CS202 - Algorithm Analysis

Tree Algorithms - Module 1

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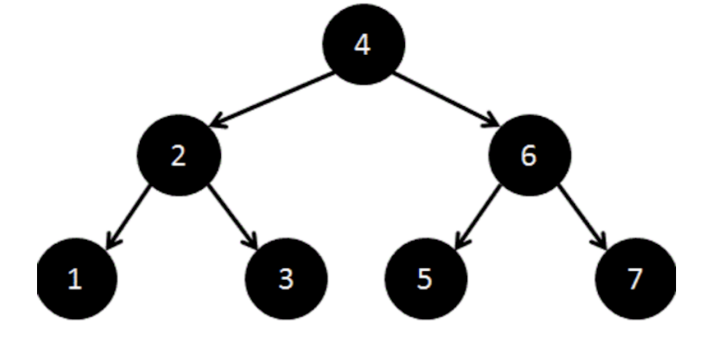
Discussion Based On ...

Sedgewick 2.4 Heap Sort

Data Structures - An overview

- So far we have seen linear structures:
  - linear: before and after relationship
  - Arrays, Stacks, and Queues
- Non-linear structure: **Trees**
  - probably the most fundamental structure in computing
  - hierarchical structure
  - Terminology: from family trees (genealogy)

Trees



Trees More Formally

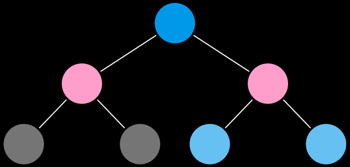
- **Definition:** A tree T is a set of nodes storing elements such that the nodes have a parent-child relationship that satisfies the following properties:
    - If T is nonempty, it has a special node, called the root of T, that has no parent.
    - Each node v of T different than the root has a unique **parent node** w; every node with parent w is a **child** of w

Trees More Formally

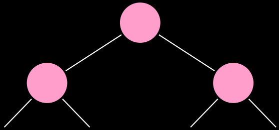
- **Recursive Definition:**
  - T is either empty
  - or consists of a node r, called the root of T, and a (possibly empty) set of trees whose roots are the children of r.

Trees - Some basic terminologies

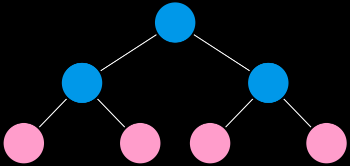
- **Siblings:** Two nodes that have the same parent are called siblings.



- **Internal nodes:** Nodes that have one or more children(s).



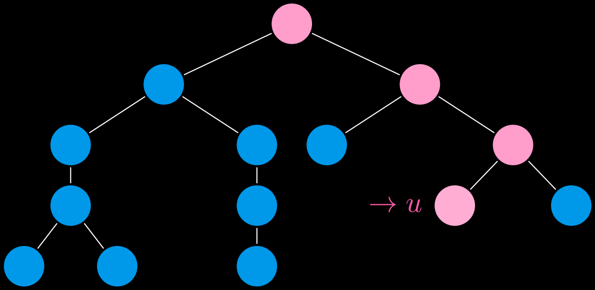
- **External nodes:** Nodes that don't have any children.



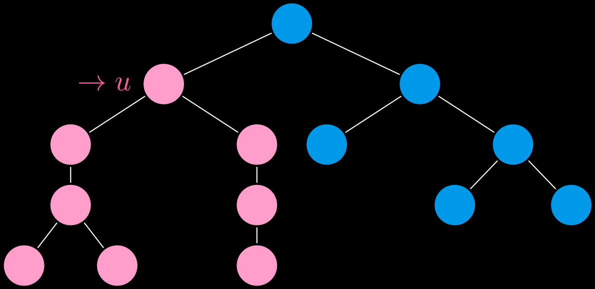
- **Ancestors:**

  Ancestors of a node u are u itself and the ancestors of its parent.

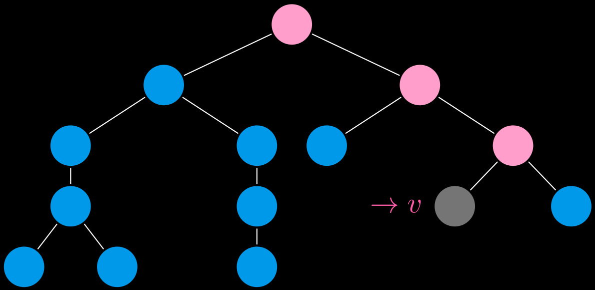
  (INCLUSIVE)



- **Descendants:**
  v is a descendants of u if u is an ancestor of v.
  (INCLUSIVE)

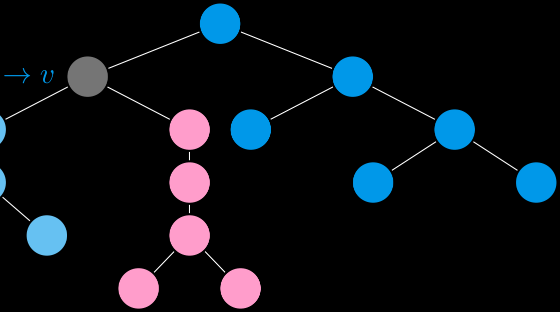


- **Depth(T, v):**
  Number of ancestors of v, excluding v itself.



