

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE NASIONALE SENIOR SERTIFIKAAT

GRADE 12/GRAAD 12

TECHNICAL MATHEMATICS P2/TEGNIESE WISKUNDE V2

.........

NOVEMBER 2023

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

CODE/	EXPLANATION/VERDUIDELIKING
KODE	
A	Accuracy/Akkuraatheid
AO	Answer only/Slegs antwoord
CA	Consistent accuracy/Volgehoue akkuraatheid
Ι	Identity/Identiteit
M	Method/ <i>Metode</i>
NPR	No penalty for rounding/Geen penalisering vir afronding nie
NPU	No penalty for omitting units/Geen penalisering vir eenhede weggelaat nie
R	Rounding/Afronding
RE	Reason/Rede
S	Simplification/Vereenvoudiging
F	Formula/Formule
SF	Substitution in correct formula/Vervanging in korrekte formule
ST	Statement/Bewering
ST/RE	Statement with reason/Bewering met rede

These marking guidelines consist of 19 pages. *Hierdie nasienriglyne bestaan uit 19 bladsye*.

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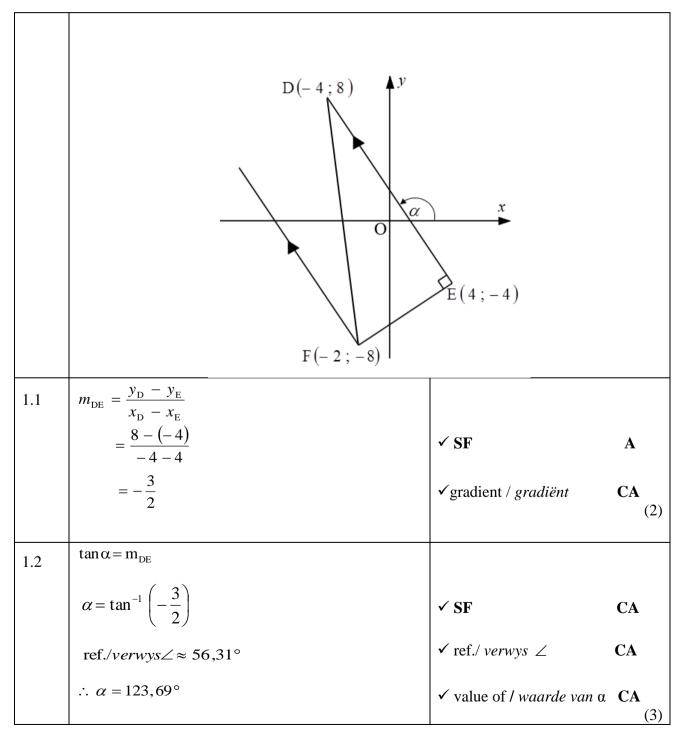
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NOTE:

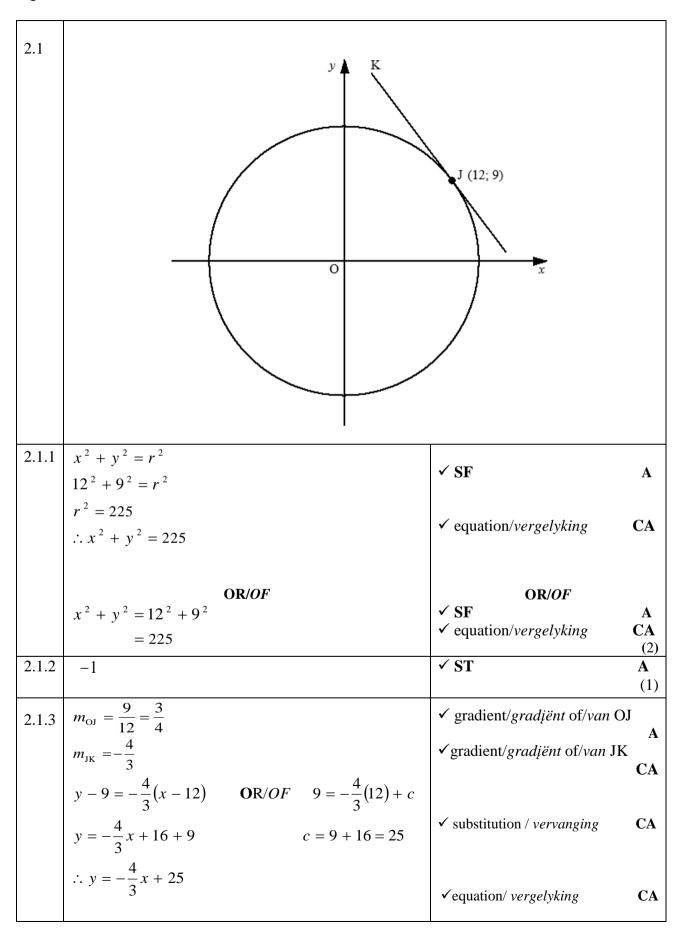
- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- The method of Consistent Accuracy marking must be applied in all aspects of the marking guideline where applicable as indicated with the marking code CA.

