CS202 - Algorithm Analysis

Merge Sort

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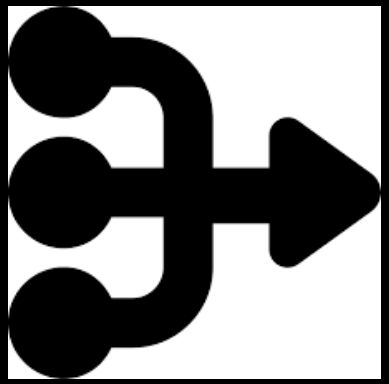
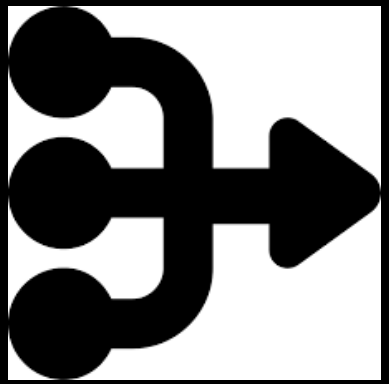
Merge Sort Algorithm



Strategy:

- **Divide:** if S has at least two elements, remove all the elements from S and put them into two sequences S_1 and S_2 , each containing about half of the elements of S.(i.e,. S_1 contains the first floor $(n/2)$ elements and S_2 contains the remaining floor $(n/2)$ elements.
- **Conquer:** Sort sequences S_1 and S_2 using Merge Sort.
- **Combine:** Put back the elements into S by merging the sorted sequences S_1 and S_2 into one sorted sequence.

Merge Sort Algorithm



Characteristics:

- sort out of ”place”, i.e., does require an additional array
- uses divide and conquer principle
- worst case running time is $O(n \times log(n))$

Merge Sort Algorithm

Merge Procedure (linear)

Algorithm - Merge(A, p, m, r)

Input: an n-element un-sorted array A of integer values, a lower bound p of the array A, and a pivot r in the array A.

Output: an n-element sorted array A of integer values.

$n_1 \leftarrow m - p$
$n_2 \leftarrow r - m$
Initialize Array L of size $n_1 + 1$
Initialize Array R of size $n_2 + 1$
for i = 0 to n_1 **do**
    L[i] $\leftarrow$ A[p+i]
end for
for j = 0 to n_2 **do**
    R[j] $\leftarrow$ A[m+j]
end for
$L[n_1 + 1] \leftarrow \infty$
$R[n_2 + 1] \leftarrow \infty$

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Merge Sort Algorithm

Merge Procedure (linear)

Initialize $i, j \leftarrow 0$
for k = p to r **do**
  if L[i] $\leq$ R[j] **then**
    A[k] $\leftarrow$ L[i]
    i $\leftarrow$ i+1
  else
    A[k] $\leftarrow$ R[j]
    j $\leftarrow$ j+1
  end if
end for

Merge Sort Algorithm

MergeSort Procedure (logarithmic)

