Written Assignment

EDU8222: Developing Critical Perspectives on Teaching Thinking Skills

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In what ways does metacognitive modelling (particularly through the teaching & modelling of the plan, monitor and evaluate approach) impact on pupil metacognition and promote their self-regulation?

How do you think this might support their future learning in your phase/subject and beyond?

How might your knowledge of metacognition and self-regulation make you a more effective teacher of your phase/subject?

Written Assignment

Introduction

Metacognition is the '*thinking about thinking*', typically dualised as a self-awareness of cognitive strategies along with self-regulation. As described by Flavell (1979, p. 909), "cognitive strategies are invoked to *make* cognitive progress [whereas] metacognitive strategies [are deployed] to *monitor* it".

The self-regulation aspect appears particularly important for the development of writing in both upper-primary and lower-high school. Indeed, students participating in an EEF-funded project that taught self-regulation benefitted from approximately nine months' further progress compared to those that didn't (Torgerson *et al.*, 2014). Although the focus of the aforementioned study is on writing only, literacy skills underpin many aspects of education, including mathematics. Research suggests "reading comprehension is needed" to successfully deal with mathematical problems (Gomez *et al.*, 2020, p. 1351), and recent work by Kim *et al.* (2024) establishes a moderate relationship between mathematics and *writing* in particular. Here, I discuss the impact of metacognition on secondary mathematics students and reflect on my own practice.

Metacognition in mathematics improves a student's ability to analyse and solve unfamiliar problems. In practice, regulation of a student's learning would see them decide on an approach that they can follow (*plan*), frequently check if they are progressing towards their intended goal (*monitor*) and reflect on the process they chose (*evaluate*). Knowing what learning strategies to use and how best to approach a task in the aforementioned way is metacognitive knowledge; the importance of explicitly teaching it is emphasised in Pintrich (2002). In contrast to this, Brown *et al.* (1981) discuss so-called *blind training*, meaning an implicit teaching of metacognition "without a concurrent understanding of [its] significance" (p. 15). Additionally, the article by Chen *et al.* (2023) declares that worked examples are superior for "prepar[ing students] to solve structurally different problems" when transferring knowledge. This gives credence to the benefits of metacognitive modelling in *either* sense of Pintrich (2002) and Brown *et al*. (1981) – it appears not to be of particular importance for transfer as to how explicit metacognition is flagged, but rather that it is exemplified before students practice it. Hence, and given that explicating metacognition appears in Part 5 of Standard 4 of the ITT Core Content Framework (DfE, 2019), I wanted to try each approach at various stages of the lesson: the implicit is used to support the explicit.

My metacognition lesson was delivered to a Year 12 A-level maths class because the placement school scheme of work leant itself nicely to embedding metacognitive practices into the mathematical content. This adheres with the EEF report's philosophy that there is no basis on which to believe that "teaching metacognitive approaches in 'learning to learn'… sessions" is beneficial (Quigley *et al.*, 2021, p. 24). Each of my two module partners taught metacognition lessons to a Year 12 biology class and a Year 9 history class, respectively. Supported also by the EEF report, Perkins and Salomon (1988) suggest that students have difficulty transferring knowledge, which includes global metacognitive strategies, between disciplines. Hence, the primary influence for choosing my Year 12 class was to garner meaningful comparisons and contrasts with others in my placement school (Appendix F).

Modelling Metacognition

According to Gall *et al.* (1990), "learning how to learn cannot be left to students. It must be taught". One of the most common strategies of self-regulation is the plan-monitor-evaluate (PME) cycle, allowing a learner to "monitor their behaviour in terms of [achieving] their goals and self-reflect on their… effectiveness" (Zimmerman, 2002, p. 66). This is the process I saw during observations of experts on the Callerton Academy ITaP day, motivating my own PME cycle (see Appendices A and B). The mathematics department at my placement school does not have a culture of *explicit* metacognition modelling *of this sort*. Given how embedded my routines have become, I anticipated a challenge with respect to how the students adapt to this new style, which is reflected in the feedback (Appendix C, Graph 5). Hence, this was accounted for in my lesson plan (Appendix A); I gradually built towards the students completing a PME cycle. That said, a stronger understanding of metacognition at that time could have alleviated these difficulties by boosting my confidence. With hindsight, another strategy would have been to explicate metacognition on more than this isolated occasion; collecting data across different lessons would have painted a more accurate picture of my teaching, and the class's acceptance, of metacognition.

The first modelling task (Appendix B, Slide 5) involved retrieval of content similar to what the pupils saw one lesson prior – this was in the style of a "We Do" strategy – with metacognitive questioning provided and verbalised during the activity. Despite meaning this to act only as a scaffold à la Rosenshine (2012) in preparation of a full PME cycle, some authors suggest retrieval practice can improve the effectiveness of self-regulation (Ariel and Karpicke, 2018), and thus the benefits here are twofold: students can focus more on the metacognitive aspect of the lesson, and the seed has been sewn (albeit covertly) for what I will soon expect students to be doing this lesson with regard to the PME cycle. On the other hand, Heyboer (2023, p. 35) advocates that *reflecting on* retrieval "did not have a significant effect" with regard to improving self-regulation. However, this study was limited in scope – just under four-times the number of subjects compared to Ariel and Karpicke, (2018) – and they proceed to comment that "*some* students *did* experience [an] increase" in self-regulation (p. 36). While this is thought-provoking, I only use retrieval for the mathematical content, discovering any additional benefit to self-regulation after the fact; this will surely be something I research further and consider in my future metacognitive teaching. This exercise aligns closely with the planning stage of the PME cycle, which I clarified verbally to the students *afterwards* (Appendix A). Although some students were initially hesitant with my metacognitive prompting questions, this didn't negatively impact their perception of the planning state (quite the opposite; see Appendix C, Graph 1).

The subsequent activity (Appendix B, Slide 6) had me explicitly introduce the PME cycle in the context of an exam question, with prompts provided (Appendix A) to guide the students. By conducting a sorting task, it minimises the mathematical cognitive load and instead spotlights metacognition. Using an exam question aligns with the theory that better metacognitive understanding can bolster assessment performance (Stanton *et al.*, 2021). The students were broadly successful in this task, a belief reinforced when I selected students to reflect on their thinking process verbally during the course of the activity (Schraw and Dennison, 1994). That said, a better proxy may have resulted from focusing on each step of the PME cycle *separately* at this initial stage – compare Graphs 1 and 2 in Appendix C, which shows a disparity between understanding of the planning and monitoring phases. Nevertheless, this introduction provided the means for the next activity (Appendix B, Slide 8).

I then showcased the full PME cycle for that same task on the whiteboard. When modelling in general, I opt for a verbal approach as a means to 'demystify' the abstraction encountered when answering mathematics problems. This is particularly helpful for low-attaining students who often find abstract thinking a challenge (Ward and Wandersee, 2002), and more generally those who are less open to alternative strategies – this is particularly relevant to the planning stage. Moreover, and in the language of Perkins (1992), this can be seen as a crucial step transitioning the pupils from tacit to aware learners. One paper claims that retrieval is "overuse[d]… in contexts where it would be better to start with extended study" (Carpenter *et al.*, 2020, p. 21). From my own experience, this can be particularly prevalent in mathematics, especially with lower-attaining students; pupils encountering a new problem often 'brain-dump' in the hopes of getting something correct, rather than first assessing what they are being asked to do and how they may go about it. This greatly informed my planning stage (see Appendix A). In Appendix E, one module partner notes this activity as a key strength of the lesson in terms of "mak[ing] clear distinctions between each phase of the cycle" but that I could improve this phase by encouraging more participation from the students ("We Do" instead of "I Do"). Although I think it important for the students to see a completed PME cycle without interruption, I agree that this suggestion is valid. Based on this feedback, a possible next step would be to partially model a PME cycle with another similar question; this minimal variation can facilitate learning as it "help[s] learners to notice what we want them to discern" (Kullberg *et al.*, 2017, p. 567) and, in this case, it would be the application of metacognitive regulation.

For its impact on metacognition itself, modelling can make students aware of the importance of thinking about their cognition in contrast with the over-significance they themselves often place on completing a process correctly. Moreover, self-regulation allowed my students to navigate towards an answer more accurately (cf. Appendix D(iii) and Appendix D(iv)); even those that couldn't obtain a correct final answer started to demonstrate an awareness of how to begin that journey. An unexpected impact (from my point-of-view) was the encouragement of metacognitive dialogue between students that followed in the pupil engagement task, which furthered their self-regulation more so than I anticipated; see the following section.