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.
- Die metode van Volgehoue akkuraatheid-nasien moet waar moontlik tot alle aspekte van die nasienriglyne toegepas word soos aangedui deur die nasienkode **CA**.



1.3	$m_{\text{parallel/ewewydig}} = -\frac{3}{2}$	✓ gradient /gradiënt	CA
	$\begin{pmatrix} 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 &$		
	$y - (-8) = -\frac{3}{2}(x - (-2)) \text{ OR}/OF - 8 = -\frac{3}{2}(-2) + c$		
	$y = -\frac{3}{2}x - 3 - 8 \qquad c = -8 - 3$		
	$\therefore y = -\frac{3}{2}x - 11$	✓ equation / vergelyking	CA
	Subst/ Vervang (-10; 5):		
	LHS / $LK = 5$		
	RHS / $RK = -\frac{3}{2} \times (-10) - 11 = 4$	✓ Subst/ Vervang (-10; 5)	CA
	∴ the point (-10; 5) does not lie on the line ∴ die punt (-10; 5) lê dus nie op die lyn nie	✓ conclusion / gevolgtrekking	g CA
	OR/OF	OR/OF	
	$m_{\text{parallel/ewewydig}} = -\frac{3}{2}$	✓ gradient /gradiënt	CA
	$m_{point/pun\&F} = \frac{-8-5}{(-10)}$	✓ SF	A
	$=\frac{-13}{8}$	✓ gradient point & F / gradiënt punt & F	nt CA
	$m_{\text{point/punt\& F}} \neq m_{\text{parallel/ewewydig}}$		
	 ∴ the point (-10; 5) does not lie on the line ∴ die punt (-10; 5) lê dus nie op die lyn nie 	✓ conclusion /gevolgtrekking	CA (4)
1.4	$EF = \sqrt{(x_F - x_E)^2 + (y_F - y_E)^2}$		
	$= \sqrt{(-2-4)^2 + (-8-(-4))^2}$ $= \sqrt{52} = 2\sqrt{13}$	✓SF	A
	$DE = \sqrt{(-4-4)^2 + (8-(-4))^2}$	✓ length/ lengte EF	CA
	$DE = \sqrt{(-4 - 4)^2 + (8 - (-4))^2}$ $= 4\sqrt{13}$	✓ length/ <i>lengte</i> DE	A
	Aron of ADEE 1 v. 2 /12 v. 4 /13	✓ SF	CA
	Area of $\triangle DEF = \frac{1}{2} \times 2\sqrt{13} \times 4\sqrt{13}$ =52 square units/vierkante eenhede	✓ area	CA (5)
			[14]

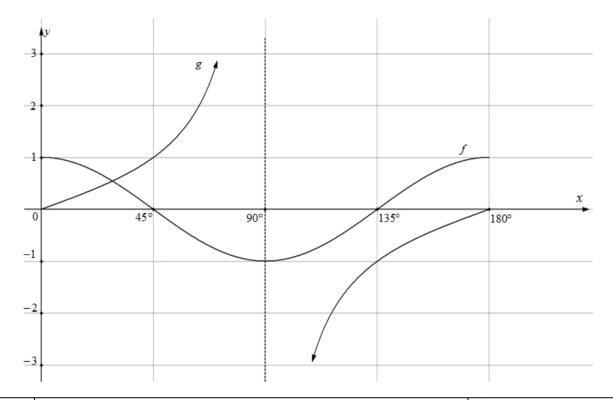


	OR/OF	07.107
	$x \cdot x_1 + y \cdot y_1 = r^2$ $12x + 9y = 225$ $9y = -12x + 225$ $y = -\frac{4}{3}x + 25$	OR/OF ✓ F ✓ subst / vervang (12; 9) ✓ subst / vervang CA ✓ equation /vergelyking CA (4)
2.2.1	$\frac{x^2}{(\sqrt{11})^2} + \frac{y^2}{8^2} = 1$	✓ standard form/ standaardvorm A (1)
2.2.2	-√11 √11 - 3	✓ x and y –intercepts/ afsnitte A ✓ elliptical shape/ elliptiese vorm CA (2)
		[10]

