Atomic Spectroscopy And The Duane Hunt Law

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This experiment was split into two parts. Atomic Spectroscopy and the Duane-Hunt Law. Both of these concepts use mathematical techniques and analysis to get to two constant in Physics. The Rydberg Constant and the Planck's constant. Looking over how this is done and the errors in the findings is also discussed in this report. The final values obtained were $2.34 \times 10^{-18} \pm 0.2$J for the Rydberg Constant and $7.169 \times 10^{-34} \pm 0Js^{-1}$ for the Planck's constant.

1 Introduction

Atomic spectroscopy is a method used to understand the composition of elements by using their electromagnetic or mass spectrum. Using the energy characteristics of photons emitted for different atoms and measuring them using scientific apparatus, you can understand what is unique about each one. In the 17th century Sir Isaac Newton showed that with a flame, a small aperture and a prism you could disperse light into their individual frequencies. After doing this, the scientific world would never be the same again as the word 'spectrum' was now defined. Through the 17th century this idea was developed by scientists like William Herschel and William Bunsen to name a few. They found that different compounds would emit different colours when heated over a flame and this was due to the emission of energy in the form of an electromagnetic wave (visible light). In the 19th century Gustav Kirchhoff realised that each element or compound had it's own unique spectrum. By studying the spectrum of an unknown source he realised that you could determine it's chemical composition. Collaborating with William Bunsen (who i mentioned earlier), they established spectroscopy as a scientific tool for examining the composition of materials. This breakthrough allowed for the discovery of the photoelectric effect which further deepened our knowledge on Quantum Physics. Atomic spectroscopy is commonly used in manufacturing and food industries in modern society. It allows companies to use technology to understand whether the batch they create contains impurities which in turn could stop contamination in the companies product.

The use of atomic spectroscopy was also used for the discovery of Planck's constant. Without this breakthrough Quantum mechanics may never have existed.

This discovery was known as the Duane-Hunt law. In atomic physics, it's defined as the relationship between the voltage applied to an X-ray tube and the maximum frequency emitted from the target. Named after William Duane and Franklin Hunt this law allowed them to understand better the makeup of our atoms.

In this report I will be discussing my investigation to determine the Rydberg constant and the determination of Planck's constant using these ideas.

2 Theory

2.1 Atomic Spectroscopy

The theory behind the Atomic Spectroscopy experiment involves multiple ideas and equations. As discussed in the introduction, the experiment involves Hydrogen so the use of the Balmer series was needed. In this series the lowest four transitions are in the visible region so when it comes to checking what wavelengths are emitted, these will be the ones needed.

To determine whether or not the experiment was accurate, the need of the Rydberg constant is used (see equation 1). This constant is then used in the energy equation to calculate the energy used (see equation 2). In this experiment however, rearranging the equation will be necessary.

$$R_y = \frac{m_o e^4}{2(4\pi\epsilon_o)^2 \hbar} \tag{1}$$

$$E_n = R_y(\frac{1}{2^2} - \frac{1}{n^2}) \tag{2}$$

As well as the equations, the experimental setup is just as important. As shown in figure 1 this shows the diagram of what happens when the light exits the low pressure glass tube.
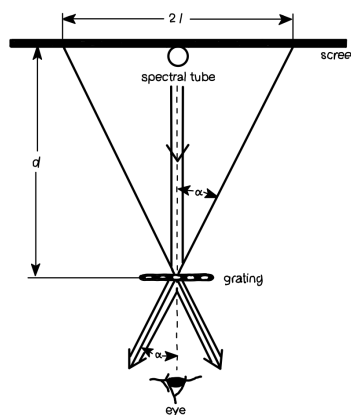
Figure 1: Diagram of atomic spectroscopy setup

2.2   Duane Hunt Law

For this section of the experiment X-rays are needed to be fired at a solid crystal of Sodium Chloride. To do this electrons inside a X-ray tube are accelerated using a Voltage that changes for repeats of the experiment. The electrons hit an anode which causes them to decelerate in a short space of time. This harsh shift in speed causes the electron to emit braking radiation. In this case, X-rays are emitted.

Braggs Law is used to convert the results of intensity vs angle into a spectrum of intensity vs wavelength. The derivation of Braggs Law is as follows:

$$n\lambda = A + B$$
$$AB = BC$$
$$n\lambda = 2AB$$
$$AB = d\sin\theta$$
$$n\lambda = 2d\sin\theta$$

Figure 2: Derivation of Braggs Law

Finally to find Planck's constant the ideology of the Duane-Hunt law is needed. It states that $eV = h\nu_{max}$. Measuring and plotting a graph of λ_{min} vs $\frac{1}{V}$ provides the opportunity to calculate the gradient and therefore the value of Planck's constant.

