1. $2010 \le \log\log n \le \log n \le \sum_{i=1}^{n} \frac{1}{i} \le \sqrt{n} \le \sum_{i=1}^{n} 1 = n \le$

$n\log n \le \sum_{i=1}^{n} i \le n^2 \le n^4 \le 2^n \le e^n \le n! \le n^n$

(See next page for more details)

2.

```
for (int i=0; i<n; i++){
    for (int j=0; j<n; j++) {
        C[i][j] = A[i][j] + B[i][j]; // 2*n^2
    }
}

for (int i=0; i<n; i++) {
    for (int j=0; j<n; j++) {
        for (int k=0; k<n; k++){
            D[i][j] = D[i][j] + A[i][k] * B[k][j];
            // 3*n^3
        }
    }
}
```

$2n^2 + 3\underline{\underline{n^3}} \rightarrow O(n^3)$

highest order

1. $\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n}$, Harmonic series

It is known that

$\log_e(n+1) < \sum_{i=1}^{n} \frac{1}{i} < \log_e(n) + 1$

therefore $\sum_{i=1}^{n} \frac{1}{i}$ is $O(\log_e n)$

$\sum_{i=1}^{n} i = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$, $O(n^2)$

e is a constant, $e \approx 2.718$

3. Running Time Analysis

Outer loop : i $0 \to n-3$

middle loop : j $i+1 \to n-2$

inner loop : k $j+1 \to n-1$

Outer loop iteration 1 :

$i=0$

middle loop iteration 1 :

$j=1$

inner loop $k: 2 \to n-1$

inner loop $(n-2)$ times

middle loop iteration 2 :

$j=2$

inner loop $k: 3 \to n-1$

inner loop $(n-3)$ times

middle loop iteration 3 :

$j=3$

inner loop $k: 4 \to n-1$

inner loop $(n-4)$ times

. . .

In the 1st iteration of outer loop, the inner loop (if statement) will be executed $(n-2)+(n-3)+(n-4)+\cdots+1 = \frac{(n-1)(n-2)}{2}$ times

Outer loop iteration 2 :

$i=1$

middle loop iteration 1 :

$j=2$

inner loop $k: 3 \to n-1$

inner loop $(n-3)$ times

middle loop iteration 2 :

$j=3$

inner loop $k: 4 \to n-1$

inner loop $(n-4)$ times

middle loop iteration 3 :

$j=4$

inner loop $k: 5 \to n-1$

inner loop $(n-5)$ times

...

In the 2nd iteration of outer loop, the inner loop (if statement) will be executed $(n-3)+(n-4)+(n-5)+\dots+1 = \frac{(n-2)(n-3)}{2}$ times

Therefore,

1st iteration of outer loop:

if statement will be executed $\frac{(n-1)(n-2)}{2}$ times

2nd iteration of outer loop:

if statement will be executed $\frac{(n-2)(n-3)}{2}$ times

3rd iteration of outer loop:

if statement will be executed $\frac{(n-3)(n-4)}{2}$ times

...

In total, if statement will be executed:

$$\underbrace{\frac{(n-1)(n-2)}{2}}_{\text{1st iteration}} + \underbrace{\frac{(n-2)(n-3)}{2}}_{\text{2nd iteration}} + \underbrace{\frac{(n-3)(n-4)}{2}}_{\text{3rd iteration}} + \dots + \underbrace{1}_{n-2\text{th iteration}}$$

$$= \frac{1}{2}\left((n-1)(n-2) + (n-2)(n-3) + (n-3)(n-4) + \dots + 2 \right)$$

$$= \frac{1}{2}\left((n-2+1)(n-2) + (n-3+1)(n-3) + (n-4+1)(n-4) + \dots + 2 \right)$$

$$= \frac{1}{2}\left((n-2)^2 + (n-2) + (n-3)^2 + (n-3) + (n-4)^2 + (n-4) + \dots \right)$$

$$= \frac{1}{2}\left(\underbrace{(n-2)^2 + (n-3)^2 + (n-4)^2 + \dots + 1^2}_{\text{Check the formula sheet in slides}} + (n-2) + (n-3) + (n-4) + \dots + 1 \right)$$

$$= \frac{1}{2}\left(\frac{(n-2)(n-1)(2n-3)}{6} + \frac{(n-1)(n-2)}{2} \right)$$

$$= O(n^3)$$