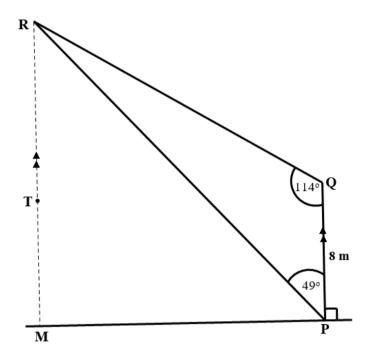
_	<u></u>	<u>, </u>	
3.1.1	$\sin(x - y) = \sin(152, 4^{\circ} - 24, 8^{\circ})$ $\approx 0,79$	✓ substitution / vervanging ✓ S	A CA (2)
3.1.2	$ \frac{1}{2}\sec\left(\frac{x}{2} + 80^{\circ}\right) \\ = \frac{1}{2}\sec\left(\frac{152,4^{\circ}}{2} + 80^{\circ}\right) \\ = \frac{1}{2}\sec156,2^{\circ} \\ = \frac{1}{2} \times \frac{1}{\cos156,2^{\circ}} $	✓ substitution / vervanging	A
	≈ -0.55	✓ S	CA (2)
3.2.1	$\sin \beta = -\frac{4}{5}$ $\csc \beta = -\frac{5}{4}$	✓ ratio / verhouding	CA (1)
3.2.2	-3 -4 5 $x^2 + y^2 = x^2$		
	$x^{2} + y^{2} = r^{2}$ $x^{2} + (-4)^{2} = (5)^{2}$ $x^{2} = 9$	✓ SF	A
	x = -3	✓ value of/waarde van x	CA
	$\tan \beta + \cos \beta = \frac{-4}{-3} + \left(-\frac{3}{5}\right)$ 11	✓ tan ratio / verh ✓ cos ratio / verh	CA CA
	$=\frac{11}{15}$	✓ S	CA (5)

3.3	$\cos x = -\sin 56.7^{\circ}$		
	$\cos x = -0,835807361$	✓ S	A
	Ref. angle $/verwhoek = 33,30^{\circ}$	✓ Ref. angle /verw hoek	CA
	$x = 180^{\circ} - 33,30^{\circ} \text{ or/of } x = 180^{\circ} + 33,30^{\circ}$	✓ 146,7°	CA
	$\therefore x = 146,7^{\circ} \text{ or/of } x = 213,3^{\circ}$	✓ 213,3°	CA
			(4)
			[14]

