Title Section

Hi I'm Brad and my presentation will be on Revisionist Integral Deferred Correction, or RIDC, and the Pseudo-Spectral method.

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Introduction

So the goal of this presentation is to be able to demonstrate how a parallel high order time integration method like RIDC can be paired with other numerical solution schemes, in this case the Pseudo-Spectral method, in order to get an approximate solution with a high order of accuracy in both time and space to a non-linear PDE with an efficient computation time.

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To demonstrate, we want to solve the 2D incompressible vorticity equation assuming periodic boundaries and arbitrary initial condition.

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Section 1 – RIDC

In section 1 I'm only going to do a brief recap on the RIDC method since Ben has just covered it in detail.

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RIDC Recap

RIDC works by splitting the time domain into a large number of equidistant points which are represented in the diagram by the nodes. These points are then grouped into equally sized intervals which span the time domain. Being a predictor corrector method, RIDC first calculates prediction values of the approximate solution before then using a correction formula to obtain corrected values. Parallelisation is possible over the correction levels because RIDC first sequentially computes only as many values in each level as are necessary, creating this staggered node setup. The remaining values in each correction level are then calculated in parallel. This process is visualised by the number inside each of the nodes which represents the ordering of their calculation.

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Why Use RIDC?

So why do we use RIDC? First, ***new slide*** it is a high order time integration method. Second, ***new slide*** as has been shown, the method is parallelisable over the correction levels. ***new slide*** Using equidistant nodes also means that the quadrature weights matrix used in the correction loops doesn't need to ever be recalculated. ***new slide*** And the final correction value at the end of every time interval is dropped down to the prediction level which improves the predictions in the next interval.

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Section 2 – Pseudo-Spectral Method

Section 2 is on the Pseudo-Spectral method which is the technique that we use in order to solve for the spatial aspect of the solution to the vorticity equation.

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Spectral Methods Intro

Spectral methods are another class of techniques used to numerically solve differential equations. They assume that the solution can be written as a linear combination of a set of predetermined basis functions. We then choose the coefficients which allow the sum to best satisfy the differential equation. Here are some examples: if the basis functions are plane waves then u is a Fourier series, if they are Legendre polynomials then u is a Fourier-Legendre series, and if the basis functions are non-linear and adaptive then u is a neural network.

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Problems with the Spectral Method

There is a problem with the spectral method that we run into when we need to multiply our spectral solution by another function. To be able to find the resulting function we must be able to find its coefficients. They can be found using this matrix-vector multiplication which is actually a convolution because of the form that the matrix V takes. Finding the coefficients this way takes computational complexity O(N^2) and we need to precompute V which adds another step to the method, both of these outcomes are undesirable.

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Pseudo-Spectral Method

We can avoid any precomputation by using the pseudo-spectral method which discretises the domain and approximates the inner product integral with an already known quadrature. We use this quadrature to then get an approximation of the coefficients.

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Fourier Pseudo-Spectral Method

By assuming the basis functions are plane waves the discrete Fourier transform acts as the quadrature for our inner product. We can then use the fast Fourier transform in order to get the approximation of the coefficients. This reduces the computation complexity from O(N^2) to O(NlogN).

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Applying the Pseudo-Spectral Method

Here the first line has the original PDE on it. Then we take the Fourier transform on both sides to obtain this equation. The terms in red are function multiplication so we can't use this method without introducing a convolution. So instead we discretise space and use the discrete Fourier transform aka the Pseudo-Spectral method.

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Why Use Spectral Methods?

So why use spectral methods? Well first off, ***next slide*** they have a very high order of accuracy because they interpolate a function globally rather than locally in say a finite element scheme. They are also, ***next slide*** computationally more efficient than finite element methods for small scale problems. Finally, compared to many other methods like RIDC, they are, ***next slide*** much easier to implement.

Section 3 – Application

So now we apply everything from section 1 and 2 to try to solve the original PDE.

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Example Problem

This is the PDE we want to solve with all of the parameters fully specified. We do this by using RIDC to time integrate the Pseudo-Spectral form ODE.

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Example Solution

Solving the PDE gives us the following time evolution of the vorticity. Using two correction levels both of order two gives an order of accuracy in time of 4 and using a 100 by 100 spatial grid in the Pseudo-Spectral method should give an order of accuracy in space of 100 for both dimensions. Since two correction levels were involved its important to note that if RIDC isn't parallelised then we should expect it to take around twice as long to compute.

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Any questions?