CS202 - Algorithm Analysis

Graph Algorithms Module-1

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Discussion Based On ...

Sedgewick 4.1, 4.2

What is a Graph?

A **Graph** is a data structure that consists of a finite set of vertices(or nodes) and set of edges which connect a pair of nodes. More formally,

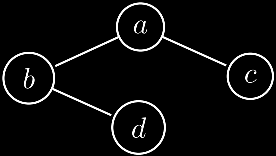
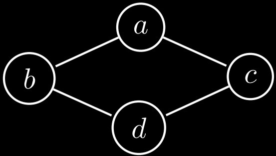
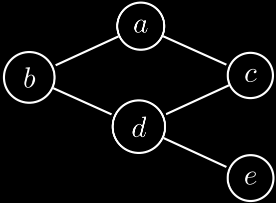
$$G = (V, E)$$

Application: Maps, Electrical Circuit, Facebook Friend List, Human Genetic data, Amazon Product data, and so on $\cdots$

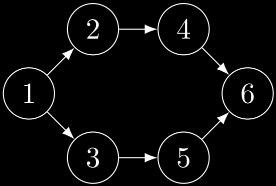
How is a Graph different from a Tree?

Tree	Graph
1. A **Tree** is a specialized case of a **Graph**. It is a connected graph with no circuits and self loops.	1. A **Graph** consists of vertices, edges, and a set representing relationship between vertices and edges.
2. There should be only one path between any two vertices.	2. There can be any number of paths between any two vertices.
3. A tree does not contain any loops and is minimally connected.	3. A Graph contains loops.

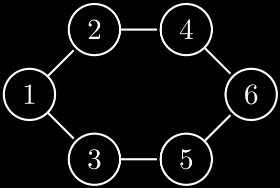
Tree Vs Graph

Tree	**Graph**
	 

Type of Graphs (1)

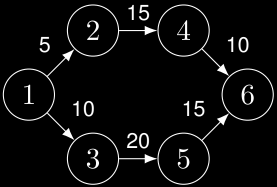


Directed (diGraph)

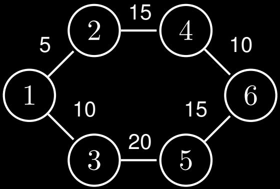


Undirected

Type of Graphs (2)

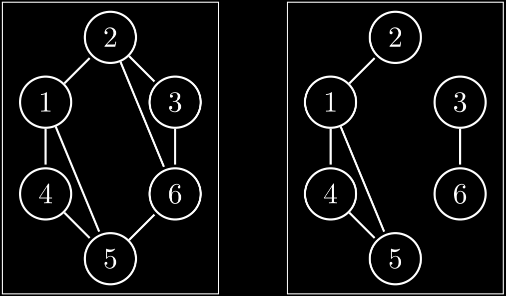


Weighted directed



Weighted undirected

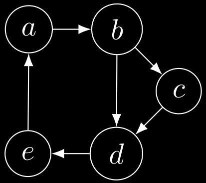
Type of Graphs (3)



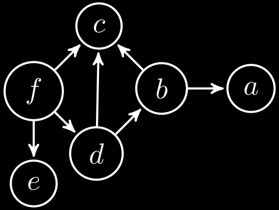
Connected **Disconnected**

A connected **Graph** consists of a path between every two vertices. It is disconnected otherwise.

Type of Graphs (4)



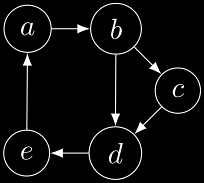
Strongly connected



Weakly connected

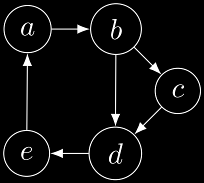
A **diGraph** is a strongly connected if every two vertices are reachable from each other. It is a weakly connected graph if the underlying undirected graph is connected.

Graph - Basic Terminology



- A **Source** vertex in a directed edge is its first endpoint. $Edge(a, b)$ has a source vertex a.
- A **Destination** vertex in a directed edge is its second endpoint (arrow pointed). $Edge(a, b)$ has a destination vertex b.
- A directed edge is said to be **Outgoing** on its source vertex. $Edge(a, b)$ is outgoing on a.
- A directed edge is said to be **Incoming** on its destination vertex. $Edge(a, b)$ is incoming on b.

Graph - Basic Terminology (2)



- **Degree** of a vertex is the total number of edges connected to a vertex. For example, $Deg(b) = 3$ and $Deg(c) = 2$

  $\forall$ vertex, $v \in$ graph G, $Deg(v) = In(v) + Out(v)$

- **Indegree** of a vertex is the total number of incoming edges connected to a vertex. $In(b) = 1$ and $In(d) = 2$
- **Outdegree** of a vertex is the total number of outgoing edges connected to a vertex. $Out(b) = 2$ and $Out(d) = 1$

Graph Properties

- Let us suppose a Graph G has V vertices and E edges, then $E < V^2$.
- If E is close to $V \times log(V)$ then the graph is called a **Dense** graph. G is too strongly connected and is in a complete form.
- If $E < V \times log(V)$ then the graph is called a **Sparse** graph.