Shown below in figure 3 is a diagram of the experimental setup for the diffraction of X-rays through a solid crystal of Sodium Chloride and how the apparatus allows the values of intensity to be calculated.
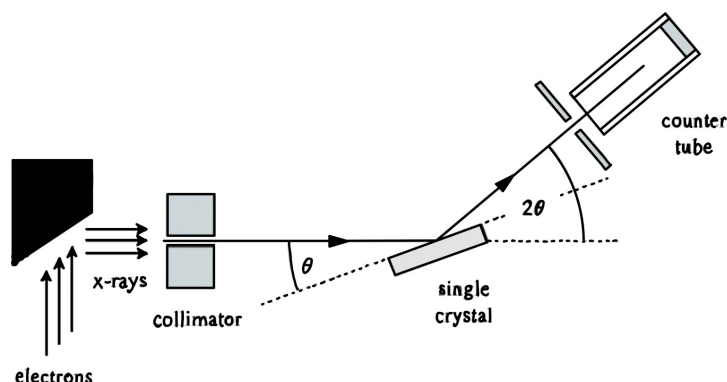
Figure 3: Experimental setup. Monochromatic X-rays are produced through the process of Braggs Law. The counter tube measures the intensity of the X-rays diffracted

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3 Methods

3.1 Atomic Spectroscopy

To start this experiment, the low pressure discharge tube containing Hydrogen is fitted into it's holder. The holder allows for the Hydrogen gas to be introduced to a potential difference. A meter ruler is placed in front of the gas tube and is centralised at 50cm.On the DC power supply (which the gas tube is now connected to), up the voltage until the gas starts to glow. The potential difference releases ionizing electrons as they are excited. Collisions leave them in various levels of excitement. As the electrons return to the ground state spectral lines are produced.

Looking through the diffraction grating, you can see that the light disperses into its component wavelengths. The first step is to calculate the wavelength of the different spectral lines. On hydrogen the three most prominent are violet,blue and red so these are used. The reason only three were measured is because it's more precise to measure light you can actually see clearly instead of a band of colour.

To be able to calculate the wavelength of the emission lines the grating equation is used (see equation 3). This equation involves a $sin\alpha$ and this is easily calculated using this equation:

$$sin(\alpha) = \frac{I}{\sqrt{d^2 + I^2}} \tag{3}$$

Where I is the half the distance of the ruler and d is the distance from the object to the diffraction grating lens. When calculating I all that needs to be done is '$50 - recordedvalue$'. Once this is deduced the next step is to use the grating equation, as shown here.

$$p\lambda = d_g sin(\alpha) \tag{4}$$

The whole experiment is revolving around wavelengths so the actual equation used is a rearranged version that looks like:

$$\lambda = \frac{d_g sin(\alpha)}{p} \tag{5}$$

where λ is the wavelength given in centimeters before conversion to nano-meters. d_g is the diffraction constant. $sin(\alpha)$ is the angle between the diffraction grating lens and the emission lines and p is the order of diffraction. The diffraction constant is 600 lines per mm as the diffraction grated lens used had this value. To find the value per mm just divide it by 600 to get the distance between two adjacent gratings in the lens.

As specified in equation two we need the energy to calculate the Rydberg constant. To do this we use the equation $E = \frac{hc}{\lambda}$, where h is Planck's constant, c is the speed of light which is

$3 \times 10^8 ms^{-1}$ and lambda which is the wavelength. Once the energy is calculated, repeating the experiment at different distances will help understand better how distance affects the outcome of the results.Measuring at three different distances (53cm,28cm and 15cm) was used.

Another thing that was useful was to repeat the steps to find the wavelength but in the negative direction on the ruler (to the left of the centre). If the values are in the uncertainty range of each other than it means that it was systematic error with the equipment setup, not the actual readings or calculations.

Finally after calculating wavelength and energy, the Rydberg constant can now be calculated. To do this a graph of $E_n vs (\frac{1}{2^2}) - (\frac{1}{n^2})$ is made. In this plot 'n' is known as the energy levels. They are different for each colour and after checking violet is n=6, blue is n=5 and red is n=3. After plotting, find the gradient to find the Rydberg constant and then check to the agreed value.

3.2   Duane-Hunt Law Method

The first step of this experiment is to set up the experimental equipment.Check the Sodium Chloride crystal is firmly secured in the holder and then turn the machine on. On the counter tube, machine values are set to allow the machine to work and to give the correct values that are needed for this specific experiment. For the first run through of the experiment the high voltage is = 35kV. The emission current is 1mA. The measuring time per angular step is 10 seconds. The angular step is 0.1 degrees.

The sensor limits are then calibrated. The lower limit is 2.5 degrees and the upper limits is 10 degrees. After pressing the coupled key to allow 2θ coupling to happen, start up the X-ray.exe software that will analysis and plot the information that is about to be measured. After making sure that the X-ray software is plotting intensity and θ press the scan key to start the machine.