Depth(T, v) = 3

- **Height(T, v):**
  Number of nodes in the longest path from v to any leaf, excluding v itself.



Height(T, v) = 4

Trees - Some basic terminologies

- What is the height of the leaf node(s)?
- The height of a tree is the height of its root.
- Height and Depth are symmetrical.
- **Proposition:** The **height** of a tree T is the **maximum depth** of one of its leaves.

Trees - Applications

- Scheduling and Priority Queue (Heap)
- Class Hierarchy in Java
- File System
- Storing hierarchies in organizations

Binary Trees More Formally

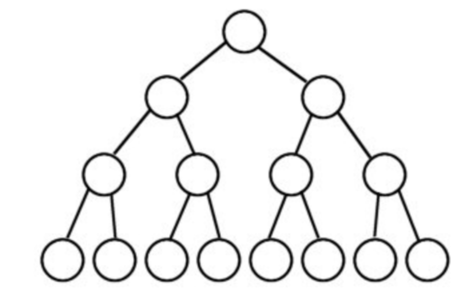
- **Definition:** A binary tree is a tree such that:
  - every node has at most 2 children
  - each node is labeled as being either a left chilld or a right child

Binary Trees More Formally

- **Recursive Definition:**
  - a binary tree is empty;
  - or it consists of
    - a node (the root) that stores an element
    - another binary tree, called the left subtree of T
    - another binary tree, called the right subtree of T

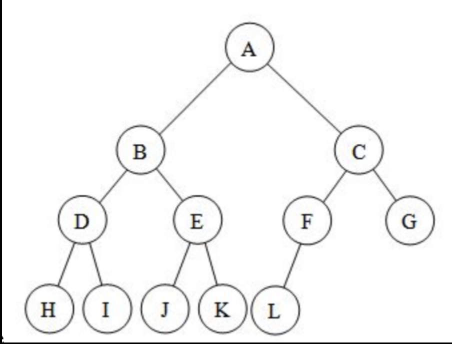
Binary Tree Examples

- A full binary tree (sometimes complete or proper binary tree or 2-tree) is a tree in which every node other than the leaves has two children.



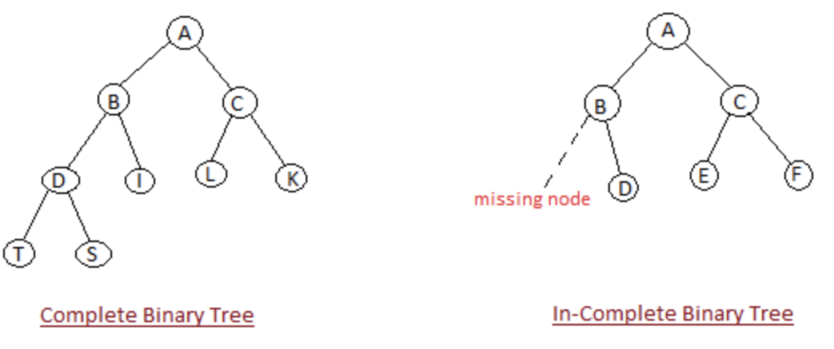
Binary Tree Examples

- An atmost complete binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.



Binary Tree Examples

- An In-Complete binary tree is a binary tree in which the properties of the complete binary tree is not true.



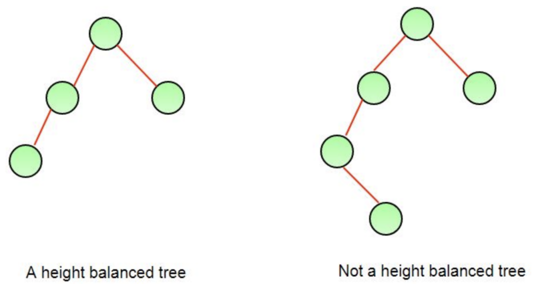
Binary Tree Examples

- **Balanced** : Difference between the height of the left and right subtree is atmost 1.
- **Unbalanced** : Difference between the height of the left and right subtree is greater than 1.

