A recursive function is one that calls itself.
Different invocations of the function have their
own local variable scope, so this is safe. As
an example, consider summing all the numbers
below (and including) some input number:

```
def add_it_up(num):
    if (num > 0):
        return num + add_it_up(num - 1)
    else:
        return 0
```
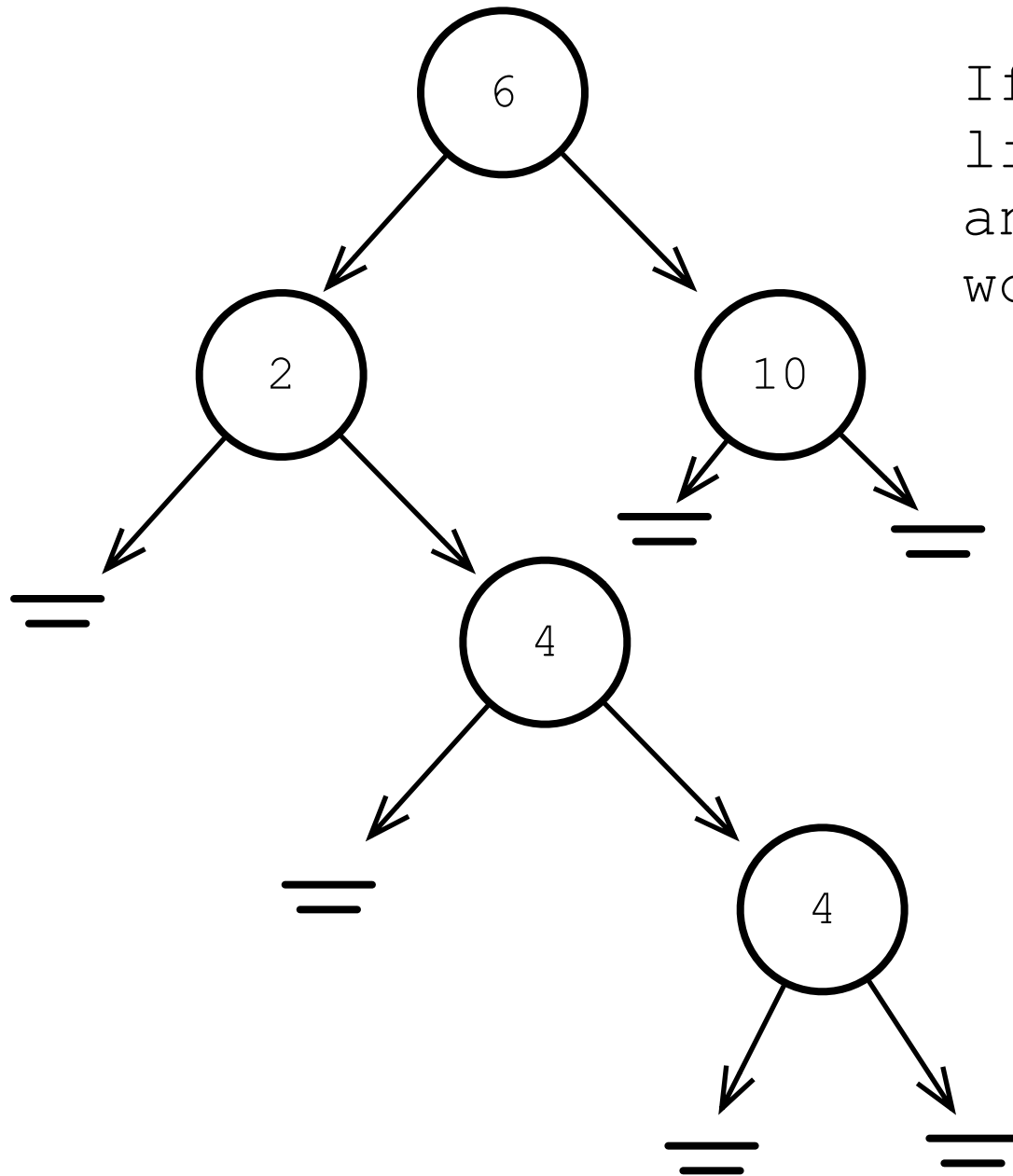
calling **add_it_up(6)** gives 21:
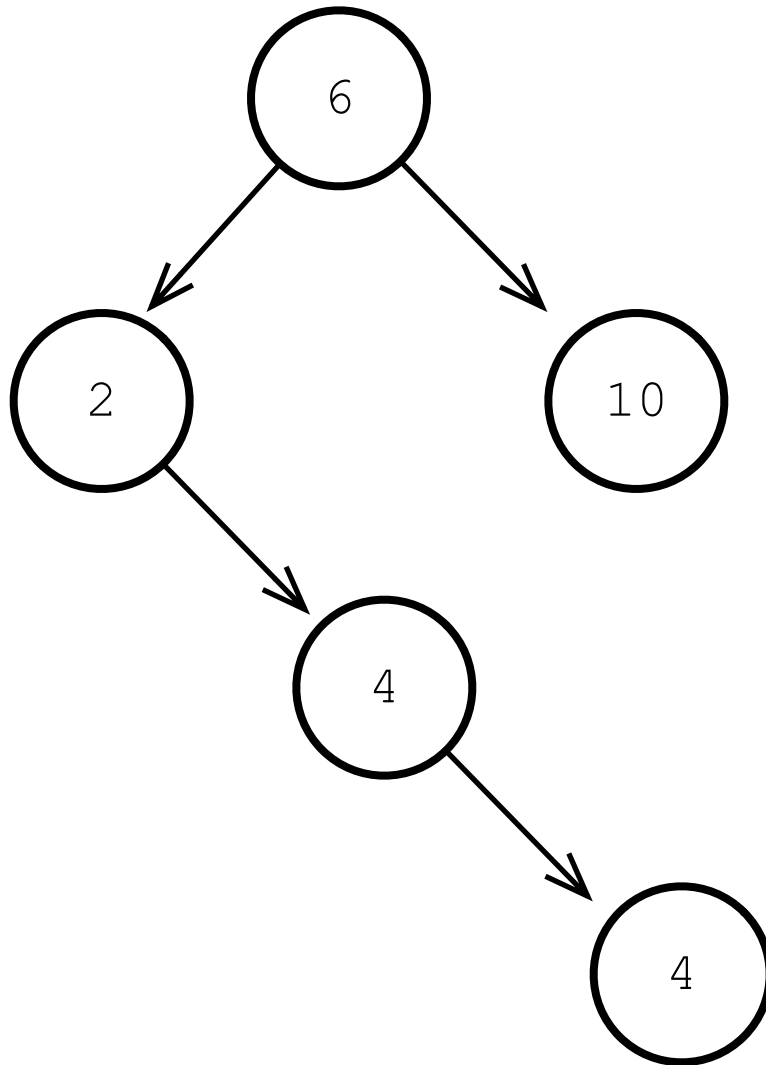
6 + 5 + 4 + 3 + 2 + 1 + 0