Pupil Engagement and Ethics

I now examine the primary activity (Appendix B, Slide 8) designed for the students to rehearse metacognition, followed by the resulting questionnaire data. By providing the prompts seen on the slide, in conjunction with those I had on the board for the earlier tasks, the students here had scaffolding to perform an explicit PME cycle but without initial mathematical support – this acts conversely to the previous activity. The eventual goal is to have students become reflective learners such that the PME cycle is "internalise[d] and automate[d]" (McCrea, 2019, p. 121), and my decision to provide question stems was noted by the observer (Appendix E). This is further supported by Graphs 1, 2 and 4 in Appendix C, showing that the vast majority of students felt comfortable with the planning and monitoring stages. That said, one student had difficulty with self-regulation during the activity; this could be a result of slight cognitive overload given the challenging mathematics given their attainment. That aside, this suggests I should have guided the metacognitive aspect more in some places as to alleviate the cognitive demands of processing metacognition – I agree with my observer's comments that more structured answer sheets would have helped in this regard (Appendix E).

Overall engagement with the activity *seemed* high, and I regret not encouraging this explicitly but my students held discussions with their peers as a means of self-regulation. Zimmerman (2002, p. 69) claims that "self-regulated learning is not asocial", and this is something I would capitalise on more when teaching metacognition in future. This could contrast with Gall *et al.*'s (1990) earlier quote, which may underplay the impact of peer-led learning compared with direct instruction.

Consequently, it seems appropriate to use both forms of learning, and my missing of this opportunity was noted in both my module partner's and the class teacher's observations; this will be something I adapt in future. Having looked at the work they produced, about half of the class demonstrated explicit PME usage – all-bar-one of which is working at an A or A* grade. Swanson (1990) suggests that metacognition and aptitude are disjoint, and that a student confident in their ability to analyse and self-regulate can, to an extent, use this to compensate for a (perceived) lack of cognitive knowledge. This is evidenced in Appendix D(iii) where a lower-attaining student achieved more than their higher-attaining but lower-metacognitive peers (Appendix D(iv)). Moreover, this reinforces the view that metacognition contributes to performance independent of ability (van der Stel and Veenman, 2010).

That said, a single lesson isn't sufficient to develop an accurate picture of metacognition and its impact. The literature posits that "explicit strategy teaching is rare" (Kistner *et al.*, 2010, p. 167), which is in complete contrast to the principles emphasised by some UK-based education bodies such as the DfE and EEF. It seems, therefore, that this is an effort to address the stated omission. Although relevant to mathematics however, this study was conducted across a somewhat modest number of twenty *German* classes. On the other hand, this work is agreeable with similar earlier findings from the United States – Moely *et al.* (1992, p. 669) determined "infrequent use of strategy suggestions and… very limited effort [was] made by most teachers" – and the lack of explicit metacognitive modelling aligns with my own experiences recounted earlier. This may account for the unease in my students captured by Graph 5 in Appendix C. A better depiction of metacognition would require embedding this *explicit* strategy in future lessons. Supported by repeated self-evaluative quizzes, this could show class progression from tacit to reflective learners more tangibly.

As per the Ethical Guidelines for Educational Research (BERA, 2024), participation in the lesson and any resulting work has been anonymised, including all questionnaire submissions as to better ensure all responses are honest. This was made clear at the start of the lesson, and towards the end just before handing out the questionnaire. Participation was not mandatory; they were informed that they could opt out if desired. During this time, I also remained at the front of the classroom with no clear view of any student's desk in an effort to bolster the integrity of each response.

Supporting Future Learning

There are several possible benefits to teaching metacognition. For instance, students can better foster the transfer of skills in many subject areas, including mathematics (Schoenfeld, 1985). It was noted earlier that Perkins and Salomon (1988) refer to transfer as being difficult to master. In fact, their sequel paper (1989, p. 23) suggests a "lack of conditions needed for transfer, rather than domain specificity, is to blame for many cases of failure of transfer". Perhaps this insinuates that only *sometimes* can metacognitive strategies be transferrable; it is my belief that this does not contradict the suggestion that "metacognitive skills are only partly general" (van der Stel and Veenman, 2010, p. 224) given that the latter paper suggests age as a factor on the argument between global versus local metacognition.

In terms of challenges to learning, metacognition can alleviate the cognitive demand often faced by the students in their particular subject. The planning phase, in particular, is an opportunity for a pupil to chunk a task into manageable parts (Sweller, 1988). Although cognitive load theory typically "assume[s] that instructors rather than novice learners" make decisions (Paas *et al.*, 2003, p. 3), Gerjets and Scheiter (2003) suggest an extension of the theory to cover the more student-led situation we see when developing metacognition. That said, caution should be taken as "metacognitive activities… have strong knowledge requirements" (Bransford and Schwartz, 1999, pp. 65-66), so embedding taught metacognition into a lesson may increase the germane load demanded of a student.

It appears also that increased metacognitive awareness can inspire more resilience when students encounter new problems. Indeed, Beilock and Willingham (2014) suggest that different strategies for approaching a task can offset mathematical anxiety. Given that metacognition develops a student's arsenal when it comes to approaching novel tasks, it is more likely to lead to successful outcomes compared to students with less metacognitive understanding (Schoenfeld, 1987); combining these ideas supports my initial hypothesis. Beyond mathematics, Covington (1992) suggests that so-called success dynamics can overcome the fear of failure in any subject. Combining this with Schoenfeld (1987), it suggests increased resilience persists beyond any one domain.

Impact

Knowledge of metacognition may impact my teaching in several ways. Firstly, it can better inform my lesson planning and sequencing because I know now how students should self-regulate. The latter was a strength of my Year 12 lesson (Appendix E) *in the context of mathematical content*, so this can be developed further if I sequence activities that require students to be evaluative.

A second benefit is on the classroom dynamics. Indeed, having an awareness of the importance of monitoring and reflection – and teaching this to the students – establishes that growth is valued more than obtaining a correct answer, that it is fine to be challenged. In parallel, metacognition also allows me to better diagnose those challenges. This was discussed broadly earlier, but as for a direct impact on my own practice, I may now be able to better discern the difference between a student having content gaps versus poor self-regulation.

To conclude, my exploration of metacognition has added depth to my understanding of how cognitive processes influence learning. I have evidenced how self-regulation is impacted even by a first-time introduction to the PME cycle, although this is limited in scope. Nonetheless, the benefits of metacognition are clear from the literature and, moving forward, this will inform my own practice through further purposeful metacognition integration to stimulate success in mathematics and beyond.

References

Ariel, R. and Karpicke, J. D. (2018). Improving self-regulated learning with a retrieval practice intervention. *Journal of Experimental Psychology: Applied*, **24**(1), pp. 43-56. doi: https://doi.org/10.1037/xap0000133.

Beilock, S. L. and Willingham, D. T. (2014). Math Anxiety: Can Teachers Help Students Reduce It? Ask the Cognitive Scientist. *American Educator* [online], **38**(2), pp. 28-32. ERIC: EJ1043398.

BERA (2024). *Ethical Guidelines for Educational Research, fifth edition (2024)*. British Educational Research Association. Available at: https://www.bera.ac.uk/publication/ethical-guidelines-for-educational-research-fifth-edition-2024-online [Accessed: 04/01/2025].

Bransford, J. D. and Schwartz, D. L. (1999). Chapter 3: Rethinking Transfer: A Simple Proposal With Multiple Implications. *Review of Research in Education*, **24**(1), pp. 61-100. doi: https://doi.org/10.3102/0091732x024001061.

Brown, A. L., Campione, J. C. and Day, J. D. (1981). Learning to Learn: On Training Students to Learn from Texts. *Educational Researcher*, **10**(2), p. 14-21. doi: https://doi.org/10.2307/1174401.

Carpenter, S. K., Endres, T. and Hui, L. (2020). Students' Use of Retrieval in Self-Regulated Learning: Implications for Monitoring and Regulating Effortful Learning Experiences. *Educational Psychology Review* **32**, pp. 1029-1054. doi: https://doi.org/10.1007/s10648-020-09562-w.

Chen, O., Retnowati, E., Chan, B. and Kalyuga, S. (2023). The effect of worked examples on learning solution steps and knowledge transfer. *Educational Psychology*, **43**(8), pp. 914-928. doi: https://doi.org/10.1080/01443410.2023.2273762.