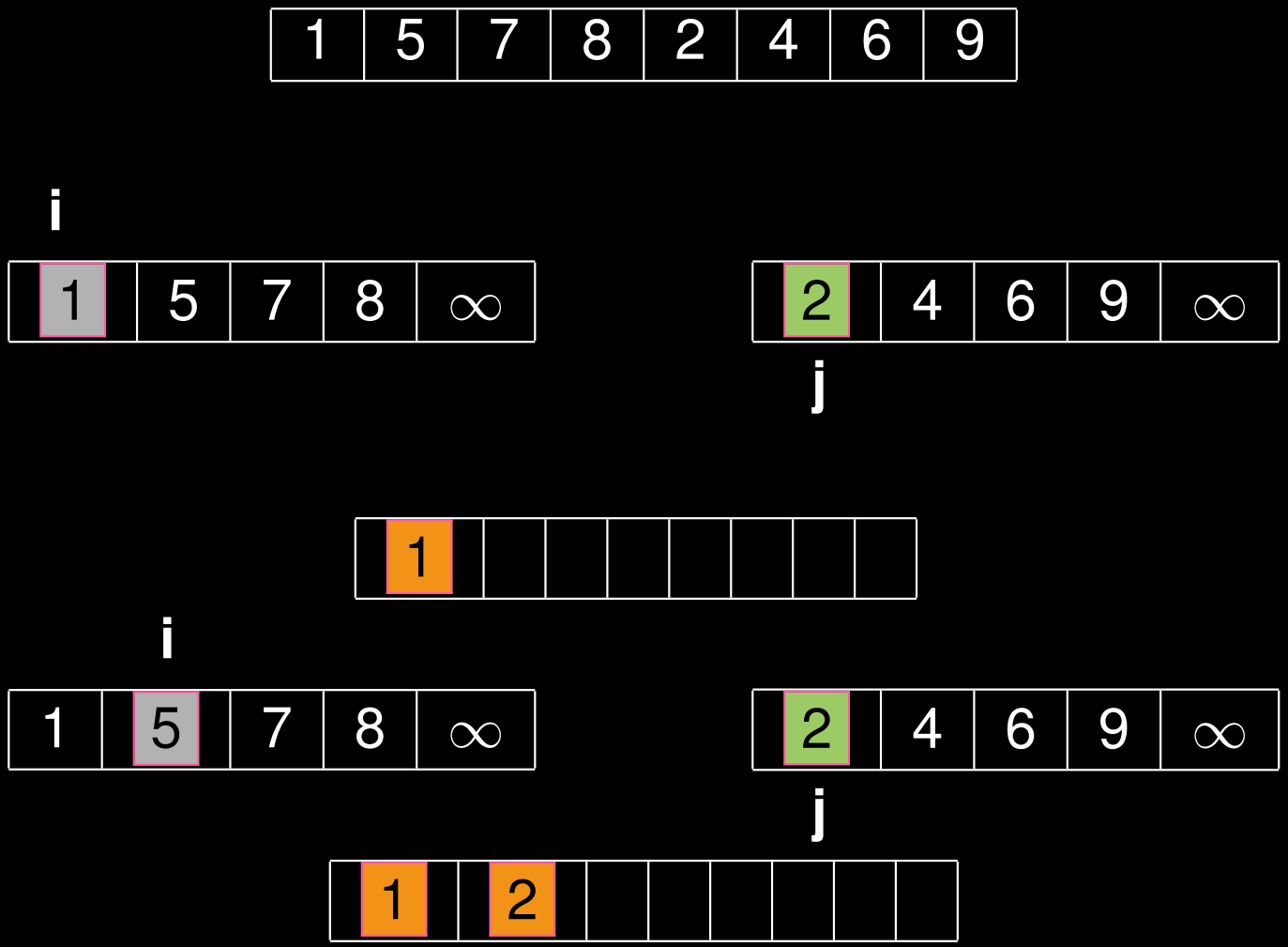
Algorithm - MergeSort(A, p, r)

Input: an n-element un-sorted array A of integer values, a lower bound p of the array A, and a pivot r in the array A.

