Simulation of Solar System

Conservation of Energy and Mars Exploration

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Introduction

In this simulation about the Solar System, a spacial system containing six celestial bodies was created. These bodies involved the Sun, a few planets, as well as a satellite launched on Earth, and were placed into their corresponding orbits as in reality, with some realistic initial velocities assigned. Then I let the system evolve with the Beeman algorithm, meanwhile experimenting on the following.

The first experiment conducted with the system was to periodically display its total energy as it evolved to verify the conservation of energy. This was also shown graphically with a plot and the standard deviation indicated.

In addition, the satellite initialized beside Earth was aimed to approach Mars and possibly to travel back then. The second experiment was then to figure out a set of reasonable initial conditions for the probe that would allow it to take place.

With such conditions and the obtained path of the satellite, some further functional information including the journey time and the amount of fuel required could be roughly calculated, which helps to make judgements about the physical realizability of the Mars exploration.

Methods

From an object-oriented perspective, intuitively I considered that this simulation of a many-body system should involve exactly two types of objects, a body and a set of bodies. Here for the evolution of the system, since only gravitational effects between each pair of bodies were considered, all algorithms would be identically applied to the Sun, the planets and the probe as they underwent updates. Therefore, they could all be handled by one common Body class, and the system then would consist of just this type of elements.

To construct this many-body system, I thus created two classes, Body class for individual bodies which stored most of their inner and real-time properties, and Many_Body_System class for the system containing a list of such bodies that is capable to evolve itself over time, to obtain some specific holistic properties, as well as to visualize the entire system in 2-dimension.

In this simulation, the bodies were initially placed in alignment (i.e., all on the x-axis), and as the system evolved, each body's position, acceleration and velocity were updated consecutively, and the method applied was the Beeman Algorithm.

The sizes of the bodies were amplified by 100~1000 times from their real sizes otherwise it would be too difficult to identify them on the plot.

Body Class

- To hold the physical properties of a body.
- To compute the kinetic energy of a body.

The Body class held not only the physical properties, but also its colour and its size, which were used to visualize a specific body. Though the bodies were given their sizes, they were actually treated as a so-called point mass when computing gravitational effects in the simulation.

Positions, velocities and accelerations of the bodies were held as 2D vectors, which was for the convenience of later calculations. And since Beeman Algorithm takes the accelerations and the velocity of the previous iterations, the corresponding variables were created as well.

Many_Body_System Class

- To hold the holistic properties of the present and previous iterations of the system.
- To update the state of the system with the Beeman Algorithm, or to evolve.
- To visualize the system in 2-dimension with animation or without animation.

The method to proceed the evolution, since involved the Beeman Algorithm, needed the attendance of time-increment (delta t), so it was initialized in the constructor to enable the system to evolve without being animated. A counter was created here as well, which would be used to record the number of iterations that the system had undergone, to then infer other information like the age of the system. The class used the counter also to determine when to update and record the kinetic, potential and total energies of the system. To be specific, those energies were recalculated every 10 iterations.

For the methods of computing the energies and accelerations, instead of returning the results, they would update these instance variables, so that the function could be called as soon as an iteration was needed.

The orbit periods of the planets are determined by the time when each of them passed through the x-axis (i.e., one's position in the y-direction became positive), since all bodies but the Sun orbited counter-clockwise. A conditional was set here to prevent repetitive computations if a body already possessed a non-zero orbit period. This brought convenience since at the beginning the periods of the planets could be set to 0 and that of the Sun/satellite could be set to, for instance, -1, to indicate that the orbit period was inapplicable to it.

For the animation part, two axes were created and animated in each figure. The first one was the visualization of the system and the second one was a plot of the total energy against time. As animating, the current total energy and time, together with the calculated standard deviation, were displayed on the second axes real-timely.

Results and Discussion

For the results we regulate the units as follows:

- Time – Earth Year (y) (Length set to be 365 * 24 * 3600 = 31,536,000s)
- Energy – Joule (J)
- Distance/Displacement – Meter (m)
- Speed/Velocity – Meters per second (m/s)

Experiment 1

Results

With each time step in {1s, 30s, 300s, 3000s, 30000s}, the system underwent 3000 iterations, and the corresponding numbers of digits conserved were {9, 6, 2, 2, 1}. Additionally, the fluctuating range in energy kept reducing as the time steps were getting smaller, as the calculated standard deviations suggested.

