so-called *box-filter* or *moving-average* algorithm, which consists of updating only the part of the computation that changes from one location to the next.

★ **(a)** Formulate such an algorithm for an $n \times n$ filter, showing the nature of the computations involved and the scanning sequence used for moving the mask around the image.

(b) The ratio of the number of computations performed by a brute-force implementation to the number of computations performed by the box-filter algorithm is called the *computational advantage*. Obtain the computational advantage in this case and plot it as a function of n for $n > 1$. The $1/n^2$ scaling factor is common to both approaches, so you need not consider it in obtaining the computational advantage. Assume that the image has an outer border of zeros that is thick enough to allow you to ignore border effects in your analysis.

3.18 Discuss the limiting effect of repeatedly applying a 3×3 lowpass spatial filter to a digital image. You may ignore border effects.

3.19 ★ **(a)** It was stated in Section 3.6.2 that isolated clusters of dark or light (with respect to the background) pixels whose area is less than one-half the area of a median filter are eliminated (forced to the median value of the neighbors) by the filter. Assume a filter of size $n \times n$, with n odd, and explain why this is so.

(b) Consider an image having various sets of pixel clusters. Assume that all points in a cluster are lighter or darker than the background (but not both simultaneously in the same cluster), and that the area of each cluster is less than or equal to $n^2/2$. In terms of n, under what condition would one or more of these clusters cease to be isolated in the sense described in part (a)?

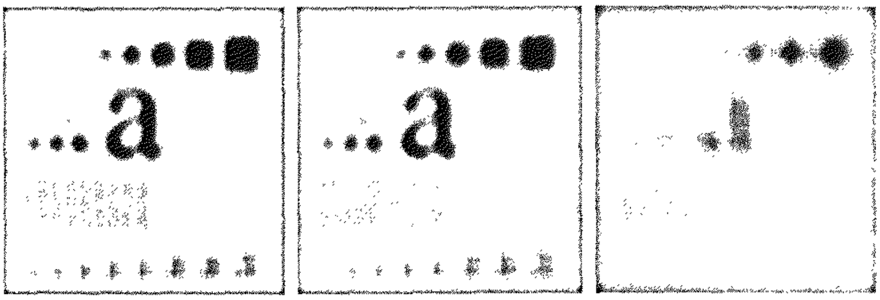
★**3.20** **(a)** Develop a procedure for computing the median of an $n \times n$ neighborhood.

(b) Propose a technique for updating the median as the center of the neighborhood is moved from pixel to pixel.

3.21 **(a)** In a character recognition application, text pages are reduced to binary form using a thresholding transformation function of the form shown in Fig. 3.2(b). This is followed by a procedure that thins the characters until they become strings of binary 1's on a background of 0's. Due to noise, the binarization and thinning processes result in broken strings of characters with gaps ranging from 1 to 3 pixels. One way to "repair" the gaps is to run an averaging mask over the binary image to blur it, and thus create bridges of nonzero pixels between gaps. Give the (odd) size of the smallest averaging mask capable of performing this task.

(b) After bridging the gaps, it is desired to threshold the image in order to convert it back to binary form. For your answer in (a), what is the minimum value of the threshold required to accomplish this, without causing the segments to break up again?

★**3.22** The three images shown were blurred using square averaging masks of sizes $n = 23, 25$, and 45, respectively. The vertical bars on the left lower part of (a) and (c) are blurred, but a clear separation exists between them. However, the bars have merged in image (b), in spite of the fact that the mask that produced this image is significantly smaller than the mask that produced image (c). Explain this.



(a) (b) (c)

3.23 Consider an application such as the one shown in Fig. 3.36, in which it is desired to eliminate objects smaller than those enclosed in a square of size $q \times q$ pixels. Suppose that we want to reduce the average gray level of those objects to one-tenth of their original average gray level. In this way, those objects will be closer to the gray level of the background and they can then be eliminated by thresholding. Give the (odd) size of the smallest averaging mask that will accomplish the desired reduction in average gray level in only one pass of the mask over the image.

3.24 In a given application an averaging mask is applied to input images to reduce noise, and then a Laplacian mask is applied to enhance small details. Would the result be the same if the order of these operations were reversed?

★ **3.25** Show that the Laplacian operation defined in Eq. (3.7-1) is isotropic (invariant to rotation). You will need the following equations relating coordinates after axis rotation by an angle θ:

$$x = x' \cos\theta - y' \sin\theta$$
$$y = x' \sin\theta + y' \cos\theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

3.26 Give a 3×3 mask for performing unsharp masking in a single pass through an image.

★ **3.27** Show that subtracting the Laplacian from an image is proportional to unsharp masking. Use the definition for the Laplacian given in Eq. (3.7-4).

3.28 **(a)** Show that the magnitude of the gradient given in Eq. (3.7-13) is an isotropic operation. (See Problem 3.25.)

(b) Show that the isotropic property is lost in general if the gradient is computed using Eq. (3.7-14).

3.29 A CCD TV camera is used to perform a long-term study by observing the same area 24 hours a day, for 30 days. Digital images are captured and transmitted to a central location every 5 minutes. The illumination of the scene changes from natural daylight to artificial lighting. At no time is the scene without illumination, so it is always possible to obtain an image. Because the range of illumination is such that it is always in the linear operating range of the camera, it is decided not to employ any compensating mechanisms on the camera itself. Rather, it is decided to use digital techniques to postprocess, and thus normalize, the images to the equivalent of constant illumination. Propose a method to do this. You are at liberty to use any method you wish, but state clearly all the assumptions you made in arriving at your design.