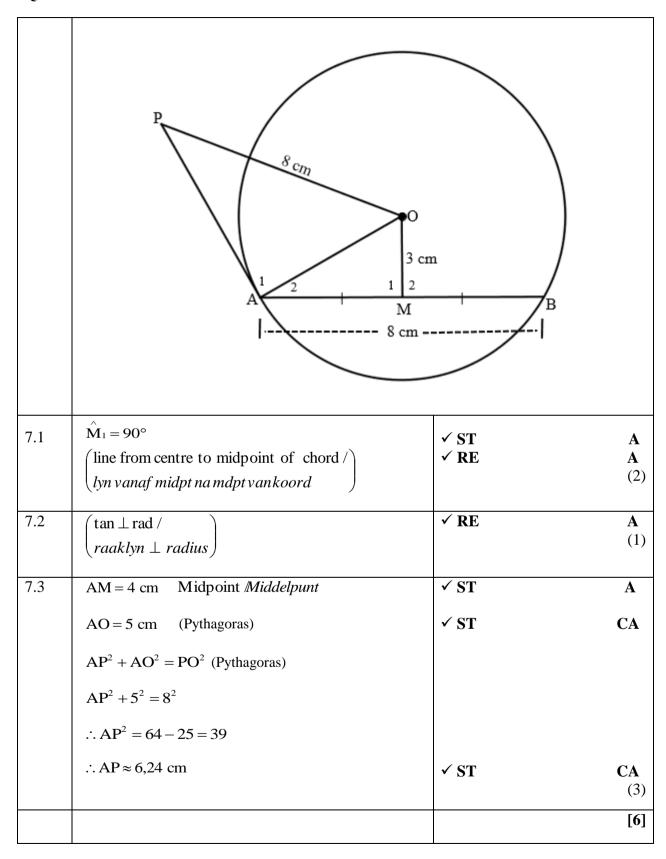
4.1.1	1	✓ I	A
4.1.2	sin A cos A	✓ reduction /reduksie	(1) A
1.1.2	603.11	reduction / reduksie	(1)
4.1.3	- cosec A	✓ reduction /reduksie	A
	$\sin(180^{\circ} + A) \cdot \cot(360^{\circ} - A) \cdot \cos(2\pi - A) + \sin^{2}(360^{\circ} - A)$		(1)
4.2.			
	$= (-\sin A) \cdot (-\cot A) \cdot \cos A + (-\sin A)^2$	$\checkmark -\sin A$	A
	$= \sin A \cdot \frac{\cos A}{\sin A} \cdot \cos A + \sin^2 A$	✓ -cot A	A
	$= \cos^2 A + \sin^2 A$	$\checkmark -\sin A \text{ or } \sin^2 A$	A
	= 1	✓ cos A ✓ cot identity/identite	A oit A
	_1	✓ S	CA
		✓ answer/antwoord	CA
			(7)
4.3.1	$\sec x(1 - \sec x)$	✓ I	A
			(1)
4.3.2	$\frac{\csc x - \csc x \cdot \sec x}{\sec x - \left(\tan^2 x + 1\right)} = \cot x$		
	LHS = $\frac{\csc x - \csc x \cdot \sec x}{\sec x - (\tan^2 x + 1)}$		
	$= \frac{\csc x (1 - \sec x)}{\cos x (1 - \sec x)}$	✓ factor/faktor (cosec	x)
	$= \frac{1}{\sec x - \sec^2 x}$		A
	$= \frac{\csc x(1 - \sec x)}{\sec x(1 - \sec x)}$	✓ I	A
		✓ S	CA
	$= \frac{1}{\sin x} \times \cos x$. 3	CA
	$= \cot x \ OR/OF \frac{1}{\tan x}$	✓ I	A (4)
	∴ LHS = RHS		(.)
			[15]