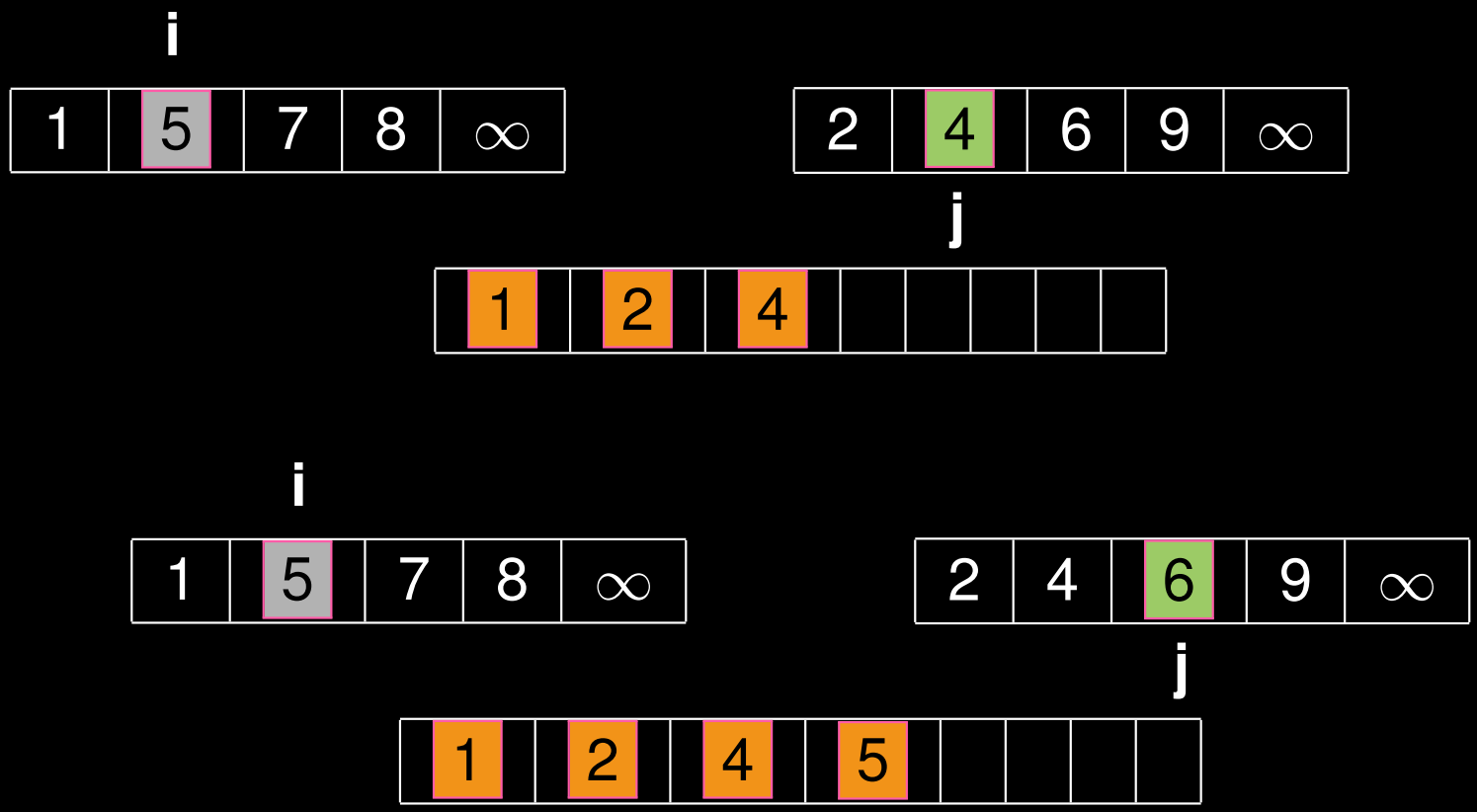
Output: an n-element sorted array A of integer values.

if p $<$ r **then**
  $m \leftarrow$ Floor $(p + r)/2$
  MergeSort(A, p, m)
  MergeSort(A, m+1, r)
  Merge(A, p, m, r)
end if

