MODULE 9
PROBABILITY

Course Title: Discrete Mathematics

Course Code: MS 101

No of Units: 3

I. Module Objectives

At the end of the course, the student should be able to:

- Define and explain basic concepts of probability.
- Compute probabilities of simple and compound events.
- Solve problems involving conditional probability.
- Apply Bayes' Theorem to solve decision-making problems in computing contexts.

II. Lecture and Discussions

Probability deals with measuring the likelihood of events. In computing, probability concepts are applied in areas such as data analytics, cryptography, algorithms, reliability testing, and network security.

A. FUNDAMENTAL CONCEPTS

1. EXPERIMENT, SAMPLE SPACE, AND EVENT

- **Experiment -** an action or process that leads to well-defined outcomes.
- **Sample Space (S) -** the set of all possible outcomes.
- **Event (E)** – any subset of the sample space.

Example 1

A coin is tossed once. Find the sample space and the event of getting heads.
Experiment: Tossing a coin
S = {H, T}
E: getting heads → E = {H}

Example 2

A die is rolled once. Find the sample space and the event of getting an even number.
Experiment: Rolling a die
S = {1, 2, 3, 4, 5, 6}
E: getting an even number → E = {2, 4, 6}

Example 3

A letter is randomly selected from the word "COMPUTER."
Find the sample space and the event of selecting a vowel.
Experiment: Selecting a letter from the word "COMPUTER"
S = {C, O, M, P, U, T, E, R}
E: selecting a vowel → E = {O, U, E}

2. PROBABILITY OF AN EVENT

The probability of event E is given by:

$$\mathbf{P(E) = \frac{n(E)}{n(S)}}$$

Where:

n(E) = number of outcomes favorable to E

n(S) = total number of outcomes in the sample space.
$0 \leq P(E) \leq 1$

Example 1

A fair die is rolled. Find the probability of getting a number less than 5.
Given:
E = {1, 2, 3, 4} → n(E) = 4
n(S) = 6

$$\mathbf{P(E) = \frac{4}{6} = \frac{2}{3}}$$

Example 2

A letter is selected from the 26 lowercase English letters. Find the probability of selecting a vowel.
Given:
n(E)=5 vowels
n(S)=26 letters

$$\mathbf{P(E) = \frac{5}{26}}$$

Example 3

A number between 1 and 100 is chosen. Find the probability that it is divisible by 7.
Numbers divisible by 7: 7,14,21,28,35,42,49,56,63,70,77,84,91,98 → 14 numbers

$$\mathbf{P(E) = \frac{14}{100}}$$

3. COMPLEMENT OF AN EVENT

The complement of event E, denoted E', consists of all outcomes in S that are not in E.

$$\mathbf{P(E') = 1 - P(E)}$$

Example 1

A die is rolled. Find the probability of not rolling a 6.
P(E)=P(rolling a 6)=1/6

$$\mathbf{P(E) = 1 - \frac{1}{6} = \frac{5}{6}}$$

Example 2

The probability that a website is secure is 0.85. Find the probability that it is not secure.

$$\mathbf{P(E') = 1 - 0.85 = 0.15}$$

Example 3

The probability of server downtime is 0.02. Find the probability that there will be no downtime.
P(server downtime)=0.02
P(no downtime)

$$\mathbf{P(E') = 1 - 0.2 = 0.98}$$

4. UNION AND INTERSECTION OF EVENTS

- For any two events A and B:

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Where:
P(A ∪ B) = probability that A or B occurs
P(A ∩ B) = probability that both A and B occur

Example 1

Given:
P(A)=0.4 P(B)=0.3 P(A ∩ B)=0.1

$$\mathbf{P(A \cup B) = 0.4 + 0.3 - 0.1 = 0.6}$$

Example 2

From a group of 50 students:
30 know Java → P(A) = 30/50
20 know Python → P(B) = 20/50
10 know both → P(A ∩ B) = 10/50
Find P(A ∪ B).

$$\mathbf{P(A \cup B) = \frac{30}{50} + \frac{20}{50} - \frac{10}{50} = \frac{40}{50} = \frac{4}{5}}$$