Binary trees are composed of nodes that have up to two children. Binary *search* trees are used to find values quickly, and are kept in a certain order (usually nondecreasing order).

binary tree node

left child   right child

←··→

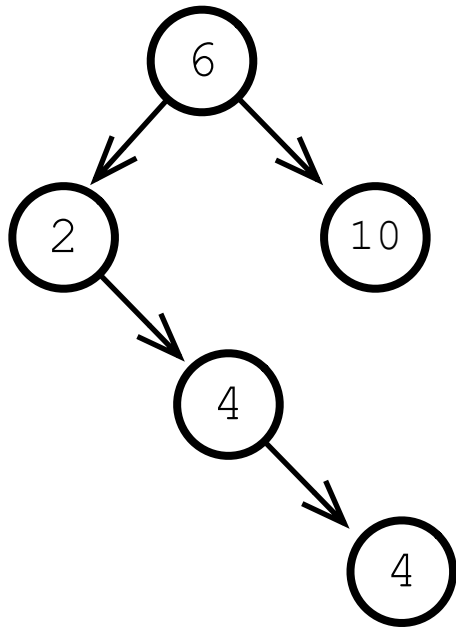smaller values   larger values
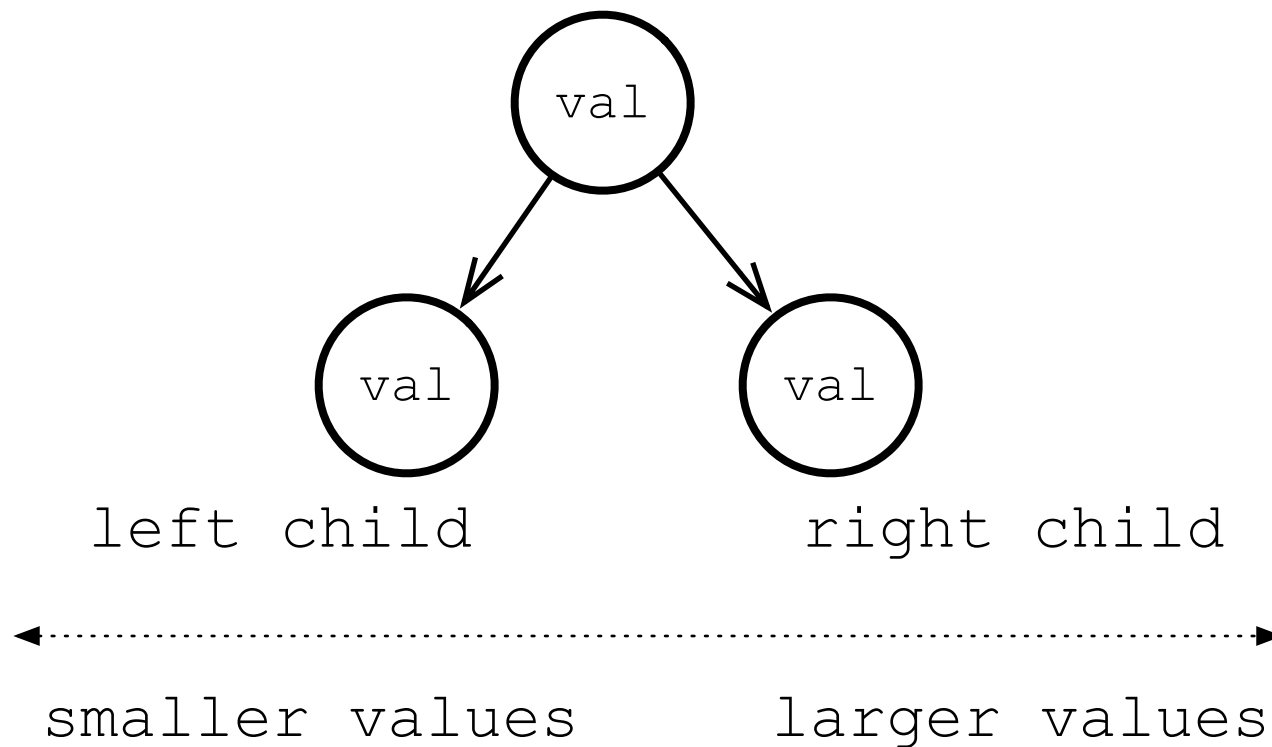
If we drew the null links between all parent and child nodes, it would be a little messy.