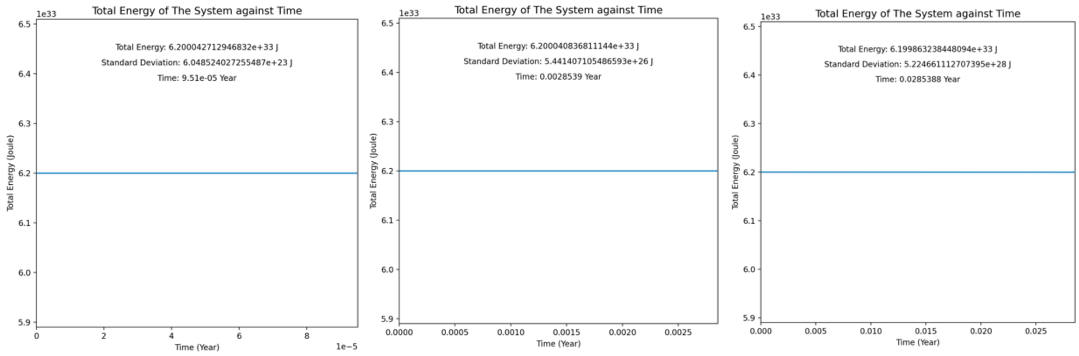


Figure 1 a): (From left to right) Energy plots with delta times in {1s, 30s, 300s} respectively. The final total energy, the overall standard deviation and the final time were indicated.

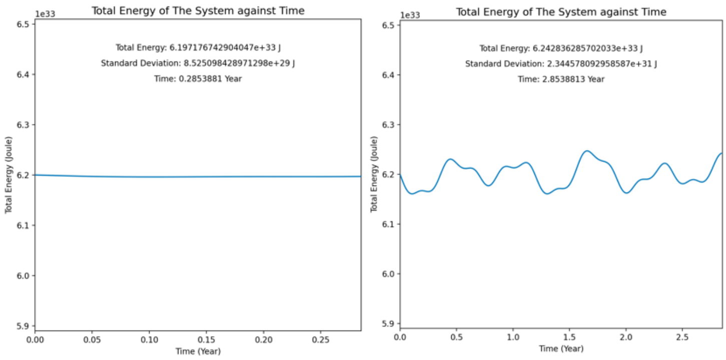


Figure 1 b): (From left to right) Similar energy plots with delta times in {3,000s, 3,0000s} respectively. The final total energy, the overall standard deviation and the final time were indicated.

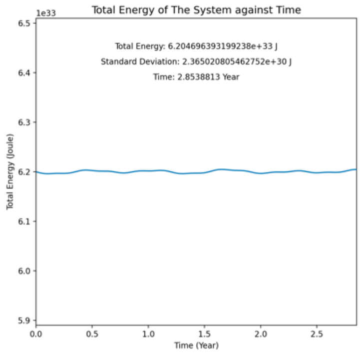


Figure 1 c): Similar Energy plot with delta time of 3,000s and 30,000 iterations.

By the way, the orbit periods calculated with all time-increments were similar and were quite accurate.

Discussion

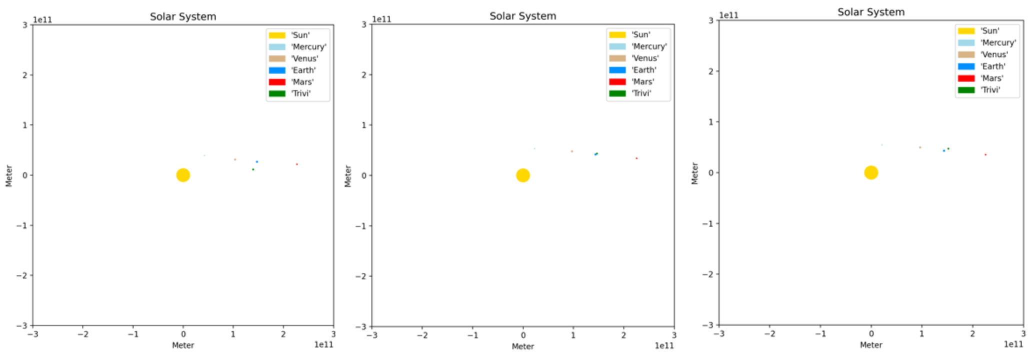


Figure 2: (From left to right) Visualized system with delta times in {300s, 3000s, 30000s} respectively after simulating for around 0.03 ~ 0.04 years. Note the positions of the satellite ‘Trivi’. With delta time 300s, it was attracted by Earth and finally moved in the opposite direction, while with 30000s it escaped rapidly. The initial speed of the satellite with respect to Earth was around 6,800m/s, which was much smaller than the initial speed needed to escape Earth.