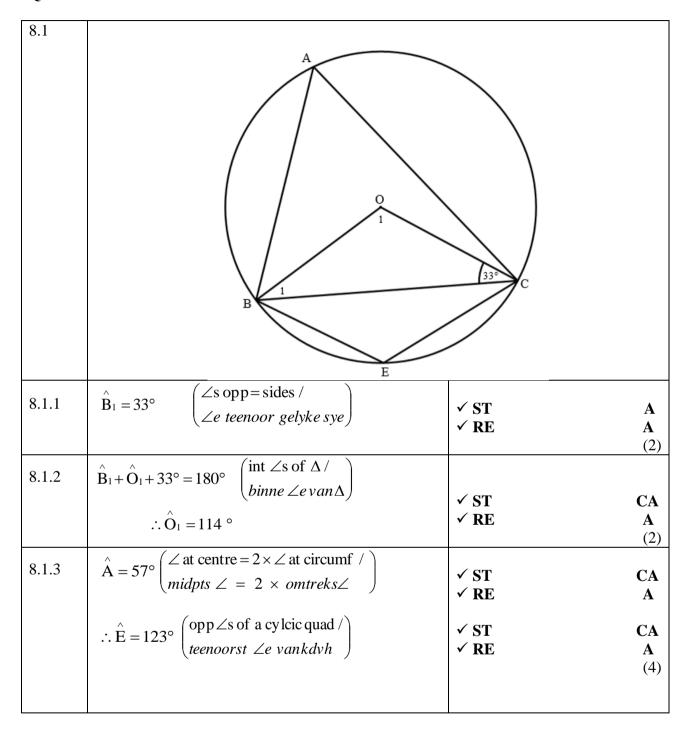


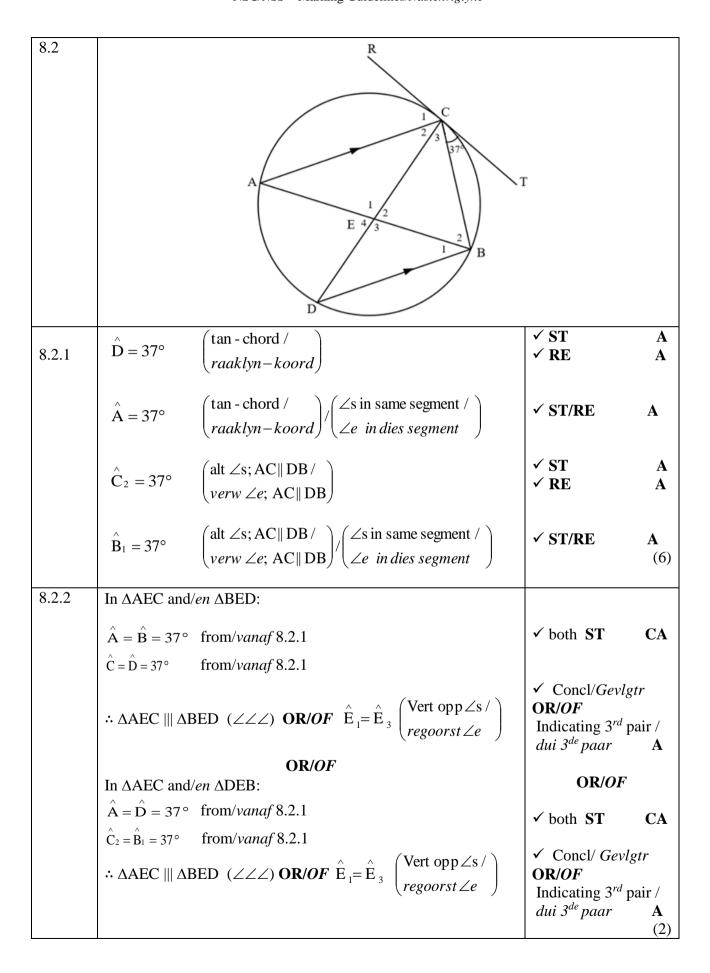
5.1.1	2	✓ value of/waarde van a A
		(1)
5.1.2	180°	✓ period /periode A
		(1)
5.1.3	$\tan x = 1$	√S A
	$x=45^{\circ}$	✓ value of /waarde van x A AO: full marks/ volpunte (2)
5 1 4	c DOD(OF	_
5.1.4	$y \in \mathbb{R} \ \mathbf{OR}/\mathbf{OF} y \in (-\infty, \infty)$	✓ range /waardevers A
5.1.5	(170 1070) 07-107-1170	(1)
5.1.5	$x \in (45^{\circ}; 135^{\circ}) \text{ OR/OF} 45^{\circ} < x < 135^{\circ}$	✓ critical values / kritiese
		waardes A
		✓ correct notation / korrekte
		notasie A
		(2)
5.2	$g(180^{\circ}) - f(180^{\circ}) \qquad \tan 180^{\circ} - \cos 2(180^{\circ})$ = 0 - 1	✓ substitution / vervanging A
	=-1 =-1	✓ S CA
		(2)
5.3	$x \in (0^{\circ}; 90^{\circ}) \text{ OR/OF } 0^{\circ} < x < 90^{\circ}$	✓ critical values / kritiese
		waardes A
		✓ correct notation / <i>korrekte</i>
		notasie A
		(2)
		[11]



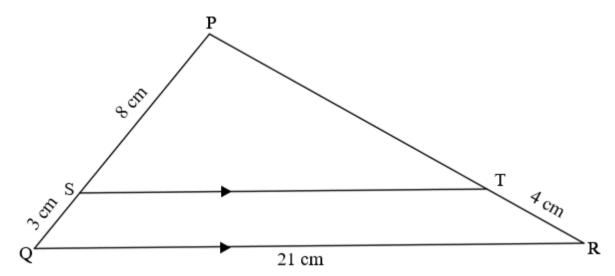
6.1	$\overrightarrow{QRP} = 17^{\circ}$	✓angle size /hoek grootte	A
	$\frac{PR}{\sin 114^{\circ}} = \frac{8}{\sin 17^{\circ}}$	✓ substitution /vervanging	A
	$PR = \frac{8\sin 114^{\circ}}{\sin 17^{\circ}}$	✓ S	CA
	≈ 25 m	✓ length / lengte	CA (4)
6.2	$\hat{RPM} = 41^{\circ}$	✓ size /grootte	A (1)
6.3	$\sin R \stackrel{\wedge}{P} M = \frac{MR}{PR}$	✓ sin ratio /verh	A (1)
6.4	$\sin 41^{\circ} = \frac{MR}{25}$	✓ substitution /vervanging	CA
	$MR = 25 \sin 41^{\circ}$	✓ length/lengte of/van MR	CA
	= 16,4 $MT = 16,4 - 5$ $= 11,4 m$	✓ length/lengte of/van MT	CA (3)
			[9]