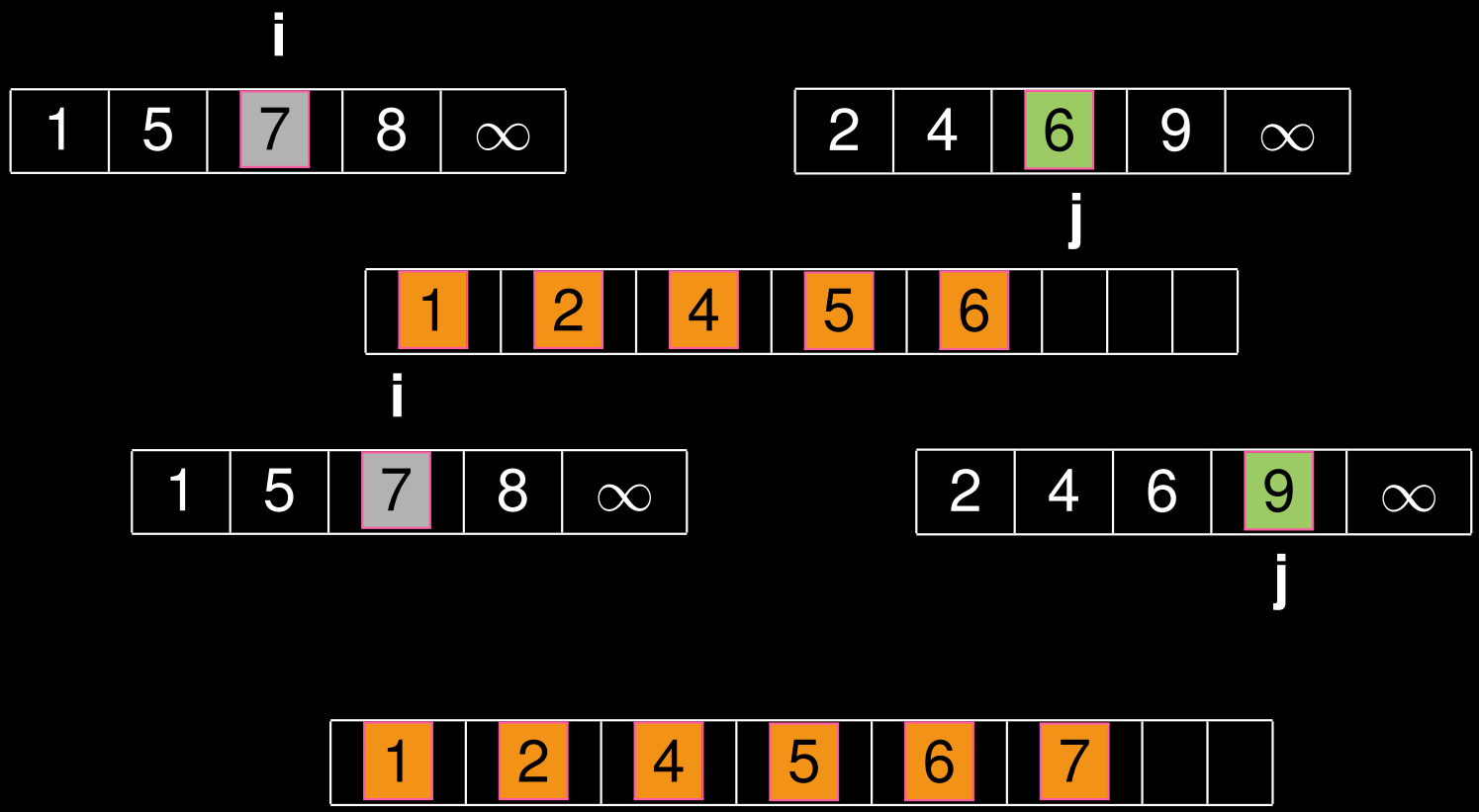
Merge Example



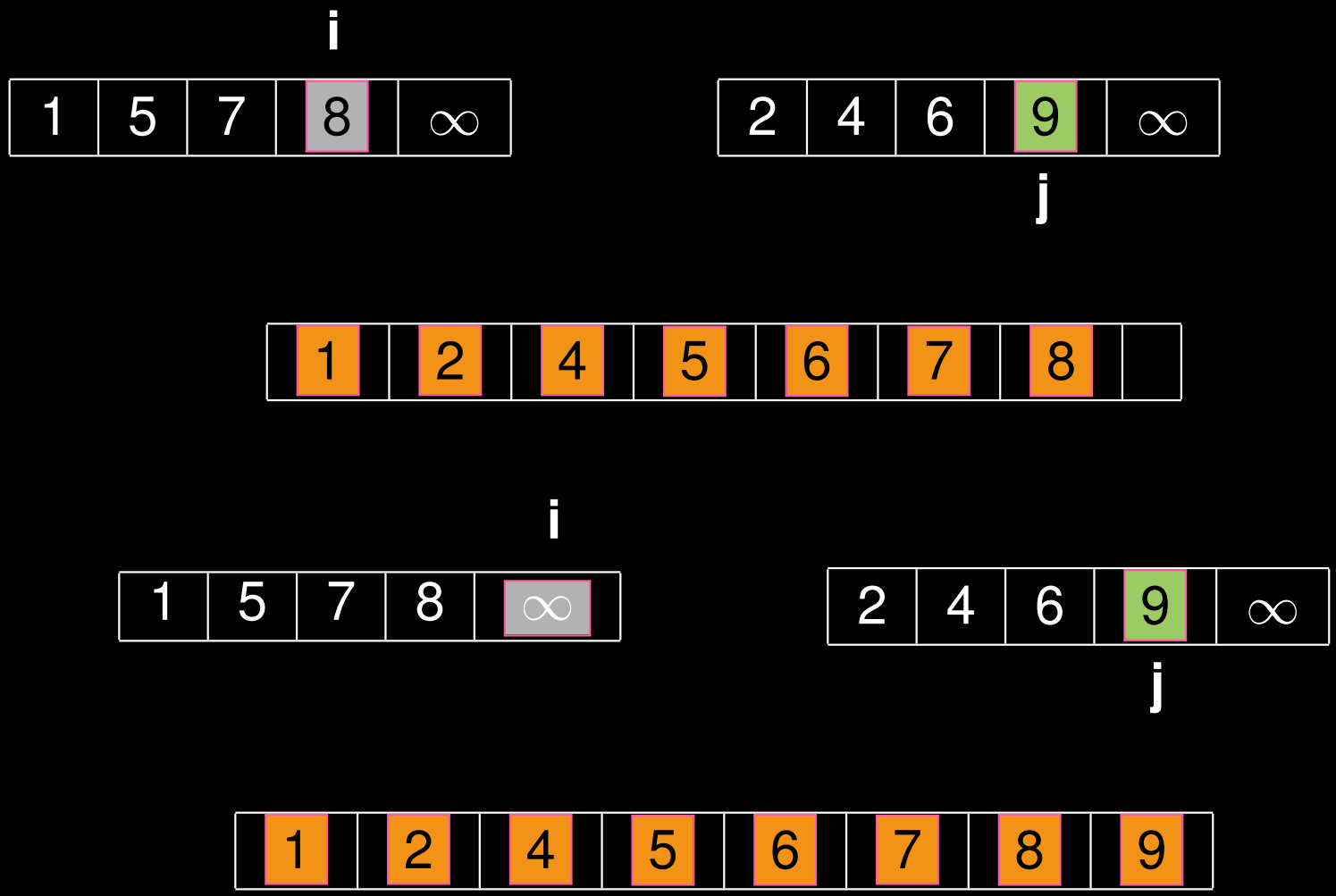
Merge Example



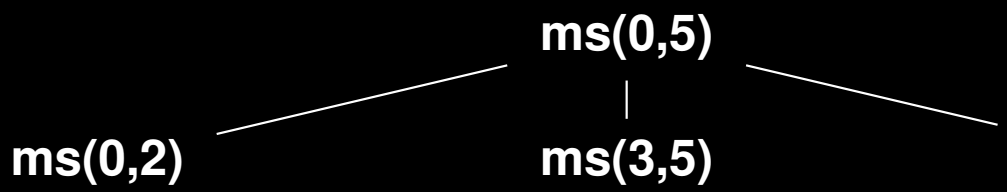
Merge Example