Covington, M. V. (1992). *Making the Grade: A Self-Worth Perspective on Motivation and School Reform*. Cambridge, NY: Cambridge University Press. ISBN: 978-1-139-17358-2.

DfE (2019). *ITT Core Content Framework*. Department for Education. Available at: https://assets.publishing.service.gov.uk/media/6061eb9cd3bf7f5cde260984/ITT_core_content_framework_.pdf [Accessed: 02/01/2025].

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist*, **34**(10), pp. 906-911. doi: https://doi.org/10.1037/0003-066X.34.10.906.

Gall, M. D., Gall, J. P., Jacobsen, D. R. and Bullock, T. L. (1990). *Tools for learning: a guide to teaching study skills*. Alexandria, VA: Association For Supervision And Curriculum Development.

Gerjets, P. and Scheiter, K. (2003). Goal Configurations and Processing Strategies as Moderators Between Instructional Design and Cognitive Load: Evidence From Hypertext-Based Instruction. *Educational Psychologist*, **38**(1), pp. 33-41. doi: https://doi.org/10.1207/s15326985ep3801_5.

Gomez, A. L., Pecina, E. D., Villanueva, S. A. and Huber, T. (2020). The undeniable relationship between reading comprehension and mathematics performance. *Issues in Educational Research*, **30**(4), pp. 1329-1354. ERIC: EJ1282087.

Heyboer, J. (2005). *The Effect of Reflection on Retrieval Practice to Self-Regulated Learning*. M.A. Thesis. Dordt University. Available at:

https://digitalcollections.dordt.edu/cgi/viewcontent.cgi?article=1172&context=med_theses&utm_source=chatgpt.com [Accessed: 29/12/2024].

Kim, Y.-S. G., Yang, D. and Hwang, J. (2024). Are Mathematics and Writing Skills Related? Evidence from Meta-Analysis. *Educational Psychology Review*, **36**(4). doi: https://doi.org/10.1007/s10648-024-09960-4.

Kistner, S., Rakoczy, K., Otto, B., Dignath-van Ewijk, C., Büttner, G. and Klieme, E. (2010). Promotion of self-regulated learning in classrooms: investigating frequency, quality, and consequences for student performance. *Metacognition and Learning*, **5**(2), pp. 157-171. doi: https://doi.org/10.1007/s11409-010-9055-3.

Kullberg, A., Runesson Kempe, U. and Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM Mathematics Education*, **49**(4), pp. 559-569. doi: https://doi.org/10.1007/s11858-017-0858-4.

McCrea, E. (2019). *Making Every Maths Lesson Count: Six Principles to Support Great Maths Teaching.* Carmarthen: Crown House Publishing Limited. ISBN: 978-1-78583-332-8.

Moely, B. E., Hart, S. S., Leal, L., Santulli, K. A., Rao, N., Johnson, T. and Hamilton, L. B. (1992). The Teacher's Role in Facilitating Memory and Study Strategy Development in the Elementary School Classroom. *Child Development*, **63**(3), pp. 653-672. doi: https://doi.org/10.2307/1131353.

Paas, F., Renkl, A. and Sweller, J. (2003). Cognitive Load Theory and Instructional Design: Recent Developments. *Educational Psychologist*, **38**(1), pp. 1-4. doi: https://doi.org/10.1207/s15326985ep3801_1.

Perkins, D. N. (1992). *Smart Schools: Better Thinking and Learning for Every Child*. New York City, NY: Free Press. ISBN: 978-1-4391-0840-6.

Perkins, D. N. and Salomon, G. (1988). Teaching for Transfer. *Educational Leadership*, **46**(1) pp. 22-32. ERIC: EJ376242.

Perkins, D. and Salomon, G. (1989). Are Cognitive Skills Context-Bound? *Educational Researcher*, **18**(1), pp. 16-25. doi: https://doi.org/10.3102/0013189x018001016.

Pintrich, P. R. (2002). The Role of Metacognitive Knowledge in Learning, Teaching, and Assessing. *Theory Into Practice*, **41**(4), pp. 219-225. doi: https://doi.org/10.1207/s15430421tip4104_3.

Quigley, A., Muijs, D. and Stringer, E. (2021). *Metacognition and Self-Regulated Learning. Guidance Report*. Education Endowment Foundation. Available at: https://d2tic4wvo1iusb.cloudfront.net/production/eef-guidance-reports/metacognition/EEF_Metacognition_and_self-regulated_learning.pdf?v=1736078277 [Accessed: 30/12/2024].

Rosenshine, B. (2012). Principles of instruction: Research-based strategies that all teachers should know. *American Educator*, **36**(1), pp. 12-19 & 39. ERIC: EJ971753.

Schoenfeld, A. H. (1985). *Mathematical Problem Solving*. Orlando, FL: Academic Press. ISBN: 978-0-12-628870-4.

Schoenfeld, A. H. (1987). “What's All the Fuss About Metacognition?”, in Schoenfeld, A. H. (ed.), *Cognitive science and mathematics education*. Lawrence Erlbaum Associates: London, pp. 189-215. ISBN: 978-1-136-46177-4.

Schraw, G. and Dennison, R. S. (1994). Assessing Metacognitive Awareness. *Contemporary Educational Psychology*, **19**(4), pp. 460-475. doi: https://doi.org/10.1006/ceps.1994.1033.

Stanton, J. D., Sebesta, A. J. and Dunlosky, J. (2021). Fostering Metacognition to Support Student Learning and Performance. *CBE—Life Sciences Education*, **20**(2). doi: https://doi.org/10.1187/cbe.20-12-0289.

Swanson, H. L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, **82**(2), pp. 306-314. doi: https://doi.org/10.1037/0022-0663.82.2.306.

Sweller, J. (1988). Cognitive Load during Problem Solving: Effects on Learning. *Cognitive Science*, [online] **12**(2), pp. 257-285. doi: https://onlinelibrary.wiley.com/doi/abs/10.1207/s15516709cog1202_4.

Torgerson, D., Torgerson, C., Ainsworth, H., Buckley, H., Heaps, C., Hewitt, C. and Mitchell, N. (2014). *Improving Writing Quality: Evaluation Report and Executive Summary*. ERIC: ED581140.

van der Stel, M. and Veenman, M. V. J. (2010). Development of metacognitive skillfulness: A longitudinal study. *Learning and Individual Differences*, **20**(3), pp. 220-224. doi: https://doi.org/10.1016/j.lindif.2009.11.005.

Ward, R. E. and Wandersee, J. H. (2002). Struggling to understand abstract science topics: A Roundhouse diagram-based study. *International Journal of Science Education*, **24**(6), pp. 575-591. doi: https://doi.org/10.1080/09500690110074017.

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory Into Practice*, **41**(2), pp. 64-70. doi: https://doi.org/10.1207/s15430421tip4102_2.

Appendix A (Lesson Plan and Metacognitive Scripting)

10:10 – 10:17 Do the exercise from the "★'ter Booklet". Encourage independent work but help where necessary whilst circulating; possibly offer general hints before showing the answers one-by-one.

10:17 – 10:18 Get students to assess their work against the solutions. Anticipate some questions about the manipulation of the trigonometric intervals.

10:18 – 10:19 Introduce the students to the metacognitive aspect of the lesson. Explain that there will be more of an emphasis on how we plan an answer, monitor our progress towards a solution and evaluate the process after the fact.

10:19 – 10:21 Recall the derivative from first principles. Have students write down the rate of change interpretation (to embed it for later in the lesson). Give some historical context; talk about the connection to infinitesimals – although not necessary, it should be sufficient to get students remembering the rate of change interpretation. We can further link this to the derivative from first principles (acting as recall from the first lesson of this chapter). In this case, it may be handy to show them

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

– which is different-yet-equivalent to the form they learn per the Key Stage 5 specification (the equivalence comes by defining $h := x - a$) – as this is much closer to the calculation of the gradient of a straight line they know from GCSE.

10:21 – 10:25 Explicate the planning stage of the PME cycle by having the students answer an exam-style question with prompting in place to get them thinking about what steps should be taken.

Script: *Read the exam question presented on the board and think about the process that we have to go through to get an answer. Ask yourselves what we mean by the rate of change. Why is it important that we have the radius, and how can we use it? Why is the derivative important to this story? Here are three prompting questions to assist you with a plan. You have two minutes to think about how these questions are helpful, and a further two minutes to answer the question.*

Annotation 1 The purpose of this exercise is to introduce the first aspect of the PME cycle, which will act as a segway into the explicit modelling and usage of the PME cycle later in the lesson. This will alleviate some cognitive load in the sense that future questions are similar to this one, and in the sense that the students will be able to focus on mastering each phase in isolation *before* combining all three.