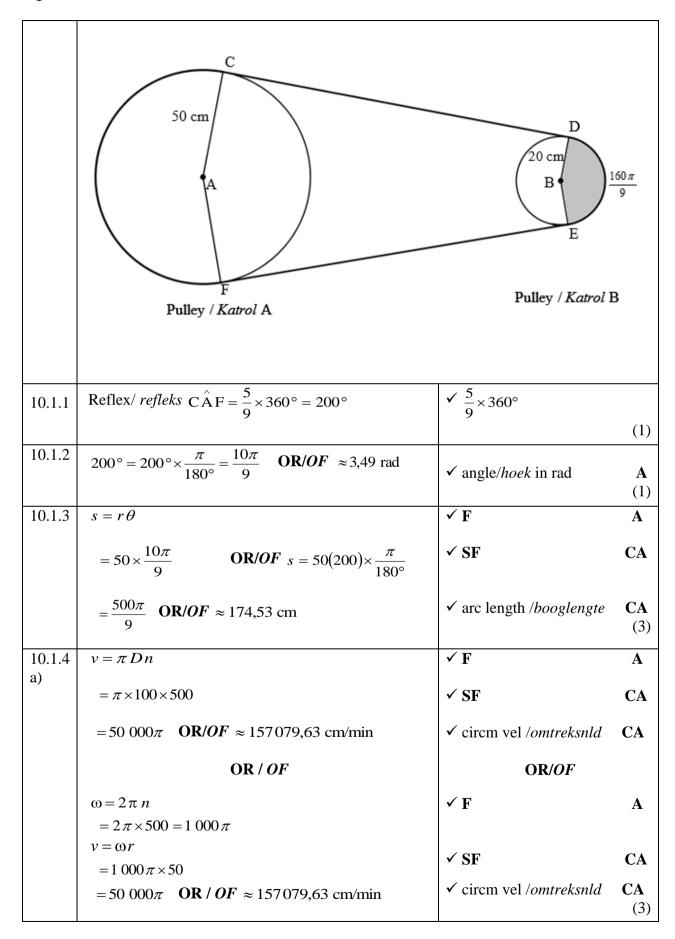




8.2.3.	$\therefore \frac{AE}{BE} = \frac{EC}{ED}$ $\therefore AE \times ED = EC \times BE$	✓ ST ✓ ST	A A (2)
8.3	T 68°	P 2 M	
	Q 323	N S	
8.3.1 a)	$\hat{Q}_1 = 32^{\circ}$ $\begin{pmatrix} SQ \text{ bisect } \angle/\\ SQ \text{ halveer } \angle \end{pmatrix}$	✓ ST	A (1)
8.3.1 b)	$\hat{P}_2 = 32^{\circ}$ $\begin{pmatrix} \angle s \text{ in same segment } / \\ \angle e \text{ in dies segment} \end{pmatrix}$	✓ ST ✓ RE	A A (2)
8.3.2	$ \hat{P} = 68^{\circ} \qquad \left(\angle s \text{ opp} = \text{sides } / \right) \\ \angle e \text{ teenoor} = sye $ $ \therefore \hat{P}_1 = 36^{\circ} $	✓ ST ✓ RE	A A
	$\hat{S}_2 = 68^{\circ} - 32^{\circ} = 36^{\circ} \begin{pmatrix} \text{ext } \angle \text{ of } \Delta \text{TQS} / \\ \text{buite } \angle \text{van} \Delta \text{TQS} \end{pmatrix}$	✓ ST	CA
	$\therefore \hat{\mathbf{P}}_1 = \hat{\mathbf{S}}_2$	✓ ST ✓ RE	A A (5)
			[26]

