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Answer 1

Logic order precedence:

- $\neg$
- $\wedge$, $\vee$
- $\implies$
- $\iff$

a)

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$(p \vee q)$	$p \wedge (p \vee q)$	$(p \wedge (p \vee q)) \wedge \neg q$	$(\neg p \wedge q) \vee ((p \wedge (p \vee q)) \wedge \neg q)$
1	1	0	0	0	1	1	0	0
1	0	0	1	0	1	1	1	0
0	1	1	0	1	1	0	0	0
0	0	1	1	0	0	0	0	0

Table 1: Truth table for the expression $(\neg p \wedge q) \vee ((p \wedge (p \vee q)) \wedge \neg q)$

This gives us $\perp$ which means contradiction.

b)

Prove that:

$$p \vee (\neg q \rightarrow (p \wedge r)) \equiv (p \vee q)$$

Proof:

1. By Implication Law of Logic we know $\neg q \rightarrow (p \wedge r) \equiv q \vee (p \wedge r)$ so rewrite it as so.
2. Put it into the expression: $p \vee (q \vee (p \wedge r))$.
3. By the Associative Law of Disjunction, rewrite as $p \vee q \vee (p \wedge r)$.
4. By the Commutation Law of Logic, rewrite as $q \vee p \vee (p \wedge r)$.
5. Apply the Absorption Law of Logic: $p \vee (p \wedge r) \equiv p$.
6. The expression simplifies to $p \vee q$, hence it is $\equiv p \vee q$ of right side.

Therefore, $p \vee (\neg q \rightarrow (p \wedge r)) \equiv p \vee q$, QED.

Answer 2

Given Predicates:

- $S(x)$: x is a student.
- $C(y)$: y is a course.
- $E(x, y)$: Student x is enrolled in course y.
- $P(x, y)$: Student x has passed course y.
- $R(y, z)$: Course y is a prerequisite for course z.

a)

$$\forall x\, \forall y\, (S(x) \land C(y) \land E(x, y) \implies \forall z\, (R(z, y) \implies P(x, z)))$$

b)

$$\exists x\, (S(x) \land \exists y\, (C(y) \land E(x, y) \land \forall z\, (C(z) \land E(x, z) \to z = y)))$$

c)

$$\forall y\, (C(y) \to \forall x\, (P(x, y) \to \forall z\, (R(z, y) \to P(x, z))))$$

d)

$$\exists x\, (S(x) \land \exists y\, (C(y) \land E(x, y) \land \neg P(x, y)))$$

e)

$$\exists y\, (C(y) \land \forall x\, (S(x) \to \neg P(x, y)))$$

f)

$$\forall y\, (C(y) \to \exists x\, (S(x) \land (E(x, y) \lor P(x, y))))$$

Answer 3

$$
\begin{aligned}
&1. \quad p \to (q \vee r) \quad (Premise - 1)\\
&2. \quad \neg r \wedge \neg s \quad (Premise - 2)\\
&3. \quad q \to s \quad (Premise - 3)
\end{aligned}
$$

Our goal is to prove $\neg p$ by $\perp$ (contradiction).

Proof

1. $\boxed{\text{Assume } p}$ (Assumption for indirect proof)
   (a) From $p \to (q \vee r)$ (Premise 1) and p, we conclude $q \vee r$ by $\to$-elimination, (Modus Ponens).
   (b) From $\neg r \wedge \neg s$ (Premise 2), we conclude $\neg r$ by $\wedge$-elimination.
   (c) $\boxed{\text{Assume } q \vee r}$ (From 1 and 2)
      i. $\boxed{\text{Case 1: Assume } q}$
         A. From $q \to s$ (Premise 3) and q, we obtain s by $\to$-elimination.
         B. But, $s \perp \neg s$ (via Premise 2 by $\wedge$-elimination).
      ii. $\boxed{\text{Case 2: Assume } r}$
         A. But, r $\perp \neg r$ (via Premise 2 by $\wedge$-elimination).
2. Since $\forall$ cases proven the $\perp$, we conclude $\neg p$ by $\neg$-introduction.

   Quod Erat Demonstratum.

Answer 4

Given Predicates:

- $P(x)$: x is a philosopher.
- $S(x)$: x is a scientist.
- $K(x)$: x knows everything.

a)

$$
\begin{aligned}
&1. \quad \exists x\,(P(x) \wedge S(x)) \quad \text{(Premise 1)}\\
&2. \quad \forall x\,(P(x) \to K(x)) \quad \text{(Premise 2)}\\
&3. \quad \exists x\,(S(x) \wedge K(x)) \quad \text{(Claim)}
\end{aligned}
$$

b)

$$\exists x\,(S(x) \land K(x))$$

1. Assume Premise 1: $P(a) \land S(a)$ for arbitrary a	Existential Instantiation
2. Assume Premise 2: $P(a) \to K(a)$ for $x = a$	Universal Instantiation
3. $S(a)$	1, $\land$-elimination
4. $K(a)$	2, Modus-Ponens
5. $S(a) \land K(a)$	1-4, $\land$-introduction
6. $\exists x(S(x) \land K(x))$	1-5, Existential-Generalization