10:25 – 10:33 Students work through a sequencing activity, in which they need to order the constituent pieces of an answer to a similar exam-style question. About half-way into the activity, pause the students and have them analyse their progress.

Script: *Okay class, I want you to put your pens down and face the front for a minute or two. Some of you are making good progress towards sequencing an answer to this question, and some may have hit a slight wall. As such, I want you ask yourselves the following questions: "are we on the way to an answer?", "did I stick to a plan similar to the one we made for the previous exam question?", "do I understand each step/calculation we are given, and how it is useful?". If you can answer in the affirmative to these questions, you are likely approaching in on an answer. On the other hand, this might reveal where you are having difficulty and better allow you to ask for help with overcoming it. Think about this for the next 30 seconds and then continue working with your partner in light of this self-reflection.*

> **Annotation 2** This is an introduction to the monitoring phase of the PME cycle without explicit labelling it as such. This, to *some* extent, is the stage the students are most likely to already do when answering questions. Upon questioning them, it seems they have never purposefully come to a halt mid-flow in order to monitor their progress.

Inform the students how this fits into the metacognitive aspect of the lesson, namely that we have planned how to answer this question and used the prompts on the slide to both inform the plan and monitor our progress during the task.

10:33 – 10:39 Model the answer in the style of a PME cycle.

Script: *Of course, the answers are important but we care more about the way in which they are obtained. Therefore, I will explicitly model the entire process.*

Reading the question, I first ask myself "have I seen a similar problem before?". The answer to this latter question is clearly "yes" because we've just done one on the previous slide. Next, "what is it I want to achieve?" and "what information do I have/need in order to get to a final answer?". We want the volume of the tank. We know that its net has an area of $54\ \mathrm{m}^2$ *and that the opposite vertical faces are square. This allows us to draw a picture* [sketch on the board] *where the height and depth are the same. The question tells us to denote the height by* x*, so this is also its depth. We do not know its length, so let's call it* y*. The volume is therefore* $V = x^2y$*. Because the volume in question is in terms of* x *only, we now want to express* y *in terms of* x*. From our diagram, the surface area is* $2x^2 + 2xy + xy$ *(recall the top is open, so there is only one horizontal rectangle). Since we are told this is* 54*, we can rearrange to obtain*

$$y = \frac{54 - 2x^2}{3x}.$$

Substituting this into $V = x^2y$ *then produces the desired outcome. I have indeed answered the question, and my strategy was minimal in the sense that every decision I made informed what maths I was doing and what I had to do the next step. But this might not always be the case and **that is fine**. The important thing is to monitor your* *progress; if you appear to reach a dead-end, trace your steps back and find an alternate path.*

The next part asks us to find the maximum or minimum volume. We are told to use differentiation, so I first ask myself "have we used the derivative to compute extreme values before?". Indeed we have, the last lesson! This is my plan: differentiate V with respect to x, equate to zero to find any stationary points and use the second derivative to find which of these (if any) is a maximum/minimum point. Well then,

$$\frac{dV}{dx} = 18 - 2x^2 = 0 \qquad \Rightarrow \qquad x = \pm 3.$$

Because x is the height of the tank, the correct stationary point is positive three. The corresponding volume is $V = 36$. As for the justification, we determine maximality or minimality by substituting the stationary value into

$$\frac{d^2V}{dx^2} = -4x.$$

Doing this gives us -12; because it is negative, we know we have a maximum. This agrees with our plan to show that the volume is maximised. We monitored ourselves during the stationary point calculation to ensure that we select the correct one. If we didn't, i.e. if we chose $x = -3$, then V would be negative which doesn't make sense in the context that it represents a volume (something physical).

Annotation 3 By demonstrating a full PME cycle, it shows the students how I am planning and monitoring the progress I am making towards an answer. In mathematics, I would argue that monitoring is slightly less overt. For example, "because the volume… is in terms of x only, we… express y in terms of x" is an example of monitoring; I am not yet at a final answer and this shows how working I have achieved 'along the way' can be used to get us to the solution.

10:39 – 10:54 Students work through some similar questions independently. Answers will be shown one-by-one throughout the practice so that students can evaluate their work. Do not insist on the full PME structure yet; this is an opportunity for students to plan if they want to, but rather they should be focused on the mathematical content and monitoring/evaluating their progress. Bring the class together if there are any common mistakes seen whilst circulating.

10:54 – 11:05 The students now have an opportunity for guided PME practice.

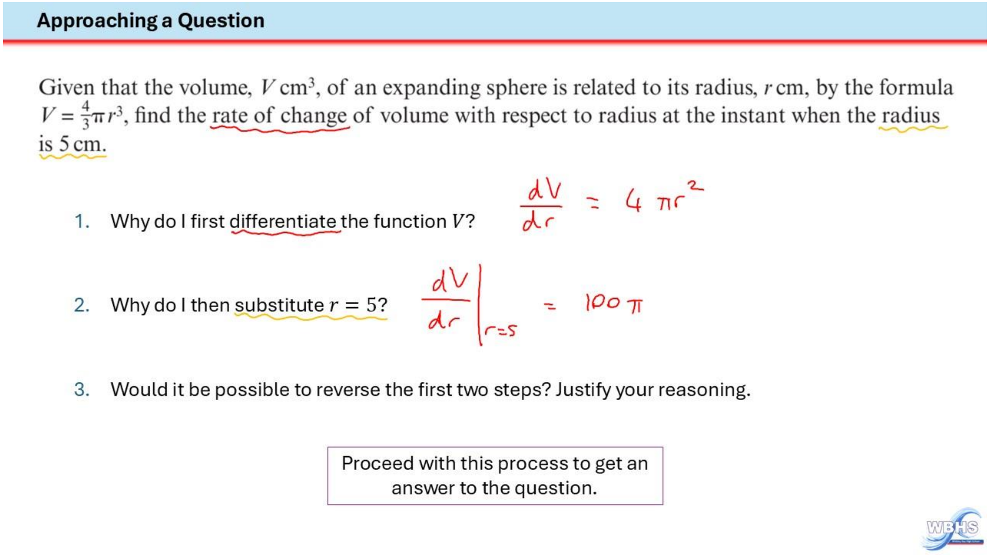
- **Plan**: students have one minute to think about what information they are told, what they might need and what steps they will take to get an answer.
- **Monitor**: after two minutes, students stop and compare with their plan what they have done. Use the prompting questions to help out anyone struggling.
- **Evaluate**: after a further two minutes of work in light of their monitoring, students see the final answer and reflect on their process.

Annotation 4 Having been shown a process on self-regulation, they can now try it out *in full* for themselves. The scaffolding leading up to this activity was important so that they are capable not only of the mathematics involved but also the metacognitive thinking required. Here, they are very much left to their own devices albeit with an explicit monitoring break to ensure they do indeed self-reflect *during* the task.

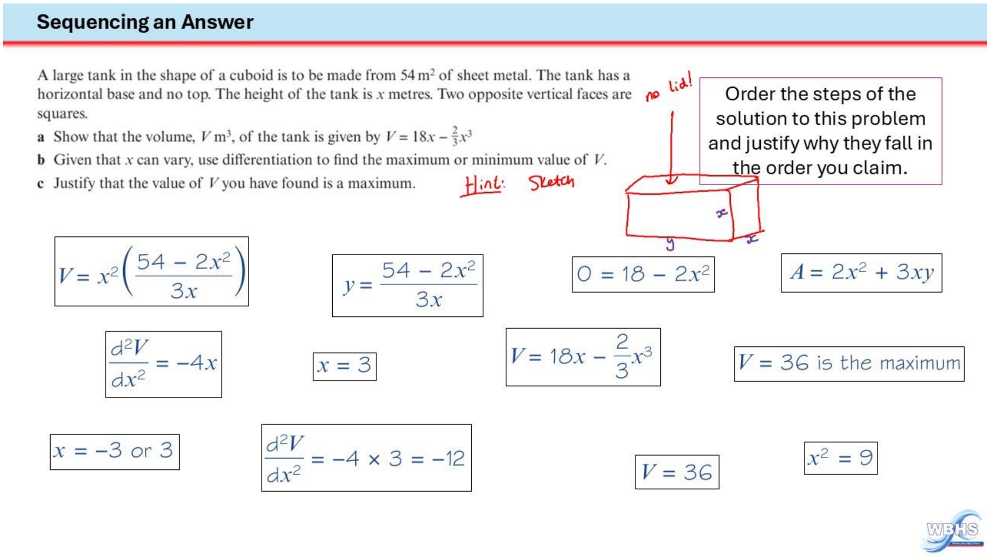
11:05 – 11:10 Students fill out the PME questionnaire.