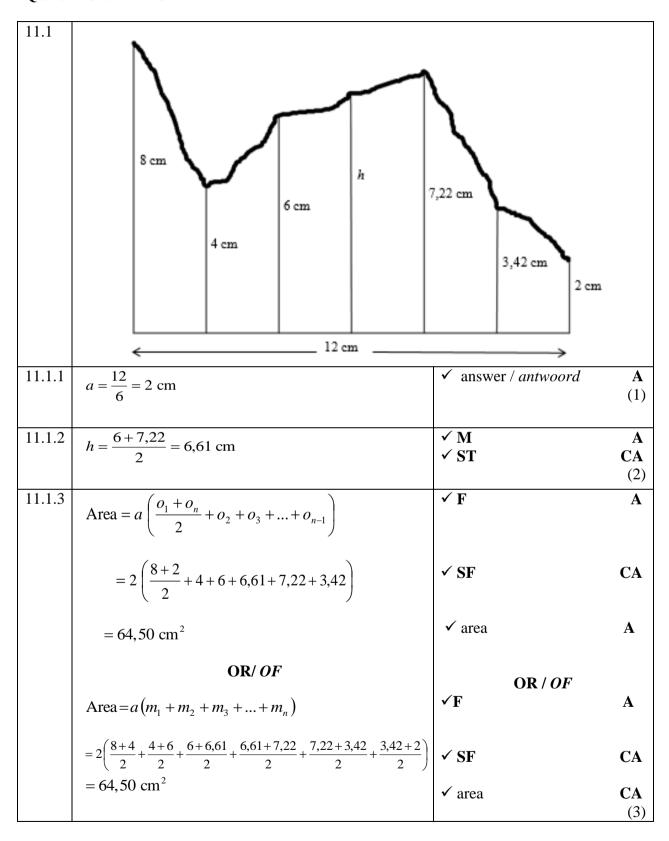


9.1	(propth,ST QR /)	✓ RE A
	ewer st,ST QR	(1)
	OR/OF	
9.2	$\frac{PT}{4} = \frac{8}{3}$	✓ Substitution /vervanging A
	∴ PT = $\frac{32}{3}$ cm OR/ <i>OF</i> ≈ 10,67 cm	\checkmark ST CA (2)
9.3	ST PS	✓ PQ A
	$\frac{ST}{QR} = \frac{PS}{PQ} \qquad (\Delta PST \parallel\mid \Delta PQR)$	\checkmark RE A (2)
9.4	$\therefore \frac{ST}{21} = \frac{8}{11}$	✓ Substitution /vervanging A
	$\therefore ST = \frac{168}{11} cm \text{ OR/OF} \approx 15,27 cm$	✓ ST CA
	11	(2)
		[7]



10.1.4	50,000 1	Languagian /hanlaidina
10.1.4 b)	$v = \frac{50\ 000\ \pi\ cm}{1\ min} \times \frac{1\ min}{60\ sec} = \frac{2\ 500\ \pi}{3}\ cm/s$	✓ conversion /herleiding A
	$1 \min \qquad 60 \sec \qquad 3$ $v_B = v_A$	✓ M (equating velocities) A
	$\therefore \pi \times 40 n = \frac{2500 \pi}{3}$	✓ SF CA
	$\therefore n = \frac{125}{6} \text{ rev/s } \mathbf{OR/OF} \approx 20,83 \text{ rev/s}$	✓ value of n /waarde van n CA
	OR/OF	
	ORIOI	OR/OF
	$v_{\scriptscriptstyle B} = v_{\scriptscriptstyle A}$	✓ M (equating velocities) A ✓ SF CA
	$\therefore \pi \times 40 n = 50000 \pi$	CA
	$\therefore n = 1 250 \text{ rpm}$	
	$\therefore n = \frac{1250 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}}$	/
		✓ conversion /herleiding A
	$\therefore n = \frac{125}{6} \text{ rev/s } \mathbf{OR/OF} \approx 20,83 \text{ rev/s}$	✓ value of n /waarde van n CA
		(4)
10.1.5	Area of sector/ rs	✓ F
	$\frac{\text{Area of sector/}}{\text{Area van sektor}} = \frac{\text{r s}}{2}$	
		✓ SF A
	$=\frac{20\times\frac{160\pi}{9}}{2}$	
	2	✓ area CA
	1600π 2 OD/OF 2.559.51 cm ²	v area
	$=\frac{1600\pi}{9} \text{ cm}^2 \text{ OR/OF} \approx 558,51 \text{ cm}^2$	
	ODIOE	\bigcirc OR/OF \bigcirc A
	OR/OF	✓ F A
	Area of sector/ $-\frac{r^2\theta}{}$	
	Area van sektor ²	✓ SF A
	$=\frac{20^2 \times \left(360^\circ \times \frac{4}{9}\right) \times \frac{\pi}{180^\circ}}{2}$	✓ area CA
	= 2	
	$=\frac{1600\pi}{9} \text{ cm}^2 \text{ OR/OF} \approx 558,51 \text{ cm}^2$	OR/OF
	9	
	OR/OF	✓ F A
		✓ SF A
	Area of sector/ $=\frac{\theta}{\pi r^2}$	✓ SF A
	$\frac{\text{Area of sector/}}{\text{Area van sektor}} = \frac{\theta}{360^{\circ}} \pi r^2$	
	$360^{\circ} \times \frac{4}{-}$	✓ area CA
	$= \frac{360^{\circ} \times \frac{4}{9}}{360^{\circ}} \pi \times 20^{2}$	(3)
	$=\frac{1600\pi}{9} \text{ cm}^2 \text{ OR/OF} \approx 558,51 \text{ cm}^2$	

