4. Let the pmf of a r.v X be given by $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x & \text{for } x = 1,2,3,.... \\ 0, & elsewhere \end{cases}$, determine the pmf of

   $Y = \begin{cases} 1 \text{ if } x \text{ is odd} \\ 0 \text{ if } x \text{ is even} \end{cases}$

5. Suppose the r.v X has a pmf given by $f(x) = \begin{cases} \frac{1}{6}\left(\frac{5}{6}\right)^x & \text{for } x = 0,1,2,3,.... \\ 0, & elsewhere \end{cases}$, Obtain the pmf of $Y = X - 1$

1.6 Change of Variable Technique

.Let X be c. r.v with pdf f(x) and let Y be a function of X, then the pdf of Y $g(y) = f(x)\left|\frac{dx}{dy}\right|$

NB If $F(x)$ is the cdf of a r.v X, then a r.v $Y = F(x)$ has a uniform distribution over $[0,1]$

Example 1

A continuous r.v X has a pdf given by $f(x) = \begin{cases} 5x^4, 0 \le x \le 1 \\ 0 \;\; otherwise \end{cases}$. Determine the pdf of $Y = x^3$

Solution

$$Y = x^3 \;\; \Rightarrow \;\; X = Y^{1/3} \;\; \Rightarrow \;\; \frac{dx}{dy} = \frac{1}{3}Y^{-2/3} \quad g(y) = f(x)\left|\frac{dx}{dy}\right| = 5\left(y^{1/3}\right)^4 \times \frac{1}{3}y^{-2/3} = \begin{cases} \frac{5}{3}y^{2/3}, 0 \le x \le 1 \\ 0 \;\; otherwise \end{cases}$$

Example 2

A r.v X has pdf $f(x) = \begin{cases} 24x^2, 0 \le x \le \frac{1}{2} \\ 0 \;\; otherwise \end{cases}$ determine the pdf of $Y = 8X^3$

Solution

$$Y = 8x^3 \Rightarrow \;\; X = \tfrac{1}{2}Y^{1/3} \;\; \Rightarrow \;\; \frac{dx}{dy} = \tfrac{1}{6}y^{-2/3} \quad g(y) = f(x)\left|\frac{dx}{dy}\right| = 24\left(\tfrac{1}{2}y^{1/3}\right)^2 \times \tfrac{1}{6}y^{-2/3} = \begin{cases} 1, \; 0 \le x \le 1 \\ 0, \; elsewhere \end{cases}$$

NB $Y = 8X^3$ is the cdf of X

Exercise:

1. For a r.v X with $f(x) = \begin{cases} 5x^4, 0 \le x \le 1 \\ 0 \;\; otherwise \end{cases}$, determine the pdf of $Y = 2\ln x$ and its range.
2. A r.v X has pdf $f(x) = \begin{cases} e^{-x}, x \ge 0 \\ 0 \;\; otherwise \end{cases}$ determine the pdf of $Y = X^4$
3. The probability density function of X is given by $f(x) = \begin{cases} \dfrac{2}{\pi(x^2+4)} & -\infty \le X \le \infty \\ 0 & otherwise \end{cases}$

   Obtain the probability density function of $Y = \tan^{-1}(\frac{x}{2})$