Appendix B (Lesson Slides)

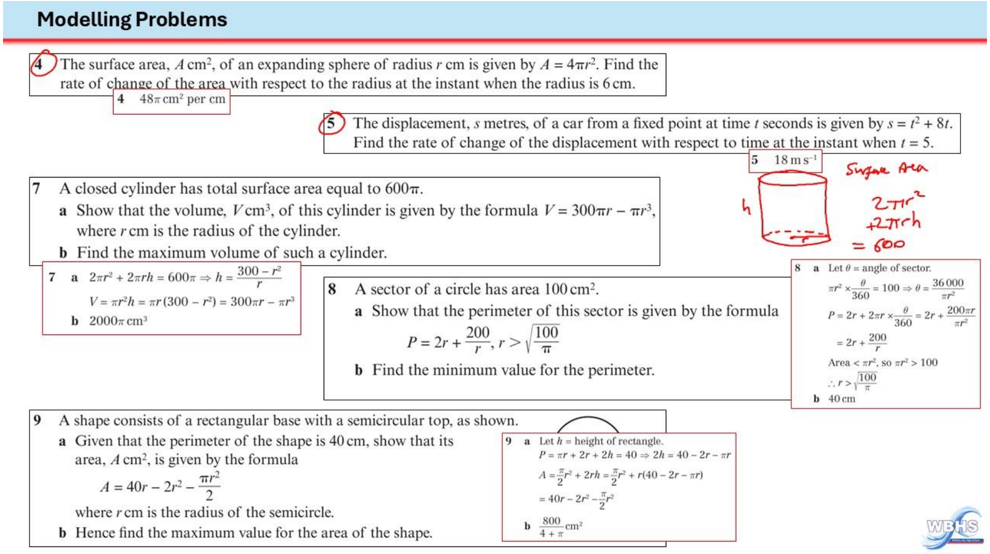
This appendix contains slides from my Year 12 metacognition lesson. Note that the only slides included are those with relevance to the PME cycle and the lesson context. Each caption provides additional details regarding the purpose of the corresponding slide.



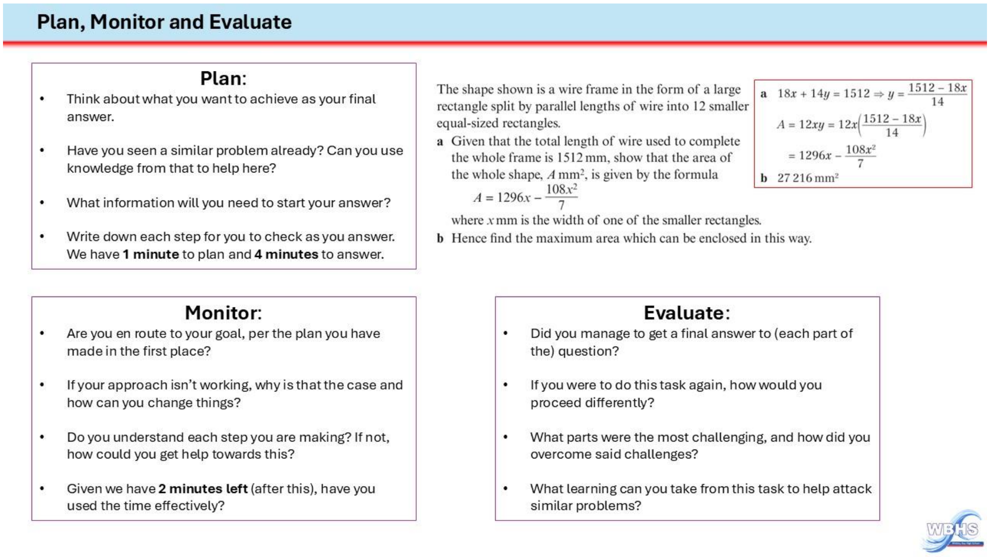
Slide 5: This is the first slide of the lesson involving metacognitive elements, in particular planning. The task was to answer the question but with the additional prompting of some planning-like questions. I cold called for answers to these extra questions – as well as the exam question – to gauge how well students could justify *why* they are doing each step.



Slide 6: This was the first time I explicitly introduced the PME cycle; this came **after** the sorting activity (the answers appeared via animation and in columns corresponding to each part **a**, **b**, **c** of the exam-style question). All parts of the PME cycle were done verbally, with me writing down the mathematics on the whiteboard. I took no input from students here, so that they could see precisely what was being done at each phase.



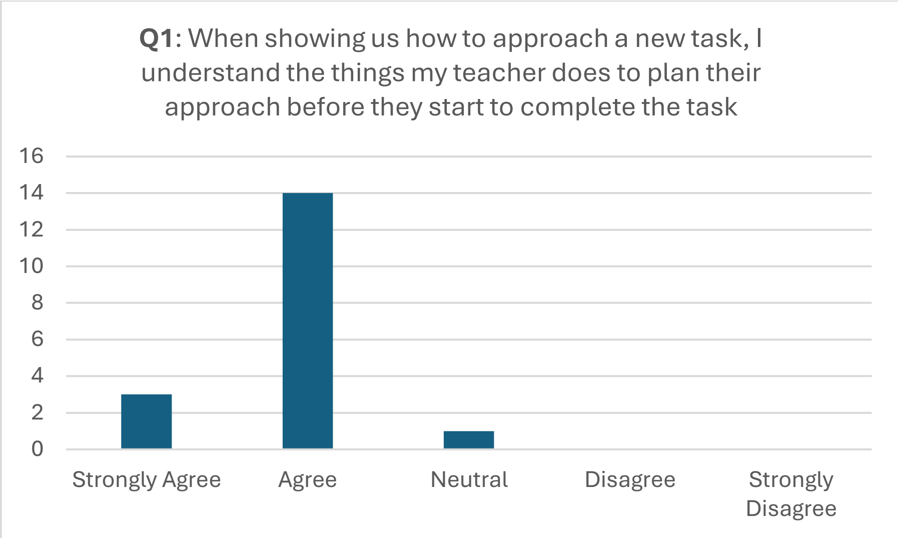
Slide 7: For context, this slide preceded the main PME activity, as a means to reduce the cognitive load demanded mathematically and instead allow the students to focus on developing metacognition. I still provided questions on the whiteboard to motivate *planning* and *monitoring* during these questions, however.



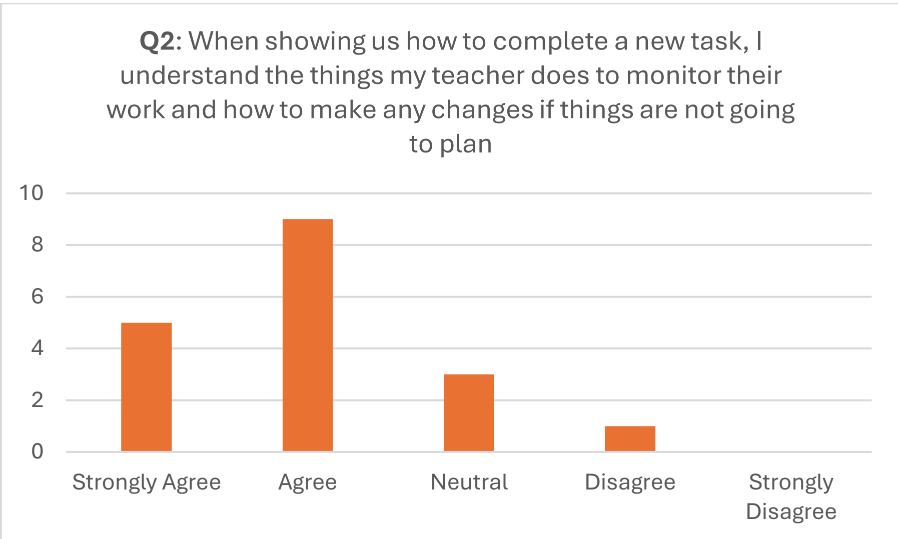
Slide 8: This was the primary activity, where students would answer an exam-style question similar to those found on Slide 7, except this time they would model a full PME cycle. Note the answers in the top-right appear via an animation to signal the beginning of the *evaluate* phase.