Example 3

In a system:
Probability of malware (M) = 0.15
Probability of phishing (P) = 0.2
Probability of both = 0.05
Find the probability that a user encounters malware or phishing.

$$\mathbf{P(M \cup P) = 0.15 + 0.2 - 0.05 = 0.3}$$

5. CONDITIONAL PROBABILITY

- Conditional probability is the probability that event A occurs given that B has occurred.

$$\mathbf{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

Example 1

In a group of 100 users: 40 use 2FA (Two-Factor Authentication). (A), 25 use password manager (B) and 15 use both. Find the probability that a user uses 2FA given that they use a password manager.

$$\mathbf{P(A|B) = \frac{\frac{15}{100}}{\frac{25}{100}} = \frac{15}{100} \times \frac{100}{25} = \frac{15}{25} = \frac{3}{5}}$$

Example 2

Among 80 students: 50 passed Math, 30 passed Programming and 20 passed both subjects. Find the probability that a student passed Math given that they passed Programming.

Solution:
P(Math|Programming) = P(Math ∩ Programming) / P(Programming)
P(Math ∩ Programming) = 20/80
P(Programming) = 30/80

P(Math|Programming) = (20/80) ÷ (30/80)
P(Math|Programming) = 20/30
P(Math|Programming) = 2/3

Example 3

In a network: 60% of users use VPN, 40% of users enable firewall, and 25% use both. Find the probability that a user uses VPN given that they have a firewall enabled.

Solution:
P(VPN|Firewall) = P(VPN ∩ Firewall) / P(Firewall)
P(VPN ∩ Firewall) = 0.25
P(Firewall) = 0.40

P(VPN|Firewall) = 0.25 ÷ 0.40
P(VPN|Firewall) = 0.625

6. INDEPENDENT EVENTS

Two events A and B are independent if the occurrence of A does not affect the probability of B.

$$\mathrm{P(A \cap B)} = \mathrm{P(A)} \times \mathrm{P(B)}$$

Example 1

A coin is flipped and a die is rolled. Find the probability of getting heads and a 4.
Given:
P(head)=1/2
P(rolling a 4)=1/6

Solution:

$$\mathrm{P(head \cap 4)} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Example 2

A letter is selected randomly (vowel probability 5/26) and a die is rolled (even probability 3/6). Find the probability of getting a vowel and an even number.
Given:
P(Vowel) = 5/26
P(Even) = 3/6

Solution:

$$\mathrm{P(both)} = \frac{5}{26} \times \frac{3}{6} = \frac{15}{156}$$

Example 3

The probability of selecting a malware email is 0.1 and a phishing email is 0.05. Assume independence. Find the probability of selecting both.
Given:
P(Malware)=0.1, P(Phishing)=0.05

Solution:

$$\mathrm{P(both)} = 0.1 \times 0.05 = 0.005$$

7. BAYES' THEOREM

Bayes' Theorem is used to reverse conditional probabilities.
Formula:

$$\mathrm{P(A|B)} = \frac{[\mathrm{P(B|A)} \times \mathrm{P(A)}]}{\mathrm{P(B)}}$$

Where:

$$P(B) = [P(B|A) \times P(A)] + [P(B|A') \times P(A')]$$

Example 1

Suppose 5% of users have malware. A scanner detects malware 95% of the time if it is present, and gives a false positive 2% of the time if malware is not present. Find the probability that a user has malware if the scanner reports positive.

Given:
P(M) = 0.05
P(¬M) = 0.95
P(Pos|M) = 0.95
P(Pos|¬M) = 0.02

Solution:

$$P(B) = [P(B|A) \times P(A)] + [P(B|A') \times P(A')]$$
$$P(Pos) = (0.95 \times 0.5) + (0.2 \times 0.95)$$
$$P(Pos) = 0.475 + 0.019$$
$$P(Pos) = 0.0665$$

$$P(A|B) = \frac{[P(B|A) \times P(A)]}{P(B)}$$
$$P(A|B) = \frac{0.95 \times 0.05}{0.0665}$$

$$P(A|B) \approx 0.7143$$

This means there is a 71.43% chance the user has malware if the scanner reports positive.