Every 10 seconds the arm inside the machine will move 0.1 degrees and will record a new intensity. After it reaches 10 degrees it will stop recording and a plot should have formed on the software. Repeat this for multiple Voltages. For this experiment values were measured at 25kV, 27.5kV, 30kV, 32.5Kv and 35kV. When all the voltages have been tested interpreting the peaks is the next step. On the Xray.exe software press F5 on the keyboard and type in the value for the lattice separation. In this case this value is 564.00pm.

The computer software then uses Braggs Law to convert from intensity and angle to $n\lambda$ and intensity.From this find a line of best fit on the graph for the background spectrum before the first peak. This is from where the graph starts to increase rapidly until the bottom of the peak. This will give a value of λ_{min}. After repeating for all voltages plot a graph of λ_{min} vs $1/V$.After finding the gradient this will equal $\frac{hc}{e}$. To find Planck's constant you rearrange the equation.

4 Results

4.1 Atomic Spectroscopy results

For the first part of the results the wavelength was found for the 3 colours of light. Using the meter ruler and measuring from the centre the values were then plugged into equation 3 to find $\sin\alpha$. After, the angle is plugged into equation 5 where the diffraction grating constant is $\frac{1}{6000}cm$ and p=1. From this, the wavelength can be found. The diffraction grating constant was found from the calibration of mercury. The diffraction grating constant is the space between two of the slits on the diffraction grate which as stated above is $\frac{1}{6000}cm$. Now wavelengths can be calculated. The results are as follows:

	25cm	38cm	53cm
Blue	358nm	408nm	420nm
Violet	300nm	340nm	353nm
Red	490nm	555nm	575nm

Table 1: Table of values for the wavelength of different colour light

Looking up the actual values of the wavelengths gave these values. Blue = 353nm, Violet = 420nm and Red = 575nm. From this it shows that the values at 53cm are closest to the actual values. To make the experiment more accurate these values will be used for the final graph and calculation of the gradient.

Next step in this experiment is to find the resultant energy from these wavelengths. You find the energy using $E = \frac{hc}{\lambda}$ and the results from this are:

	25cm	38cm	53cm
Blue	5.55×10^{-19}	4.87×10^{-19}	4.73×10^{-19}
Violet	6.63×10^{-19}	5.85×10^{-19}	5.63×10^{-19}
Red	4.06×10^{-19}	3.58×10^{-19}	3.46×10^{-19}

Table 2: This table shows the energy in joules for different colours of light

The final step before plotting is to calculate the change in energy levels for each of the colours when they get excited and transition energy levels. For a hydrogen atom, violet goes from n=6 to n=2, blue goes from n=5 to n=2 and red goes from n=3 to n=2. Using these values and substituting them into $(\frac{1}{2^2} - \frac{1}{n^2})$ numbers are acquired that can be used to plot a graph of En vs $(\frac{1}{2^2} - \frac{1}{n^2})$.

Finally plotting these numbers and using python to get an accurate best fit line the graph is produced. Shown below in figure 4.
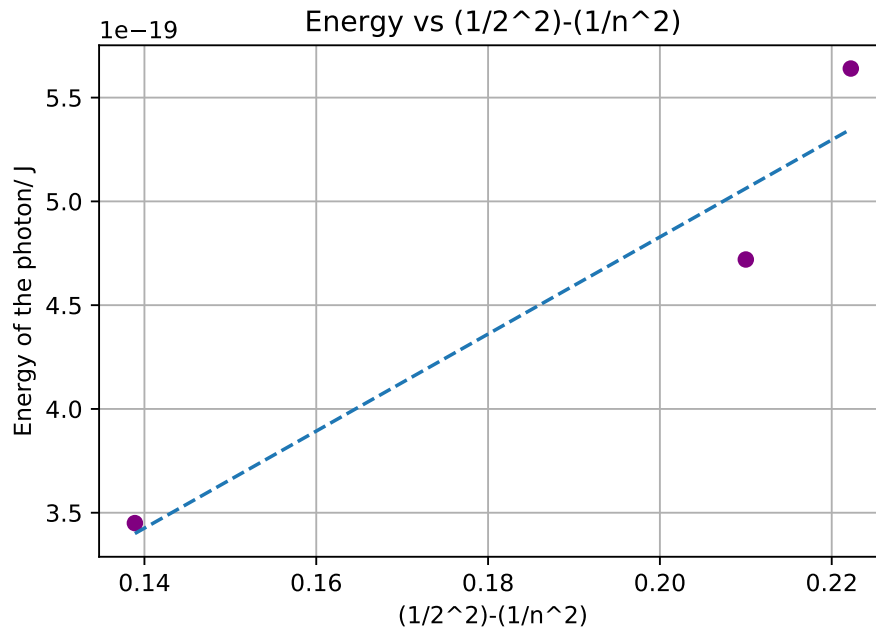
**Figure 4:** Graph showing the relationship between energy in joules and change in energy levels.