Depend on the balancing scheme. Later.

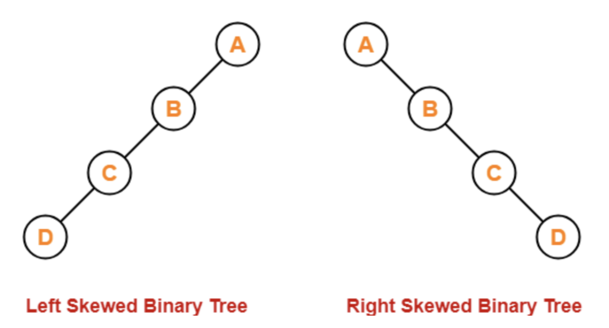
Binary Tree Examples

- **Balanced Vs Unbalanced**



Binary Tree Examples

- A skewed binary tree is a binary tree that satisfies the following 2 properties:
  1. All the nodes except one node has one and only one child.
  2. The remaining node has no child.

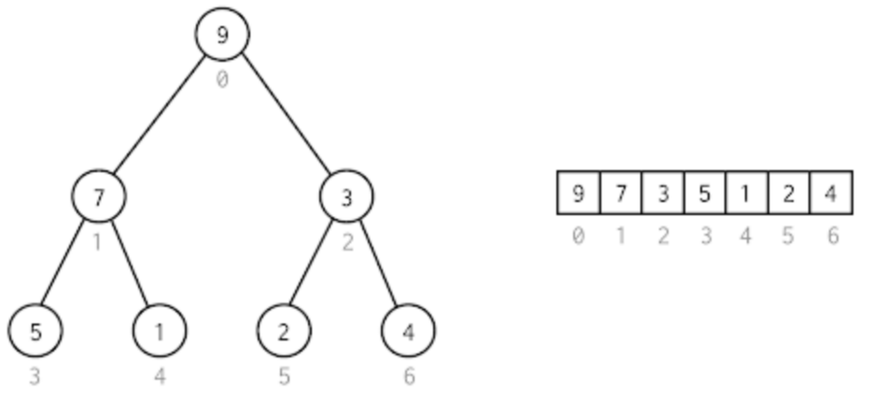


Properties of Binary Trees

- In a binary tree
  - level 0 has $<= 1$ node
  - level 1 has $<= 2$ nodes
  - level 2 has $<= 4$ nodes
  - $\ldots$
  - level i has $<= 2^{\text{i}}$ nodes

How to store a binary tree in a program?

- An array can be used to represent the binary tree structure.



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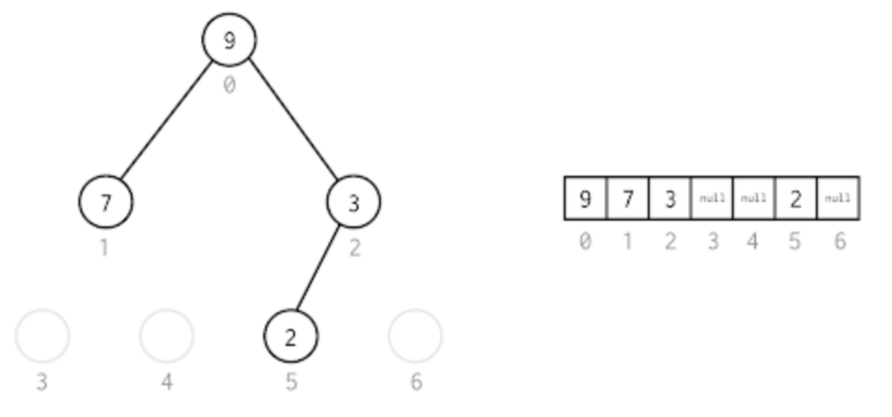
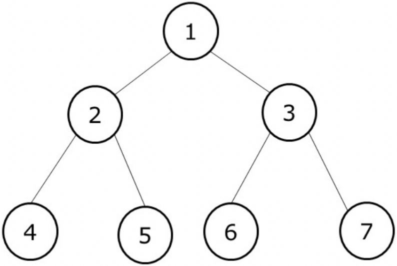


How to traverse a Tree?

- What is Level Order Traversal here?



Preorder	[1,2,4,5,3,6,7]
Inorder	[4,2,5,1,6,3,7]
Postorder	[4,5,2,6,7,3,1]

Next Steps

- Heap Sort
- Binary Search Tree

Reading Assignment

Sedgewick 2.4 Heap Sort

Questions?

Please ask if there are any Questions!