Example 2

8% of network traffic is malicious. A detector flags malicious traffic 90% of the time if malicious, and 5% falsely when not malicious. Find the probability that traffic is malicious if flagged.

Given:
P(M)=0.08
P(¬M)=0.92
P(Flag|M)=0.90
P(Flag|¬M)=0.05

Solution:

$$P(B) = [P(B|A) \times P(A)] + [P(B|A') \times P(A')]$$
$$P(Flag) = (0.90 \times 0.08) + (0.05 \times 0.92)$$
$$P(Flag) = 0.072 + 0.046$$
$$P(Flag) = 0.118$$

$$P(A|B) = \frac{[P(B|A) \times P(A)]}{P(B)}$$
$$P(A|B) = \frac{0.90 \times 0.08}{0.118}$$

$$P(A|B) = \frac{0.072}{0.118}$$

$$P(A|B) \approx 0.6102$$

This means there is a 61.02% chance that the traffic is malicious if flagged.

Example 3

20% of users are premium. 85% of premium users enable 2FA, while 40% of regular users do. Find the probability that a user is premium given that they enabled 2FA (Two-Factor Authentication).

Given:
P(Premium)=0.2
P(Regular)=0.8
P(2FA|Premium)=0.85
P(2FA|Regular)=0.4
P(Premium|2FA)=0.17/0.49≈0.3469

Solution:

$$P(B) = [P(B|A) \times P(A)] + [P(B|A') \times P(A')$$
$$P(2FA) = (0.85 \times 0.2) + (0.4 \times 0.8)$$
$$P(2FA) = 0.17 + 0.32$$
$$P(2FA) = 0.49$$

$$P(A|B) = \frac{[P(B|A) \times P(A)]}{P(B)}$$
$$P(A|B) = \frac{0.85 \times 0.2}{0.49}$$

$$P(A|B) = \frac{0.17}{0.49}$$

$$P(A|B) \approx 0.3469$$

This means there is a 34.69% chance a user is premium given that they enabled 2FA (Two-Factor Authentication).

III. Application/Activity

Instruction: Solve the following problems on yellow pad paper. Show complete step-by-step solution

Activity 1: Basic Probability

Instructions: Solve the following problems on yellow pad paper. Show complete step-by-step solutions.

1. A number from 1 - 60 is chosen. Find the probability that it is divisible by 4.
2. A deck of cards is shuffled. Find P(drawing a red card).
3. Find the probability of selecting a consonant from the word "INFORMATION".

Activity 2: Compound and Conditional Probability

Instructions: Solve each problem and write the complete solution.

4. In a class of 80 students, 50 know HTML, 40 know CSS, and 25 know both. Find the probability that a randomly chosen student knows HTML or CSS.

5. Find the probability that a student knows HTML given they know CSS.
6. Two dice are rolled. Find P(sum is at least 10).

Activity 3: Bayes' Theorem
Instructions: Solve each problem and write the complete solution.

7. In a network, 6% of files are infected. A detector correctly flags infected files 97% of the time and wrongly flags clean files 2% of the time. Find P(infected|flagged).

8. In a website, 30% of users are new. 80% of new users enable notifications, and 40% of returning users do. Find P(new|notifications).

IV. Other References

- Rosen, K. H. (2019). Discrete Mathematics and Its Applications (8th ed.). McGraw-Hill Education.
- Epp, S. S. (2020). Discrete Mathematics with Applications (5th ed.). Cengage Learning.
- Johnsonbaugh, R. (2018). Discrete Mathematics (8th ed.). Pearson.

V. Conclusion / Summary

Probability provides tools to measure uncertainty. Understanding events, complements, unions, intersections, conditional probability, independence, and Bayes' Theorem supports accurate analysis of real-world problems. In IT, these concepts apply in security, spam detection, reliability analysis, and algorithm performance.

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