From this I used python to calculate the gradient. From the method section it was discussed that the gradient is the Rydberg constant. The gradient for this experiment is 2.34×10^{-18}. The actual value for the Rydberg constant is 2.18×10^{-18}. Errors will be analysed in the error analysis section but for now this is the value I got.

## 4.2   Duane Hunt Law results

After setting up the equipment and calibrating the machine, results can be recorded. After the machine finished and the graph has been converted using the lattice spacing the graphs were produced for the associated voltages.

Each run through takes about 16 minutes. For this experiment and after lots of trials and testing (which will be discussed about later in the report), it was clear that the background spectrum had to be recorded. The best fit line was also calculated on the X-ray.exe software. Using a built in tool, you can highlight specific parts that are important for the best fit line. This is what has been done below with these results.
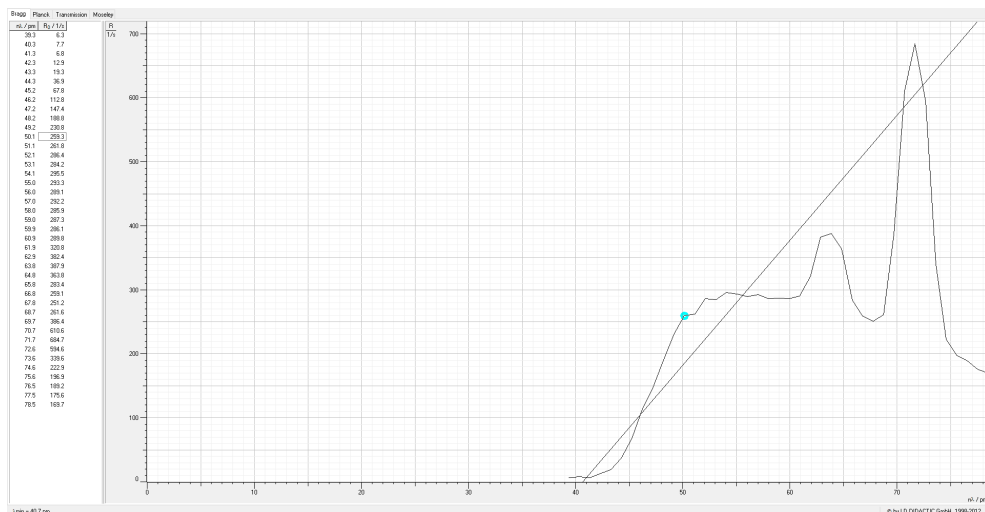
Found below is the plots of $n\lambda$ vs intensity for these voltages.
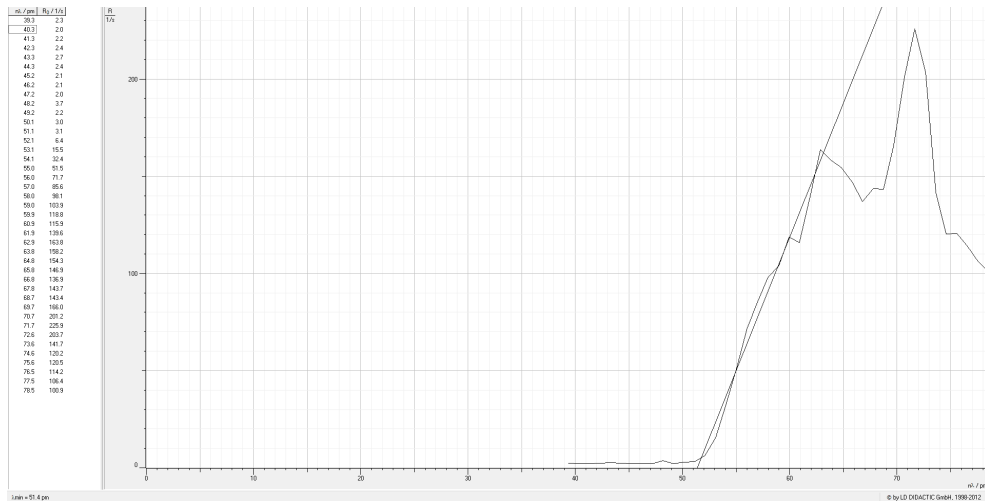
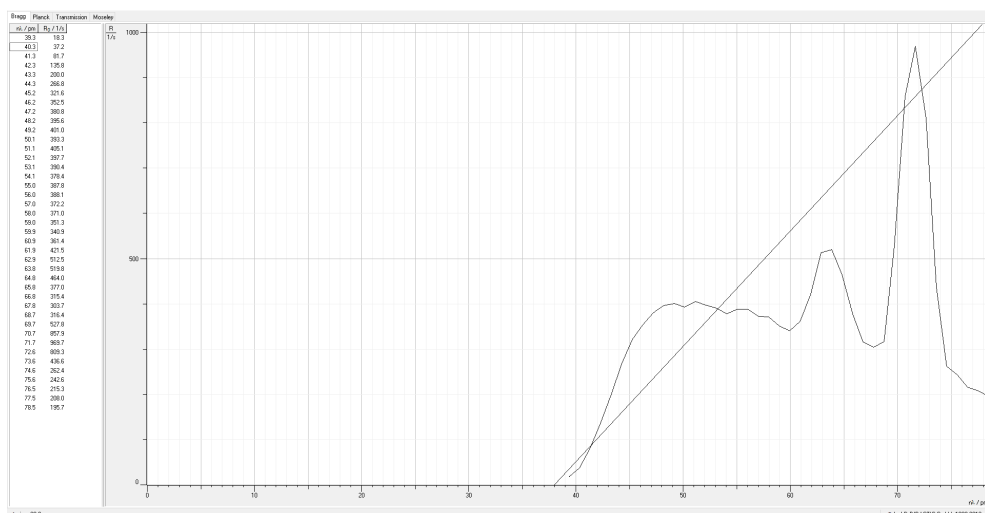**Figure 5:** Graph for the 25kV background spectrum
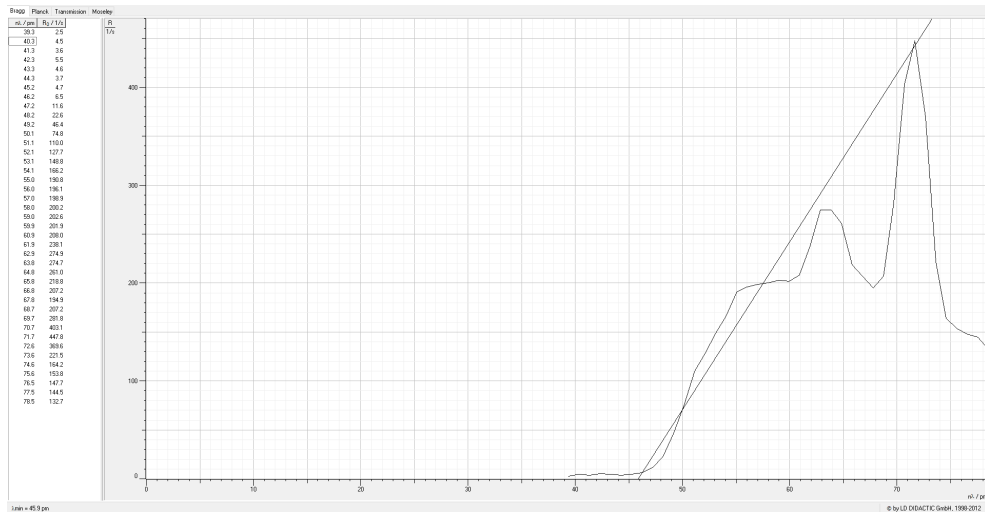
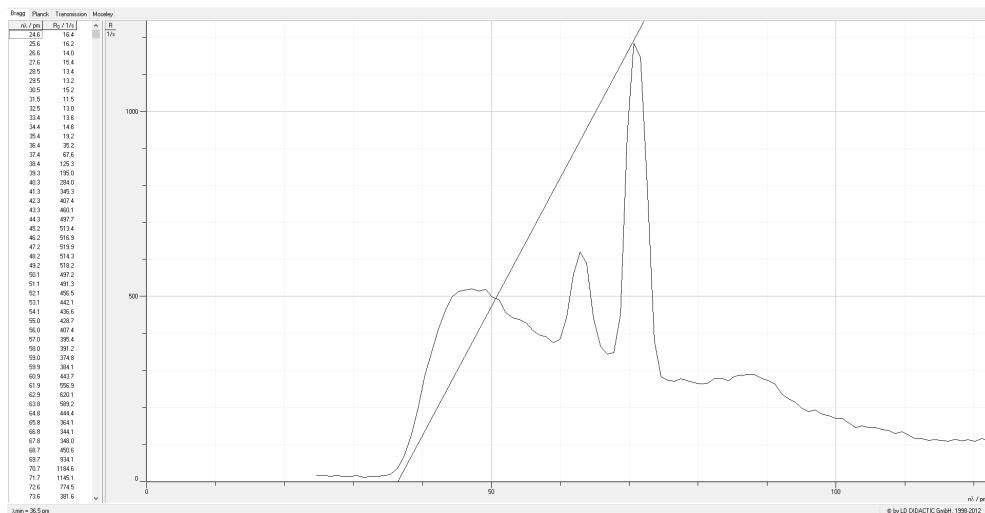**Figure 6:** Graph for the 27.5kV background spectrum