It could be seen from the plots that the tendency of the total energy being conserved was getting increasingly obvious as the time steps became smaller, as the plotted lines were smoother, and the standard deviations declined significantly. It should be claimed that the energy was conserved more for small time-increments not simply because the system experienced a shorter time, as Figure 1 c) showed the energy with delta time 3,000s indeed fluctuated much less than that with 30,000s in the same time period.

For the largest experimented time step, 30,000 seconds, it is clear from the plot that the total energy fluctuated with an intensity of around 10^{32}J, which is approximately 2% on its scale. However, this time-increment had actually been quite large, as the satellite 'Trivi' (the satellite was for experiment 2, but I just mentioned it here to indicate the large time steps could mess up results) could leave Earth from its surface without even reaching any cosmic velocities (Figure 2). Thus, such fluctuations were acceptable, I suppose. More importantly, with the least shown delta time 1s, the fluctuating range reduced to around $\pm 2.5 * 10^{24}$ J, merely 0.00000004% on its scale. Furthermore, as the time-increment was set to 1 μs (10^{-6} s), all digits of the total energy were saved and the overall standard deviation non-periodically jumped between 0 and {some number} * 10^{18} J, which I think could be attributed to the round-off errors in floating computations. With the above, it was reasonable to confirm the system did conserve its total energy.

By the way, it seemed that whether the total energy was above the average or below it depended on the bodies' positions in their orbits. I guess it was relevant to how far the bodies were from the others, but I didn't have enough time to experiment on that, and it seemed to be insignificant as smaller time steps were applied.

Experiment 2

Results

In respect to the satellite, I gave it the name 'Trivi' and a mass of 1,000kg, and initialized it from the edge of the atmosphere of Earth, with a vector velocity of around (8,730.10m/s, 2,924.86m/s). After a period of 0.83999 years (around 10 months plus 2 days) in the simulation, the probe got close to Mars and entered the orbit of Phobos (orbit radius: ~9,377km), and the nearest distance observed between it and Mars was around 5,000 km. Then I continued the simulation for a period of 10 years, and the satellite was not detected to approach Earth, or enter the orbit of the Moon again.

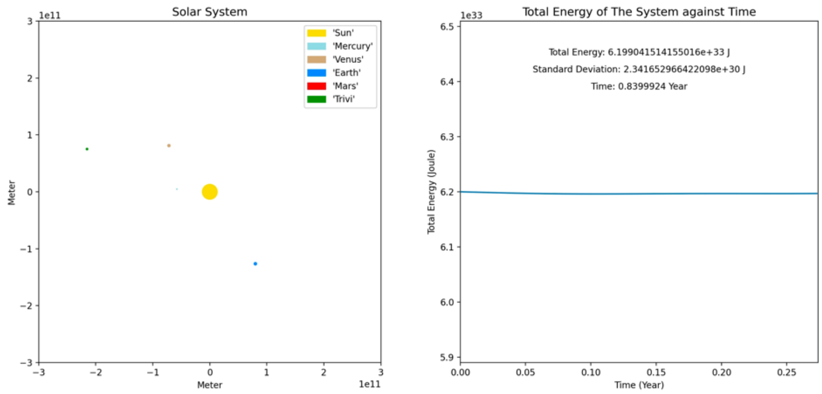
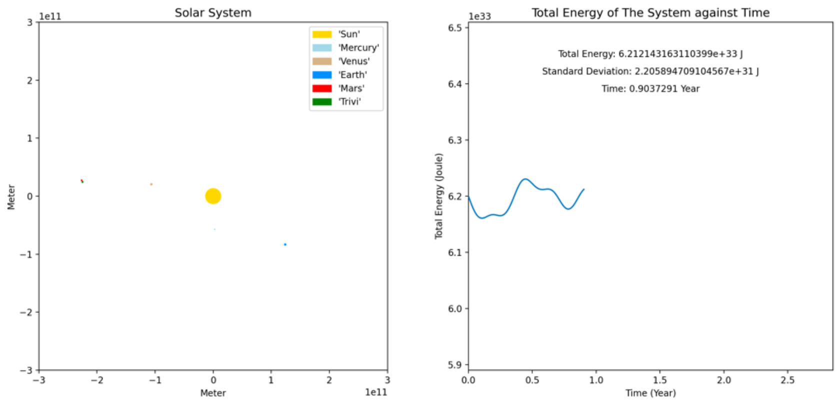


Figure 3: Visualized system and the energy plot with time increment 3,000s at the time when the probe reached the nearest distance between it and Mars. The probe and Mars had overlapped each other on the plot, but actually they were still 5,000km far.

