Numerical Optimization Techniques

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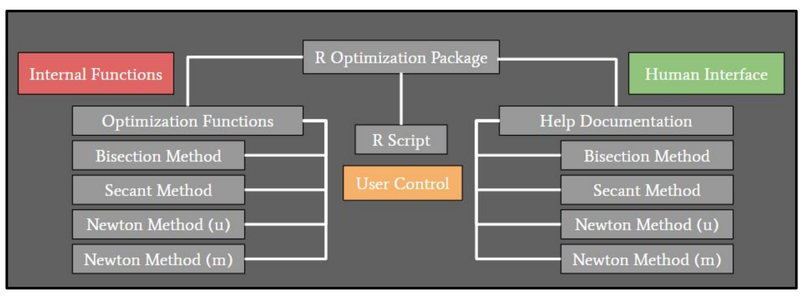
Introduction

- The goal of my project was to create an R package with multiple functions that optimize mathematical equations using numerical estimation techniques
- Optimization refers to the search for optima (global and/or local minima and maxima)
- Helpful for mathematicians, statisticians, and modelers

System Architecture

- Bisection Method, Secant Method, univariate Newton's Method, and multivariate Newton's Method
- Help page documentation



Project Outcomes

- Bisection Method is working
- Secant Method is working
- Univariate Newton's Method is working
- Multivariate Newton's Method is working
- My code is fully commented
- Useful help pages are included for every method
- Optimization methods seem to struggle finding global optima if local optima are nearby the initial starting value(s)
- Unable to implement Nelder-Mead Algorithm due to time constraints

Project Reflections

- **The primary goal of this project was to learn the math behind optimization techniques, how to create functions in R, and how to organize an R package**
- **I learned to break down code into smaller components and then build towards greater function complexity**
- **I realized the importance of prioritizing quality over quantity**

Usage and Equity

- Government funding, Free Education, Universal Healthcare, and Basic Needs are all optimization problems
- Statistical studies using optimization (i.e. least squares or maximum likelihood) influence policy regarding gender or race disparities
- Optimization can be used to find critical thresholds or equilibria in Applied Mathematical Models (i.e. population growth, species competition, or disease models)

Numerical Optimization Graph Examples

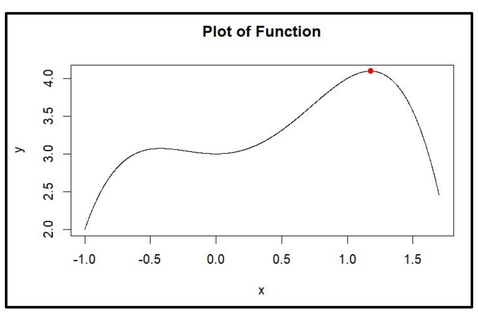


Figure 1: Polynomial Function optimized using Bisection Method

$$y = -x^4 + x^3 + x^2 + 3$$

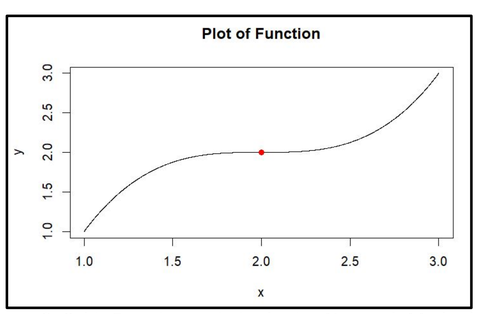


Figure 3: Cubic Function optimized using Newton's Method

$$y = (x-2)^3 + 2$$

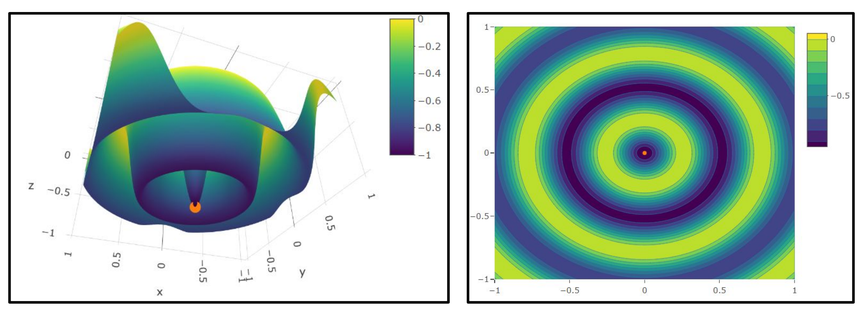


Figure 2: Drop-Wave Function optimized using multivariate Newton's Method; 3D plot (left), Contour plot (right)

$$f(\mathbf{x}) = -\frac{1+\cos\left(12\sqrt{x_1^2+x_2^2}\right)}{0.5(x_1^2+x_2^2)+2}$$

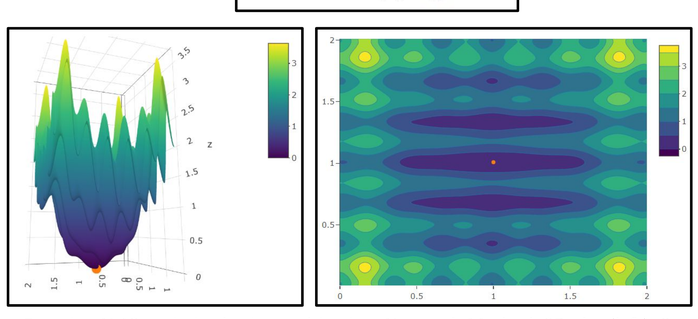


Figure 4: Levy Function N.13 optimized using multivariate Newton's Method; 3D plot (left), Contour plot (right)

$$f(\mathbf{x}) = \sin^2(3\pi x_1) + (x_1-1)^2\left[1+\sin^2(3\pi x_2)\right] + (x_2-1)^2\left[1+\sin^2(2\pi x_2)\right]$$

References / Acknowledgements

- Thank you to my advisor, Professor Leatherman, for helping me learn about optimization and assisting me with issues while coding
- Thank you to Professor Skon for reviewing my project and helping me with GitHub
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- [2] "Virtual Library of Simulation Experiments:" *Optimization Test Functions and Datasets*, https://www.sfu.ca/~ssurjano/optimization.html
- [3] https://github.com/BradenZeman/Optimization-Project