Appendix C (Student Feedback Data)

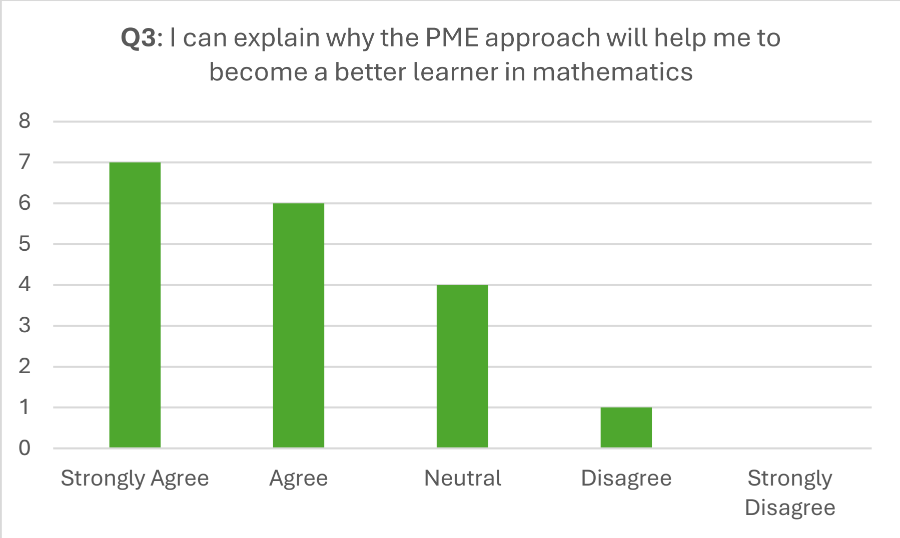
This appendix collates the responses received from the student feedback forms at the end of my Year 12 metacognition lesson. Each question is replicated verbatim as they appeared on the sheet (they are essentially as written on the provided template).



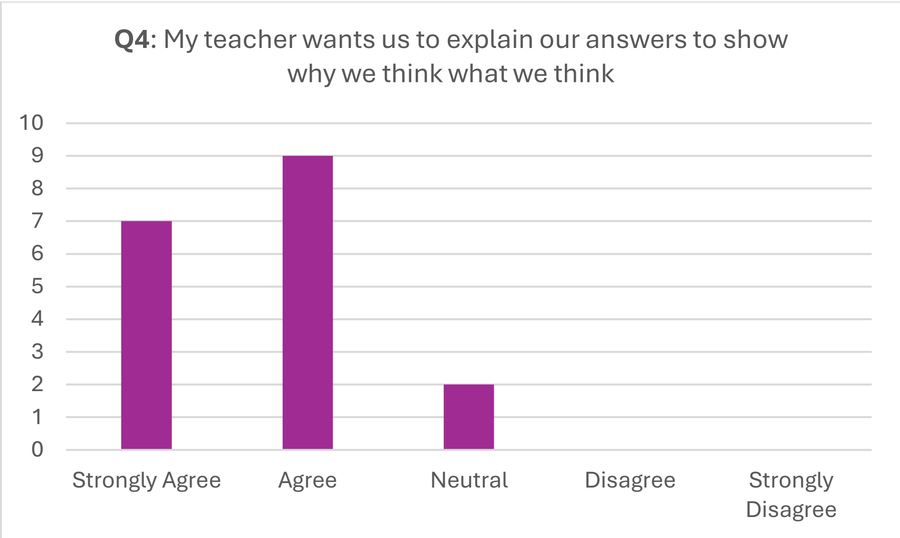
Graph 1: This suggests that the vast majority of students understood the planning phase. It was expected, as there are a lot of natural questions one can ask themselves before starting to answer a problem (typically, some of these are done automatically and internally, without explicit mention of it being a metacognitive strategy).



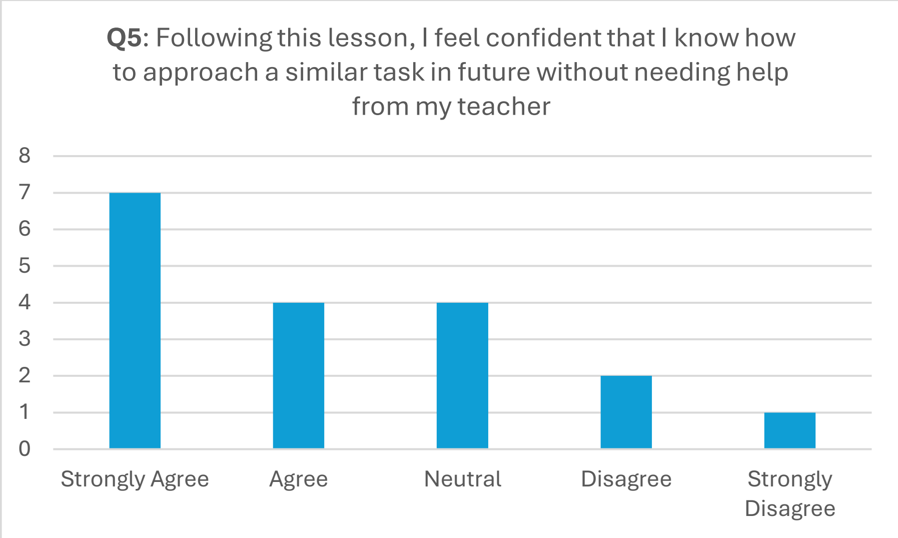
Graph 2: The data here suggests that a majority, still, understood the monitoring phase. Note there were two additional "Strongly Agree" students in comparison with Graph 1, but one student vocalised their misunderstanding of the *monitor* phase. Personally knowing said student, it is more likely that this comes from explicating metacognition rather than not knowing how to monitor, as they often do demonstrate self-regulation.



Graph 3: This graph shows that many of the students understood the benefits of the PME cycle on their learning and answering of questions. However, just over 27% of them were either "Neutral" or "Disagree" with the statement. Perhaps this indicates that I should reiterate and emphasise the *why* more often instead of focusing on the *what*.



Graph 4: Although 16 of the 18 students felt that I insisted they explain their working, the fact that two students were indifferent is enough for me to demand change when I next model metacognition. This may be partly down to a slight 'crunch' as the end of the lesson was rapidly approaching during the *evaluate* phase. It may have been beneficial to instead get the students to do a PME cycle earlier (in addition to this one).



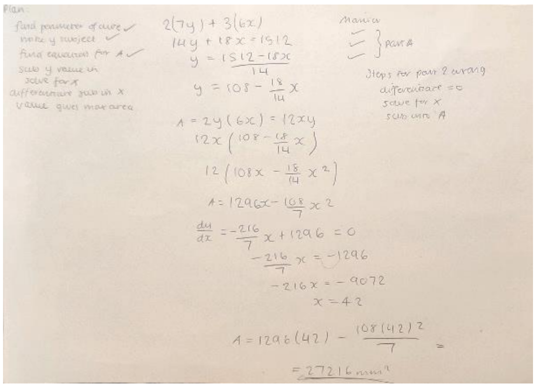
Graph 5: This graph is telling. Although many students were confident with each of the three stages, some are much more hesitant if they were to do the full PME cycle in totality *independently*. I regret not asking an additional sixth question of the form "following this lesson, I feel confident that I know how to approach a similar task in future **with** guidance from my teacher". Nevertheless, this question had the joint-highest number of "Strongly Agree" responses. Some caution should be taken as the students were likely in a rush to leave the class when they were answering the end of the survey, but the fact there is such a spread of results seems to indicate the survey was taken honestly (to an extent).