10.2	4,6 m	1,8 m
10.2.1	h = 1.8 + 0.72 = 2.52m	✓ value of $h / waarde van h$ A
10.2.1	n = 1.0 + 0.72 = 2.32111	
10.0.0		(1)
10.2.2	$4h^2 - 4dh + x^2 = 0$	✓ F A
	$4h^2 - 4dh + x^2 = 0$	✓ SF CA
	$4(2,52)^{2} - 4d(2,52) + (4,6)^{2} = 0$ $-10,08d = -46,5616$	✓ S CA \checkmark value of d / waarde van d
	$d \approx 4.62 \text{ m}$	CA
		(4)
		[20]
		[26]



11.2	$Volume_{Ball A} = \frac{4}{3} \pi (11)^3$	✓ SF A
	$= \frac{5324}{3} \pi \text{ cm}^3$ $\therefore \text{ Volume}_{\text{Ball B}} = \frac{1}{2} \times \frac{5324}{3} \pi$	✓ vol of ball A / vol van bal A CA
	$=\frac{2662}{3}\pi \text{ cm}^3$	✓ vol of ball B / vol van bal B CA
	$\therefore \frac{4}{3}\pi x^{3} = \frac{2662}{3}\pi$ $x^{3} = \frac{1331}{2} \text{ OR/ } OF \approx 665,5$	✓ S CA
	$x = \sqrt[3]{\frac{1331}{2}}$ OR/ OF $x = \sqrt[3]{665,5}$ $\approx 8,73$ cm	✓ value of <i>x Iwaarde van x</i> CA (5)
11.3	7,81	cm
11.3.1	5 cm)
	$S.A = \pi r^2 + \pi r l$	
		✓ SF A
	$S.A = \pi r^2 + \pi r l$	✓ surface/buite area CA
11.3.2	S.A = $\pi r^2 + \pi r l$ = $\pi (5)^2 + \pi (5)(7,81)$	
11.3.2	S.A = $\pi r^2 + \pi r l$ = $\pi (5)^2 + \pi (5)(7,81)$ = $\frac{1781}{2} \pi \mathbf{OR} / \mathbf{OF} \approx 201,22 \text{ cm}$	✓ surface/buite area CA (2) ✓ new/nuwe radius A
11.3.2	S.A = $\pi r^2 + \pi r l$ = $\pi (5)^2 + \pi (5)(7,81)$ = $\frac{1781}{2} \pi \mathbf{OR} / \mathbf{OF} \approx 201,22 \text{ cm}$ $r_{new/nuwe} = 5 \times 1, 2 = 6 \text{ cm}$ $h_{new/nuwe} = 6 \times 0, 9 = 5,4 \text{ cm}$ $\therefore l_{new/nuwe} = \sqrt{5,4^2 + 6^2} \approx 8,07 \text{ cm}$	✓ surface/buite area CA (2) ✓ new/nuwe radius A
11.3.2	S.A = $\pi r^2 + \pi r l$ = $\pi (5)^2 + \pi (5)(7,81)$ = $\frac{1781}{2} \pi \mathbf{OR} / \mathbf{OF} \approx 201,22 \text{ cm}$ $r_{new/nuwe} = 5 \times 1, 2 = 6 \text{ cm}$ $h_{new/nuwe} = 6 \times 0, 9 = 5,4 \text{ cm}$	✓ surface/buite area CA (2) ✓ new/nuwe radius ✓ new height /nuwe hoogte ✓ new slant height /
11.3.2	S.A = $\pi r^2 + \pi r l$ = $\pi (5)^2 + \pi (5)(7,81)$ = $\frac{1781}{2} \pi \mathbf{OR} / \mathbf{OF} \approx 201,22 \text{ cm}$ $r_{new/nuwe} = 5 \times 1,2 = 6 \text{ cm}$ $h_{new/nuwe} = 6 \times 0,9 = 5,4 \text{ cm}$ $\therefore l_{new/nuwe} = \sqrt{5,4^2 + 6^2} \approx 8,07 \text{ cm}$ $\therefore SA_{new/nuwe} = \pi (6)^2 + \pi (6)(8,07)$	✓ surface/buite area CA (2) ✓ new/nuwe radius ✓ new height /nuwe hoogte ✓ new slant height / nuwe skuinshoogte CA