Figure 7: Graph for the 30kV background spectrum

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Figure 8: Graph for the 32.5kV background spectrum

Figure 9: Graph for the 35kV background spectrum

From these graphs, the gradient can be calculated from the line of best fit. These gradients equal the minimum wavelength the X-rays are diffracted from the multiple voltages tested. The gradients are as follows in table 3:

| Voltages	λmin
25kV	51.4pm
27.5kV	45.9pm
30.0kV	40.7pm
32.5kV	38.0pm
35kV	36.5pm

Table 3

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Now with this information a graph of $\frac{1}{V}$ vs λmin can be made. This is plotted because the relationship between these two values equals a useful expression that can be used to calculate Planck's constant. The values that are gonna be plotted are in table 4 below.

1/Voltage	λmin
1/25kV	51.4pm
1/27.5kV	45.9pm
1/30.0kV	40.7pm
1/32.5kV	38.0pm
1/35kV	36.5pm

Table 4

The graph was plotted on Python and the line of best fit was also calculated using this coding language. From the plot, it is clear that there is a linear trend and that the line of best fit isn't going to form a unusual result. This is because it passes very neatly through the regions near all the points. Therefore, there are no anomalies.
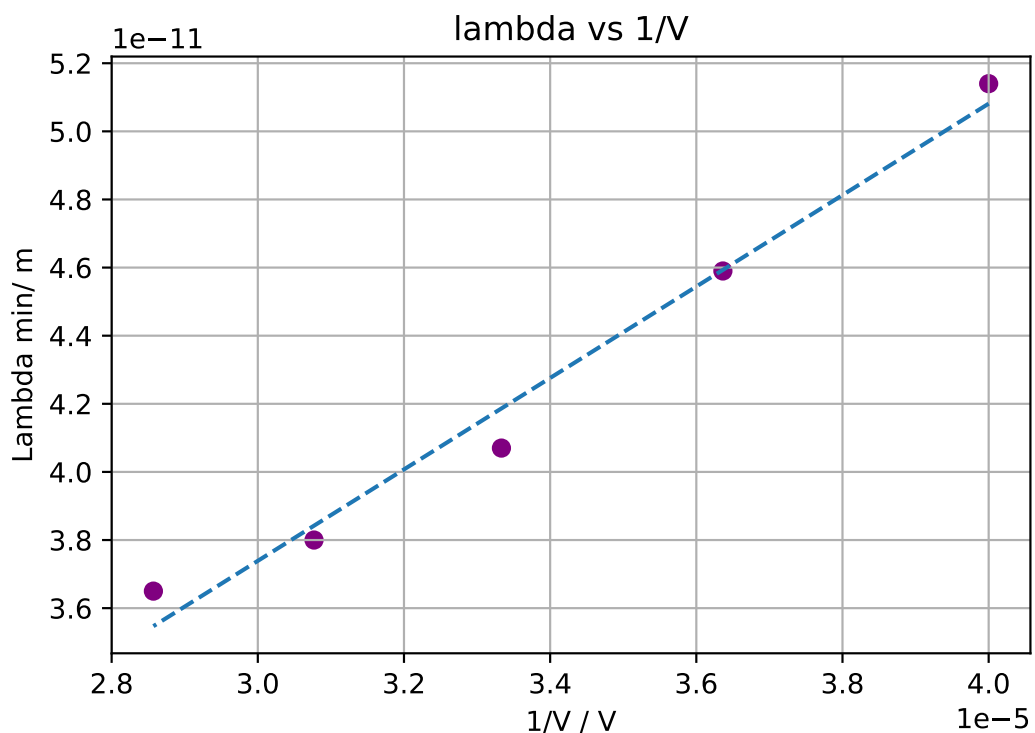
Figure 10: Graph to calculate the Planck's Constant

Now the gradient can be calculated. This value is $V\lambda = 1.3424 \times 10^{-6}$. As discussed earlier this is equal to $\frac{hc}{e}$. Rearranging this equation shows that the value of Planck's constant in this experiment is:

$$\frac{v\lambda \times e}{c} = \frac{1.3424 \times 10^{-6} \times e}{c} = 7.169 \times 10^{-34} Js^{-1} \tag{6}$$

The actual value of Planck's constant is $6.626 \times 10^{-34} Js^{-1}$. The uncertainties and error analysis will be discussed below but for now this is the result of the Duane-Hunt Law experiment to calculate Planck's constant.

5 Supplementary Testing For Results

In addition to the standard experiment that was run, there were some things that were done to make sure the experiment was accurate. These extra tests turned out to be very useful as it showed what may have been a bad idea to test and whether the equipment was set up in the correct way.