Merge Example



Merge Sort Example

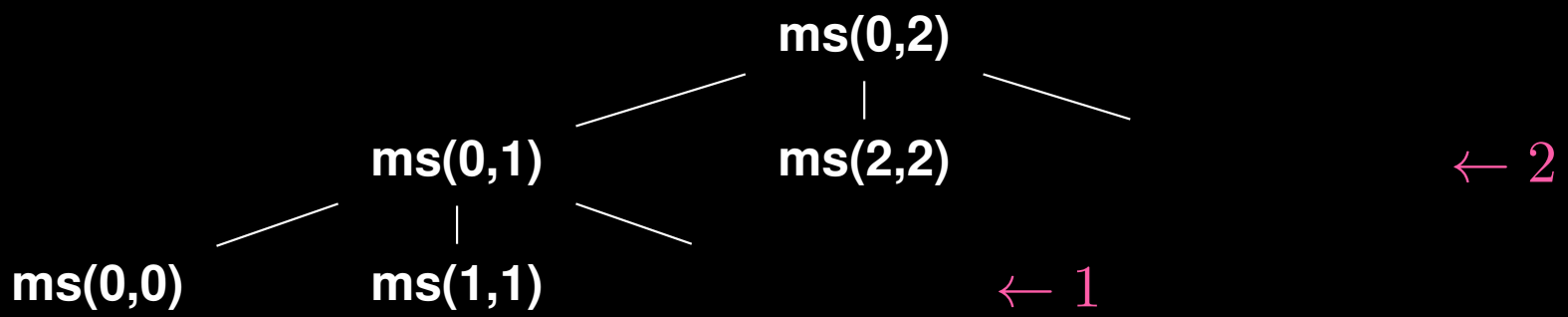


$\leftarrow 5$

Let us complete the tree?

Merge Sort Example

9	6	5	0	8	2