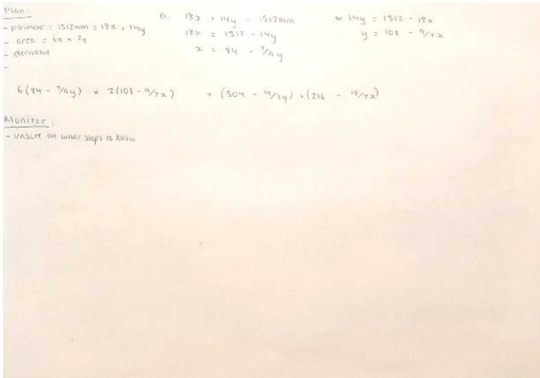
	Q1	Q2	Q3	Q4	Q5
Strongly Agree	3	5	7	7	7
Agree	14	9	6	9	4
Neutral	1	3	4	2	4
Disagree	0	1	1	0	2
Strongly Disagree	0	0	0	0	1

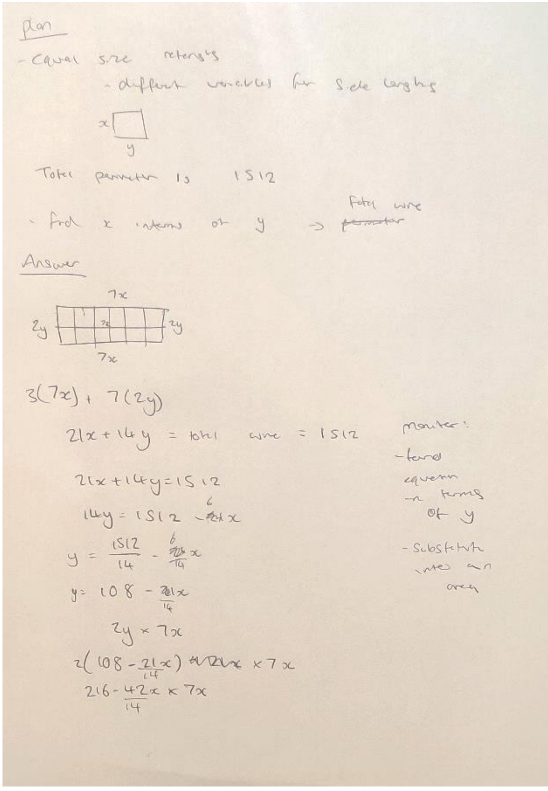
Table 1: This table consists of the raw data used to create Graphs 1 to 5 above. Each entry is the number of students that selected the corresponding option on the questionnaire.

Appendix D (Student Work)

(i) Good Evidence of PME

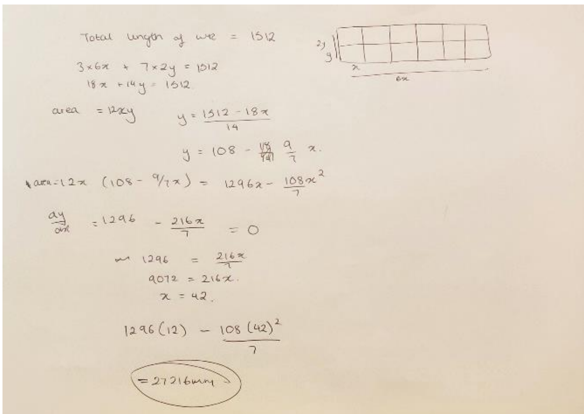






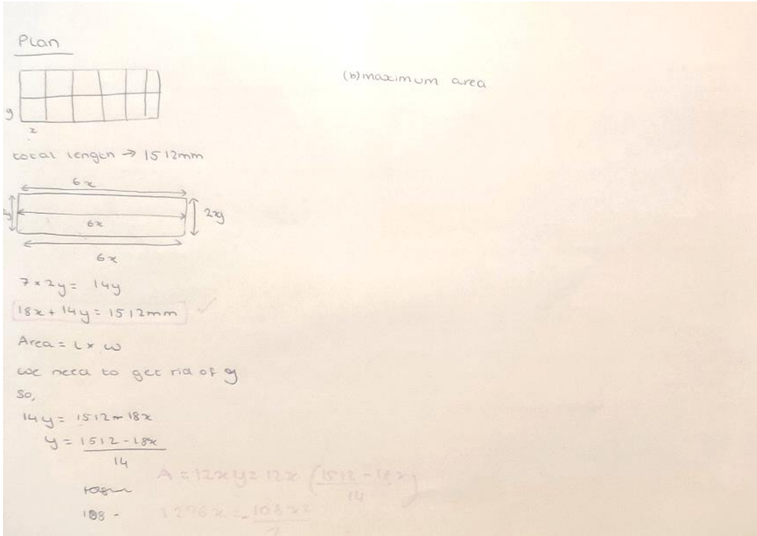
Work 1 These pieces of student work show clear evidence of the planning stage. The bottom-left also demonstrates that monitoring has taken place, as they are "unsure on what steps to follow". Although some evaluation was done verbally at the end of the task and lesson, not much was written down. One way to counter this next time is discussed as a result of the observer feedback (Appendix E); namely, to provide a structured sheet on which the students can answer the question but with explicit planning, monitoring and evaluating sections in which they can track their metacognition.

(ii) Little Evidence of PME



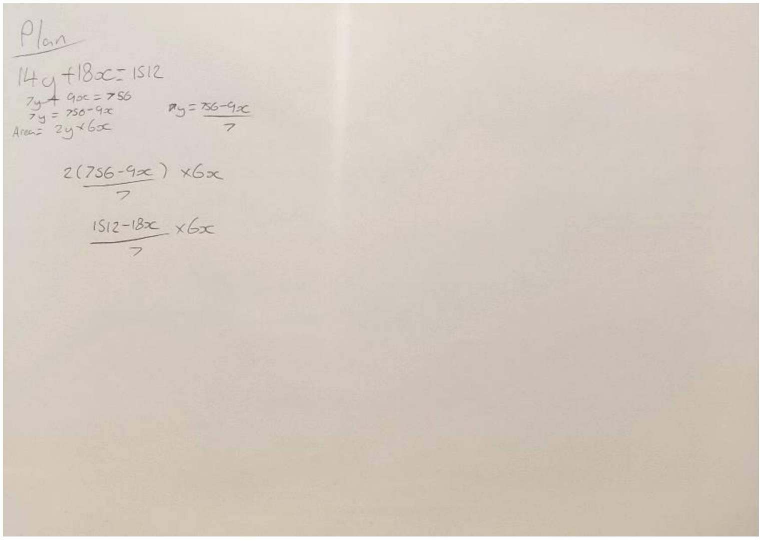
Work 2 This is the work of an A* grade student, which shows much less evidence of the PME cycle. In particular, the student has not explicated any of the stages, although they do annotate a drawing which could be interpreted as a *de facto* plan. Nevertheless, the student surpasses the work seen in Appendix D(i) in terms of mathematical content, which could mean that their planning and monitoring is more instinctive and already attuned, whether they know it or otherwise.

(iii) Lower-Attaining Student with More Metacognitive Evidence



Work 3 This is the work of a B grade student. Not only does it show signs again of (*de facto*) planning via the annotated diagram, but there is clear evidence of implicit monitoring throughout, by writing useful formulae and assessing that they "need to get rid of y". Moreover, there is some evaluative process demonstrated: in rather faint purple pen towards the bottom of their working, the student has written the correct answer to compare with their own work. The result is not a full evaluation on-paper, but what they have written could lead to a constructive self-reflection.

(iv) Higher-Attaining Student with Less Metacognitive Evidence



Work 4 This is the work of an A grade student. There is no evidence of planning, monitoring nor evaluating throughout their work on the exam-style question. Although Work 3 and Work 4 arrived at the same point at first glance, the latter has made an error and their answer is incorrect. Perhaps if the student felt more confident with the PME cycle, they would have been able to perform better both in a metacognitive sense, and hence mathematically. On the other hand, this issue could also reside at the level of subject content, so further (one-to-one, with this student in particular) analysis would be needed to understand what is happening in this case.

Appendix E (Observation Notes)

Herein lies the comments from the observation notes of one of my module partners. I have added some comments regarding suggestions for the future and verbal feedback they gave me after the lesson. In conjunction with the data from Appendix C, this will help improve my understanding of metacognitive pedagogy in mathematics.

Comment 1 The idea for a print-out would have structured the students' metacognitive processes much better. In reality, I had them write their answers on the blank sides of their questionnaires so that I could ensure the working was handed in along with the data.

The 'Active Ingredients' for mentors to notice when observing the metacognitive modelling sequence

1. Narrate the thought processes about how an expert thinker plans (thinks through) their approach at each stage of the plan, monitor, review process.
2. Offer explicit justification at each stage of the plan, monitor, review process.
3. Ask questions that bring pupils' attention to the how (the thinking process) not just the what (of the subject content) at each stage of the plan, monitor, review process.
4. Develop structures to promote and develop metacognitive talk in the classroom. This might involve equipping pupils with the language to talk metacognitively through providing sentence stems or using specific vocabulary.