5.1 Atomic Spectroscopy

For this experiment there were some worry that the experiment was giving faulty results. Before calculating the wavelengths, testing the angle on the opposite side of the hydrogen tube to what was originally tested could show whether the errors produced would be systematic or not. From figure 1, it is clear that the angle will be the same. Using equation 3 and measuring with the exact same conditions these values were found:

	Original angle	Angle on opposite side of the hydrogen tube
25cm	Blue = 0.22	Blue = 0.22
25cm	Violet = 0.18	Violet = 0.17
25cm	Red = 0.29	Red = 0.32
38cm	Blue = 0.25	Blue = 0.25
38cm	Violet = 0.20	Violet = 0.20
38cm	Red = 0.33	Red = 0.35
53cm	Blue = 0.25	Blue = 0.27
53cm	Violet = 0.21	Violet = 0.21
53cm	Red = 0.35	Red = 0.38

Table 5: Table showing the difference between the actual angle and the opposite angle.

From this it was easy to understand that the experiment was working as it should. If the angles didn't match in an appropriate range, reconstructing the apparatus would have to have taken place. The angles were roughly similar so when it comes to error analysis later the errors can be refined.

5.2   Duane-Hunt Law

In this part of the experiment there was a lot of trial and error before getting to a final result. Firstly, figuring out the range of voltages to use was a necessity. It turns out after doing multiple tests, it was found that 20kV was the cutoff voltage where no peaks occurred. As the whole experiment revolves around the first peak on the graph it was clear that any voltages under this wouldn't be used. See figure 11.
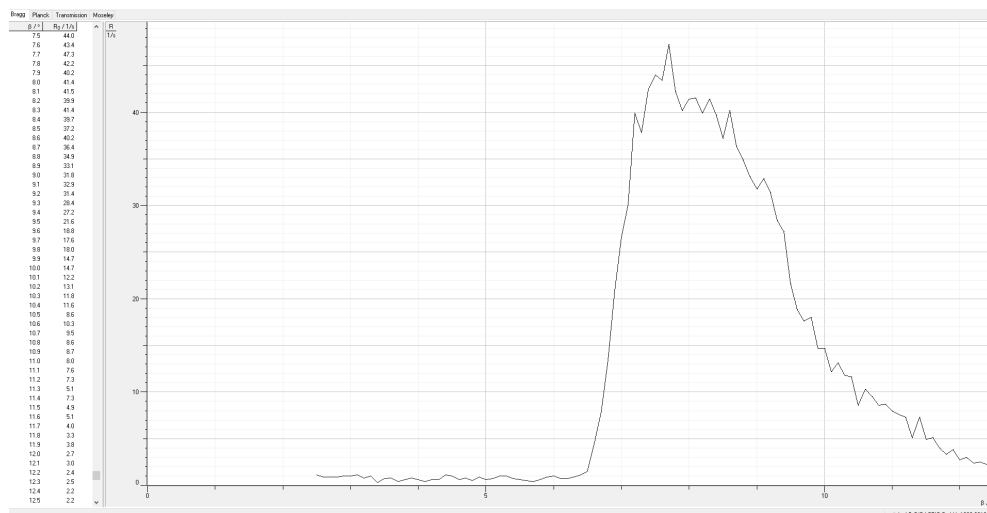

Figure 11: 20kV showing no peak

From this graph it clearly shows no peak between 6.4 degrees or 7.2 degrees. When running this experiment this is where the peak is known to occur due to the calibration so the fact that it wasn't there allowed the experiment to be tweaked to allow more accurate results for the final calculations.

6   Error analysis

6.1   Atomic Spectroscopy

For this experiment a lot of it included manual readings. This in turn can create a vast amount of uncertainty in the results. This would occur for the first reading of where the spectrum of colours are on the meter ruler. Using an automated reading software instead could be used but this will be discussed in the discussion.

When measuring in this experiment a meter ruler was used. The uncertainty on a meter ruler for this case is dependent on two measurements. These are where the measurement starts and where the final reading is. The uncertainty for one measurements on a meter ruler is $\pm0.5mm$ but as there is two values and they need to be subtracted to find the actual measurement so for the uncertainty you add them. This is is $\pm1mm$.

For the next part $\sin(\alpha)$ has uncertainty. Using equation 3 the uncertainty in d^2 and I^2 is ±0.2 I used propagation of errors to calculate that $\sin(\alpha)$ has an uncertainty of $\pm0.1degrees$.

Now calculating the uncertainty in the wavelength involved using equation 5. The uncertainty in the diffraction grating constant is zero as its a constant given on the lab equipment. The order of diffraction also has no uncertainty so the value of uncertainty for wavelength is $\pm0.2mm$. Energy will have the same uncertainty.

To find the Rydberg constant the gradient is calculated. Energy has an uncertainty of $\pm0.2mm$ and the other value has no uncertainty. This shows that the final uncertainty in the Rydberg constant is ±0.2

The value recorded in this experiment is therefore $2.34 \times 10^{-18} \pm 0.2$J so that means it is in the uncertainty value.