Let us complete the tree?

Merge Sort Example

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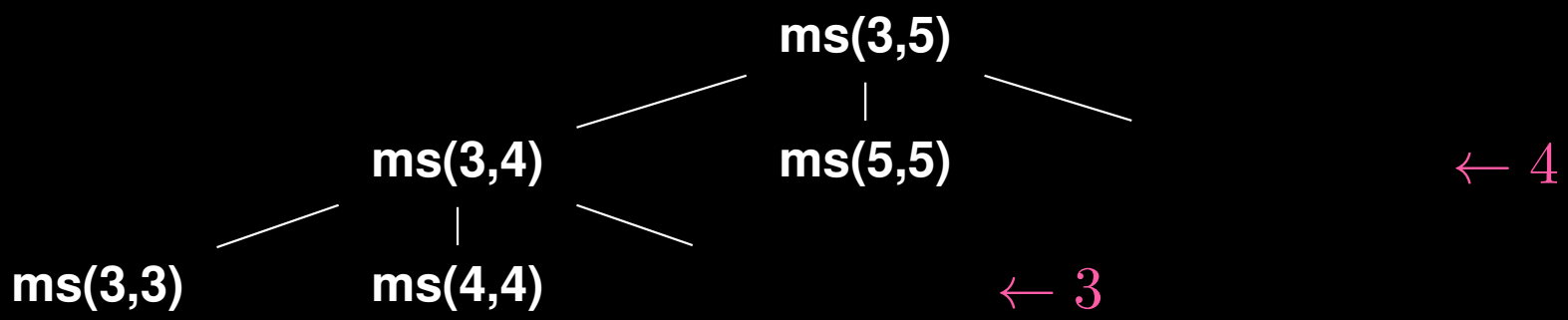
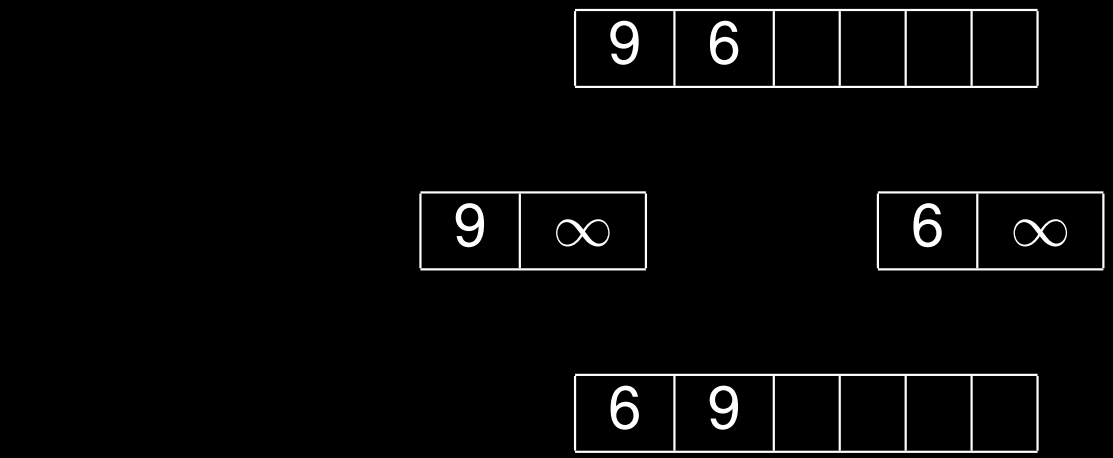