Discussion

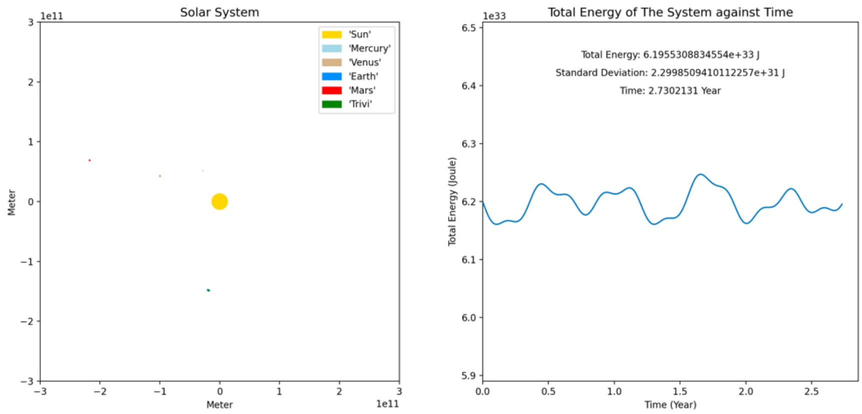
The journey time of 'Trivi' was around 10 months, while those of the Viking Probes were 11 months and 12 months respectively, which seemed to be reasonable since the initial scalar velocity of 'Trivi' was 9,207.03m/s, which was almost the speed needed to escape Earth from its surface. However, it was 'launched' at the edge of Earth atmosphere instead of at the surface, so I guess this velocity was, in fact, larger than what is required to get close to Mars, and that might be the reason why its journey time was slightly longer than the Viking Probes'.

When approximating a reasonable first guess for the launching scalar speed, I initially planned to launch the probe on the surface and took the second cosmic speed into consideration. I indeed figured out such conditions that allowed the probe to get close to Mars in around 11 months and return to Earth in 33 months (Figure 4 a) and b)). However, I didn't realize that they merely looked 'close' on the plot, but actually were still more than 10^9m far from each other. Also, I used 30,000s as the delta time, which was so large that the satellite was launched at the surface with a speed (6,800m/s) much smaller than the second cosmic speed but successfully escaped Earth. This set of information was then dropped in Experiment 2 but used in Experiment 1.



👈 Figure 4 a): With delta time 30,000s, 'Trivi' seemed to be close to Mars, but was actually quite far still.

Figure 4 a): Similarly, 'Trivi' seemed to be close to Earth, but was actually quite far still. 👉



Actually, when experimenting with the paths of the probe without visualization, I found that the system was quite sensitive to the initial conditions. In particular, with an initial horizontal velocity of 8,731.2 m/s, the satellite could be at least around 78,000 km far from Mars. However, if that velocity was decreased by 1 m/s to 8,730.2 m/s, the nearest distance between the probe and Mars could reach around 7,500 km, which had dropped one order of magnitude and entered the orbit of Phobos (the inner moon of Mars). Furthermore, as the initial horizontal velocity was decreased by 0.1m/s to 8,730.1 m/s, as finally adopted, this distance became around 5,000 km, while it would rise again if that velocity continued declining.

This kind of sensitivity seemed to be the feature of a nonlinear chaos system, which kind of made sense since a many-body system contained multiple linear systems (2-body systems) interacting with the others. However, I was not sure whether this kind of sensitivity would remain if we had particularly small time-increments, since large ones brought more errors. However, if the delta time became that small, not only the round-off errors would affect the experiment to a large extent, but also the simulation would be extremely slow, so that to verify this sensitivity on tiny scales was difficult to perform in this simulation.

Since the system was so sensitive to the initial conditions, the set of all possible paths of the probe, if visualized, should be dense in space, and there should be one that would allow the probe to return Earth after approaching Mars.

Conclusions

In summary, the many-body system has been appropriately simulated since it evolved in a very similar way as what would be expected in a real special system. The experiments were conducted with proper conditions and valuable results were obtained. For the first experiment, the total energies were conserved with increasingly small fluctuations as the sizes of the time-increments reduced. Then for another one, the assigned probe 'Trivi' entered the orbit of Phobos and approached Mars in a similar time as the journey times of the Viking Probes, but did not return to Earth in 10 years. Additionally, it was revealed that this many-body system was quite sensitive to the initial conditions, which might imply that it would be some kinds of nonlinear chaos system. The codes were not competent to do accurate computations on tiny scales and I believe there must be some underlying principles for a many-body system, but I don't have enough time now, so let's end here.