6.2   Duane-Hunt Law

For this part there isn't much uncertainty to discuss. As it was all done using computer software and scientific instruments that are calibrated. The only uncertainty would be when fitting the best fit line. The voltage would have zero uncertainty as it is programmed on computer software and the wavelength would have no uncertainty meaning the total uncertainty would be zero. The value recorded in this experiment is therefore $7.169 \times 10^{-34} \pm 0Js^{-1}$ so that means it is out of the uncertainty value as Planck's constant is actually $6.626 \times 10^{-34}Js^{-1}$.

7   Discussion

7.1   Atomic Spectroscopy

This experiment went very well in my opinion. Not only did the final result occur in the uncertainty range, the whole layout of the equipment also seemed to be accurate. This surprised me as there were a few things when recording the data that could've been better to maybe even give an even better result. The apparatus given to us was limited so in an advanced laboratory dedicated to this field there may have been better measuring equipment to have been used.

The fixings on the experiment setup was not ideal. The ruler that was used was stuck to the prop using blue tack. Even though this worked this may have led to unwanted movement in the ruler. As finding the centre point of the ruler and lining it up with the diffraction grate was really important, it may have been better to clamp it in place. Secondly, the ruler used could've been a more accurate one. Using a ruler with smaller increments would make the results more precise. This however may not be the case as human error comes into account because the readings were all done by eye. Finally, the last thing that I would change if I did the experiment again would be to setup the experiment on a longer platform. From this experiment it was made obvious that the further away the grating is from the object being measured, the more accurate the final results will be. This experiment was hindered as the table was only 58cm long. That is the reason that value was used. If the table was longer the results would be even closer to the actual value of the Rydberg constant.

Overall, this experiment was a success. Even though there is key areas that would be improved it still worked considerably well. If repeating the experiment was an opportunity in the future the ideas stated above would be taken into consideration. Using different inert gases as well may add to the challenge and would really solidify the experimental setup and analysis that took place.

7.2 Duane-Hunt Law

Finding Planck's constant from X-rays and a crystal of sodium chloride was a really eye opening experiment. The fact that you can obtain a value so small and precise from X-rays and angles just goes to show how important experimental setup and analysis has to be. Even though the value produced after this experiment wasn't in the uncertainty value it still was impressive to get to such a small number that was in the same order of magnitude as the actual value.

When doing the experiment there were some things that could be improved next time. A huge improvement would be to do repeats of the experiment adding even more voltages into the range. As the experiment took 16 minutes, each run through and analysis of the data took a fair amount of time. Re-running the test for more than five voltages would have taken up too much time. The second part that would make it better would be to increase the time at which the crystal moves. Increasing the time it takes before the crystal is moved and decreasing the angle at which it would move would allow for more points on the graph to have been plotted. This in turn would generate a line of best fit that would be strong as it had been calculated using even more points.

To check the accuracy of the best fit, the use of other software such as Matlab or Excel could have been an opportunity to develop the analysis a bit further.

8 Conclusions

8.1 Atomic Spectroscopy

To conclude, after calculating the Rydberg constant from a spectrum of light from an inert gas I have gained ample knowledge in this field of Physics. With my results being in the uncertainty range and having reasons and ideas of how to improve the experiment it is clear that this experiment went to plan and that if there was a chance to repeat the experiment the value recorded would be even closer to the agreed value. To finish, this was my final result. $2.34 \times 10^{-18} \pm 0.2J$

8.2 Duane-Hunt Law

To finish this whole report off, I really enjoyed this part of the experiment. Finding the constant that jump started the quantum era was an amazing opportunity for me. Calculating it to the same order of magnitude and to a high level of precision made it a very fun process from start to end. If there was more time, extending the voltage range and also increasing time under scanning would ultimately contribute to better results and perhaps even a result that is the same as the agreed value. To finish this was the result that I had obtained. $7.169 \times 10^{-34} \pm 0Js^{-1}$

References

[1] University Of Exeter Physics, *Atomic Spectroscopy Manuscript*

[2] University Of Exeter Physics, *Duane-Hunt Law Manuscript*

[3] `https://www.azooptics.com/Article.aspx?ArticleID=1645`(accessed $12^{th}October$).

[4] `https://chem.libretexts.org/Courses/Solano_Community_College/Chem_160/Chapter_07$\%$3A_Atomic_Structure_and_Periodicity/7.03_The_Atomic_Spectrum_of_Hydrogen`(accessed $12^{th}October$).

[5] `https://en.wikipedia.org/wiki/Balmer_series#:~:text=The$\%$20visible$\%$20spectrum$\%$20of$\%$20light,quantum$\%$20number$\%$20n$\%$20equals$\%$202.`(accessed $20^{th}October$).