Graph Representation

So how do we represent a Graph in a Program?

- **Adjacency Matrix**
- **Adjacency List**

- Adjacency Matrix is used primarily to represent a Dense graph.
- Adjacency List is used primarily to represent a Sparse graph.

Graph Representation

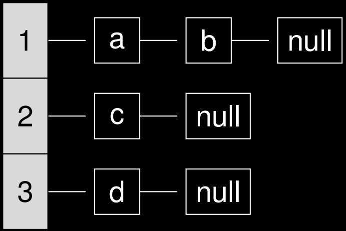
Adjacency Matrix

$$
\begin{array}{cc}
\begin{matrix} a & b & c & d & e \end{matrix} & \\
\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix}
\end{array}
$$

Space Complexity = $O(V^2)$

$$
\begin{pmatrix} a[i][j] = 1 & \text{if i and j are adjacent} \\ = 0 & \text{otherwise} \end{pmatrix}
$$

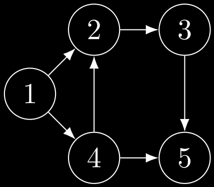
Adjacency List



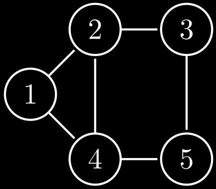
Space Complexity = $O(V + 2E)$

An array of lists of vertices. The list item **(i)** contains vertex **(j)** if there exists an $Edge(i, j)$ in Graph G.

Graph Representation - Example 1

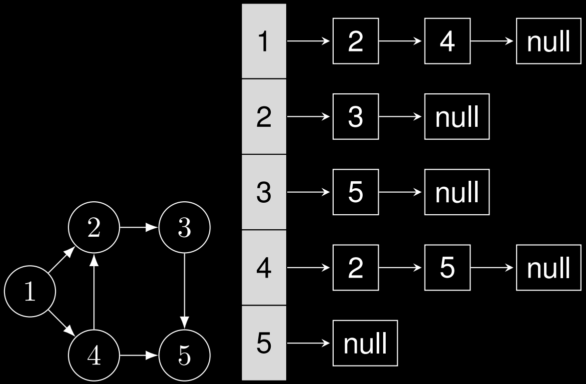


$$\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\end{array}$$

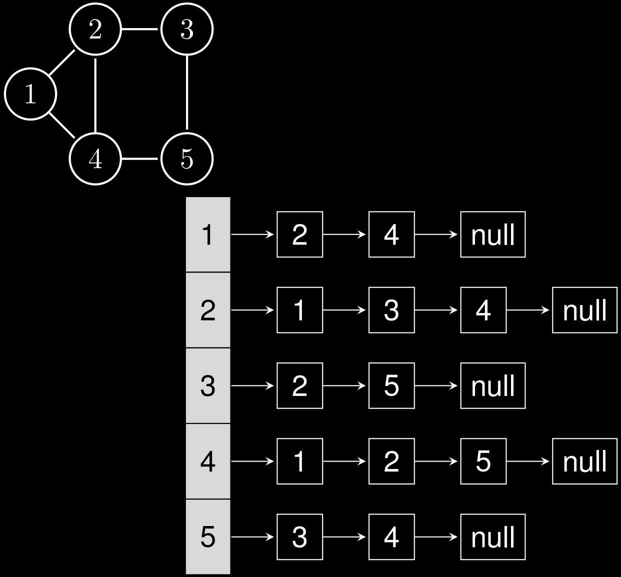


$$\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\end{array}$$

Graph Representation - Example 2

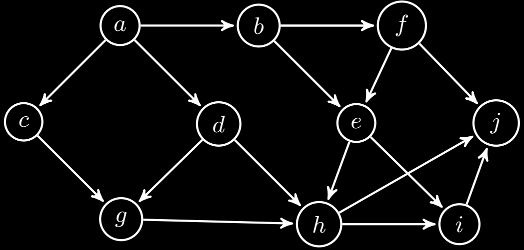


Graph Representation - Example 3



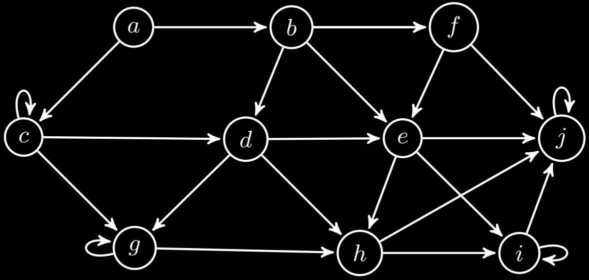
Try out 1

- **Identify** the in-degree and out-degree of all vertices (a to j) in the Graph provided below:



Try out 2

- **Draw** the adjacency matrix and adjacency list representation of the Graph provided below:



Next:

- **Graph Traversal Algorithms:**
  BFS and DFS
- **Graph Shortest Path Algorithms:**
  Dijkstras algorithm.

Reading Assignment

Sedgewick 4.1, 4.2

Questions?

Please ask if there are any Questions!