Plan	Monitor	Evaluate
What statements does your trainee make to illustrate their thought processes when planning explicit? What questions does your trainee ask the pupils to get them involved in the thought processes that planning involves?	What statements does your trainee make to illustrate their thought processes when monitoring task completion explicit? What questions does your trainee ask the pupils to get them involved in the thought processes that monitoring task completion involves?	What statements does your trainee make to illustrate their thought processes when evaluating the completed task explicit? What questions does your trainee ask the pupils to get them involved in the thought processes that planning involves?
• "Have we seen similar questions?" • "What do we want to achieve?" • "We want… and we currently know that… so we could do this:…"	• "At this point, we should maybe recall the formula for…" • "Are you close to an answer? Why?" • "How can you tell if your work thus far is correct?" • "Who can you ask for help if you're stuck?" • "What happens to $x = -3$ and why?"	• "How successful were we in answering the question?" • "Were there any steps I struggled with? • "I felt very confident with… because…" • "Next time, I will look out for…"

Pupils' developing metacognitive awareness and self-regulation	Strengths of their practice
Pupils' developing metacognitive awareness and self-regulation: What do you notice in terms of the language that pupils are using that could be interpreted as a proxy for metacognitive regulation? "I think we should start by drawing a diagram" "This looks very similar to one from the last slide" "I've managed to get an expression for the volume but how do I get rid of y?" "I found this quite difficult, how did you manage it?" (to another student)	Strengths of their practice: The sequencing of the lesson was very good, gradually building up to the PME task. You made clear distinctions between each phase of the PME cycle. You used a lot of questions at each stage. Students took this on board in their task. Good idea having the metacognition questions on the board to guide students. Next steps to improve metacognitive modelling: Make a print-out for them to answer on, instead of blank paper (guide their thinking more). And could this be done more collaboratively? Some students seemed to do this naturally but you didn't really encourage it explicitly.

Comment 2 The observer fed back to me afterwards that the majority of the comments they heard from students fell into the *plan* phase of PME, with *some* things demonstrating monitoring. Evaluation was less frequent, unfortunately.

Appendix F (Module Partner Data)

In this appendix, I reproduce the data from my module partner's survey of their Year 9 history class (with their permission). Relevantly, van der Stel and Veenman (2010) investigate general and subject-specific metacognitive development in history and mathematics, expecting that "[subject]-specific metacognitive skills would tend to generalize [sic] during development" (p. 224) but being unable to confirm this hypothesis. This seems to suggest that some metacognitive strategies remain specialised and thus the impact of teaching metacognition in one subject fails to translate to another.

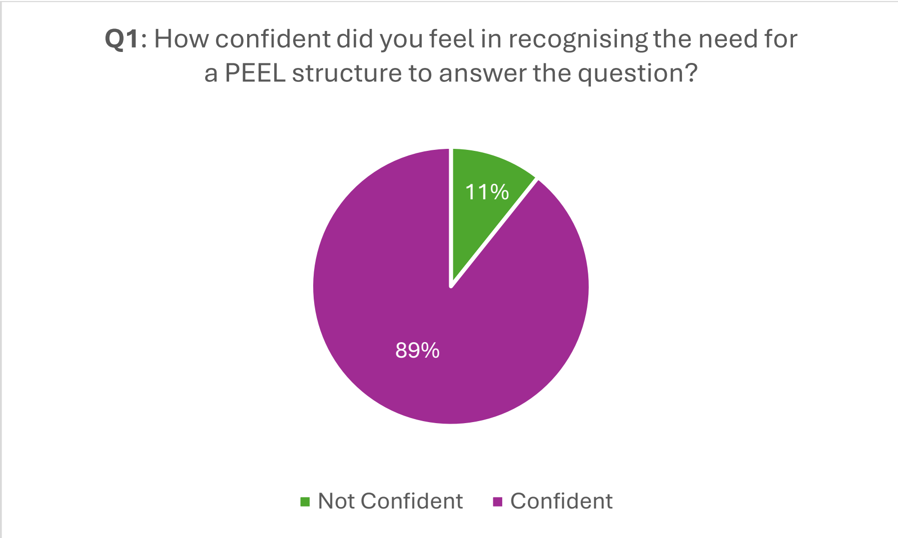


Chart 1: This pie chart shows that the vast majority of the 28 surveyed students felt confident that they could recognise the correct answer structure to their assessment question. This data aligns nicely with mine in the sense that a significant majority were confident when it came to the *planning* phase of the PME cycle.

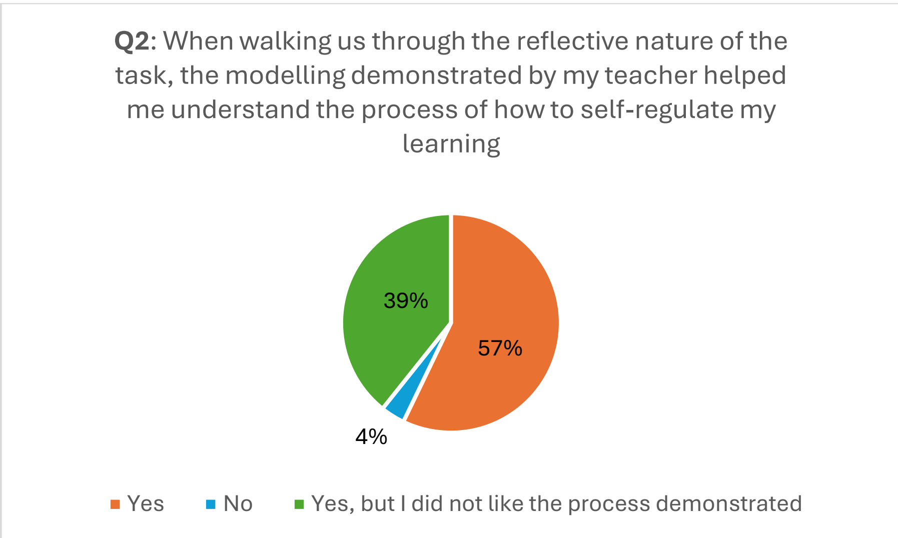


Chart 2: Here again, there is some consistency with the data from my Year 12 class in that there was scepticism regarding the *modelling* phase. However, this Year 9 class was larger (28 students compared to my 20) and the percentage of those that didn't understand modelling was lower (4% compared to my approximate 9%). That aside, there is some cross-curricular and cross-age agreement on a broad scale.

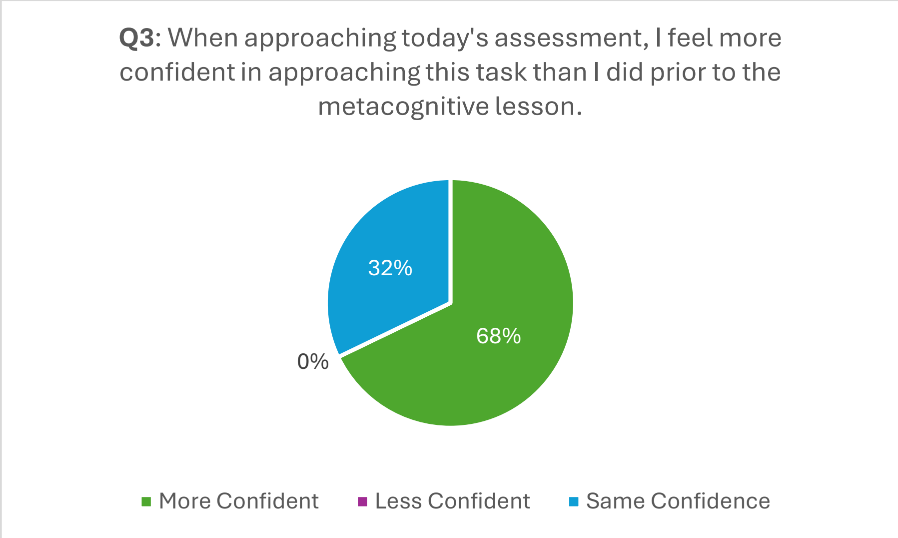


Chart 3: This shows that all students didn't get any less confident upon completion of the lesson. Sadly, this hasn't got a true comparison to my data (Appendix C, Graph 5). Indeed, the aforementioned phrasing of my question was loaded with the suggestion of independence, something that this question has tactfully avoided. On the other hand, a student with zero (resp. maximum) confidence may still have no (resp. full) confidence at the end and thus fall into the "Same Confidence" category, so it could be difficult to decipher the meaning here.

With hindsight, it may have been prudent for my module partner and I to each explore metacognition with our respective Year 9 and Year 12 classes and compare data as a means of supporting or rejecting the aforementioned conjecture of van der Stel and Veenman (2010). Further comparisons could have been made with my second module partner but I did not manage to share data with them at the time of writing.