The emission rate of ionizing photons into the IGM per baryon can be written as:

$$\dot{n}_{\mathrm{ion/b}} = \frac{d}{dt}\left[\bar{\rho}_m^{-1}\int_{M_{\min}}^{\infty} dM_h \frac{dn}{d\ln M_h} f_{\mathrm{b}} f_* N_{\gamma/b} f_{\mathrm{esc}}\right] . \tag{80}$$

Here $\bar{\rho}_m$ is the mean matter density, and the average source (galaxy) properties are expressed in terms of:

- f_{b} – the baryon fraction of a halo, in units of the cosmic mean value, Ω_b/Ω_m
- f_* – the fraction of halo baryons ending up in stars
- $N_{\gamma/b}$ – the number of ionizing photons produced per stellar baryon
- f_{esc} – the fraction of produced ionizing photons which escape the galaxy into the IGM

and the lower limit of integration, $M_{\min}$, corresponds to the minimum halo mass capable of hosting a star forming galaxy (i.e. the product $f_{\mathrm{b}} f_* N_{\gamma/b} f_{\mathrm{esc}} = 0$ for $M_h < M_{\min}$).

Typically, these astrophysical parameters are combined into a single ionizing efficiency:

$$\zeta = 20\left(\frac{N_\gamma}{4000}\right)\left(\frac{f_{\mathrm{esc}}}{0.1}\right)\left(\frac{f_*}{0.05}\right)\left(\frac{f_{\mathrm{b}}}{1}\right) . \tag{81}$$

If ζ is independent of halo mass and time, eq. (80) simplifies to include the time derivative of the collapse fraction:

$$\dot{n}_{\mathrm{ion/b}} = \zeta \frac{df_{\mathrm{coll}}(> M_{\min}, z)}{dt}. \tag{82}$$

More generally, one could parametrize ζ with a power-law scaling with the halo mass, $\zeta = \zeta_0\left(\frac{M_h}{M_{\min}}\right)^\alpha$, in which case eq. (80) becomes:

$$\dot{n}_{\mathrm{ion/b}} = \frac{d}{dt}\left[\frac{\zeta_0}{\bar{\rho}_m}\int_{M_{\min}}^{\infty} dM_h \frac{dn}{d\ln M_h}\left(\frac{M_h}{M_{\min}}\right)^\alpha\right] , \tag{83}$$

with $\alpha = 0$ reducing to eq. (82).

In principle, direct observations of galaxies can be used to constrain some of the above parameters. The abundance matching technique applied to $z \sim 8$ LBGs, discussed in the previous chapter can be used to motivate the scaling $f_{\mathrm{b}} \propto M_h^{0.2-0.4}$, assuming a mass-independent mapping from the observed 1500 Å luminosity to the ionizing luminosity (e.g. Trenti et al. 2010; Greig & Mesinger 2015; Atek et al. 2015; Sun & Furlanetto 2015). However, f_{esc} likely has an opposite scaling, increasing towards smaller halo masses (e.g. Wise & Cen 2009; Paardekooper et al. 2015), perhaps even showing a non-monotoic evolution (e.g. Xu et al. 2016). Moreover, the IMF, which sets $N_{\gamma/b}$, could be more top-heavy in low-mass, poorly-enriched galaxies. For the purposes of EoR modeling, we only care about the product, ζ.