OK done

Merge Sort Example

Merge(0,0,1)

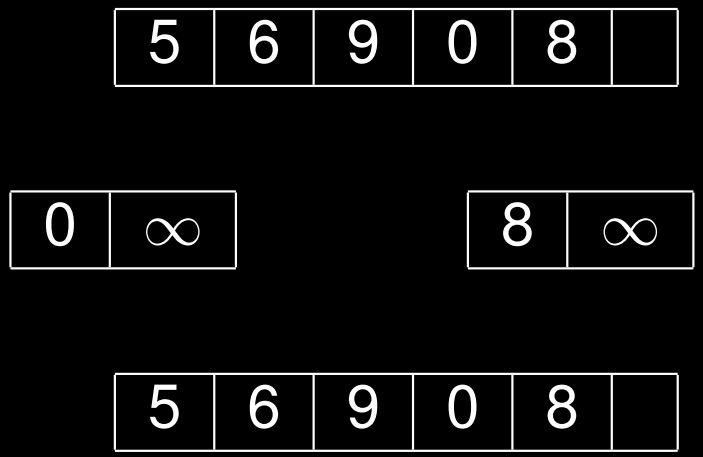


Merge Sort Example

Merge(0,1,2)

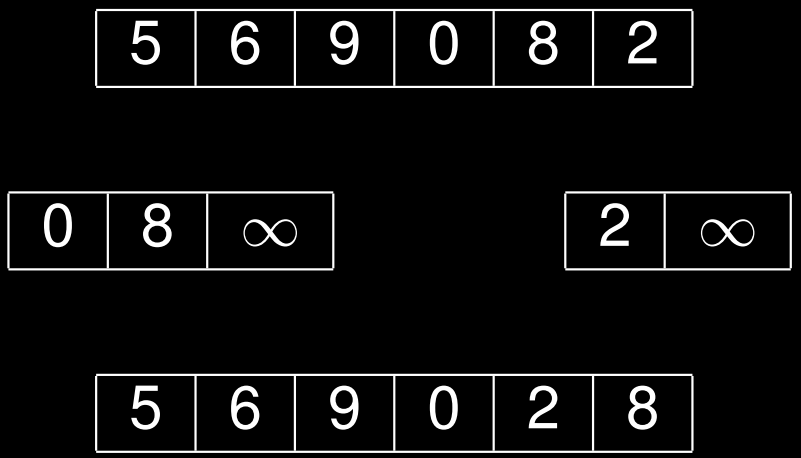
Merge Sort Example

Merge(3,3,4)



Merge Sort Example

Merge(3,4,5)



Merge Sort Example

Merge(0,2,5)

Done Sorting!

Merge Sort Algorithm - An analysis

- Space complexity: O(n)
- Time complexity: O$(n \times logn(n))$ for Worst, Average, and Best case

Reading Assignment

Sedgewick 2.2 Merge Sort

Questions?

Please ask if there are any Questions!