Binary trees are almost always drawn without the null links. It is assumed, for example, that the '2' node has a null left child, and that the lower '4' node has two null children.

Nodes have the following order:

left children are all **less** than the parent node.

right children are all **greater than or equal to** the parent node.

Because nodes have this structure and ordering rule, we can confidently apply **traversal** algorithms to them. We can traverse in three ways, but for our purposes, only one of them is useful. They are:

```
pre-order walk:  visit, go left, go right
in-order walk:   go left, visit, go right
post-order walk: go left, go right, visit
```
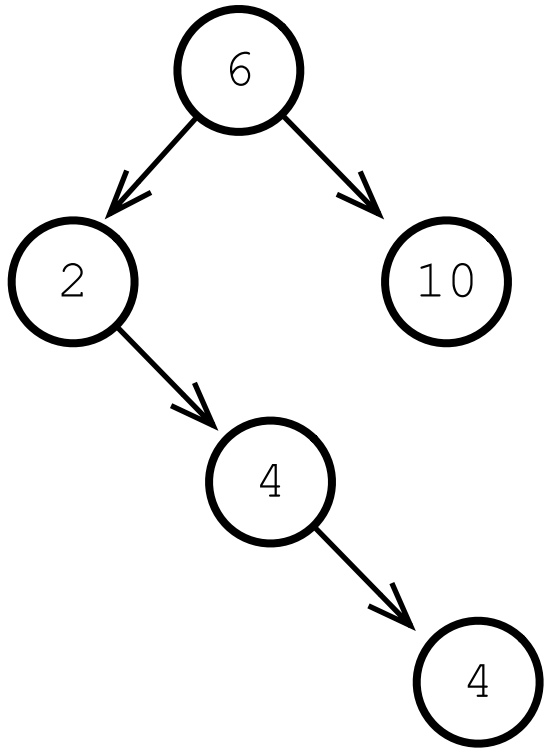
In-Order is the useful one for the BST assignment, because it gives us the sort order we want. Here's the code outline:

```
def in_order(node):
    if (node is not None):
        in_order(node.left)
        visit(node)
        in_order(node.right)
```
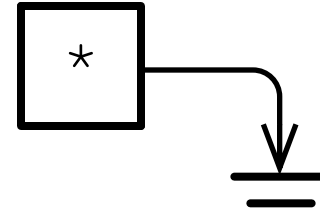
Notes:
Sanity checking (is it null?) happens inside the function.

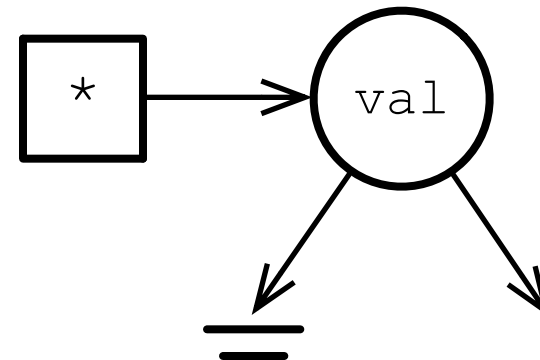It is recursive because it calls itself.

The 'visit' line can be replaced with whatever you need, like printing out the node, or adding it to a list of values that are in the proper sort order.
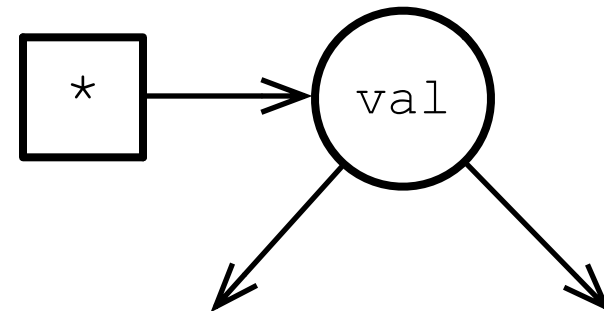
Insert has three possible conditions:
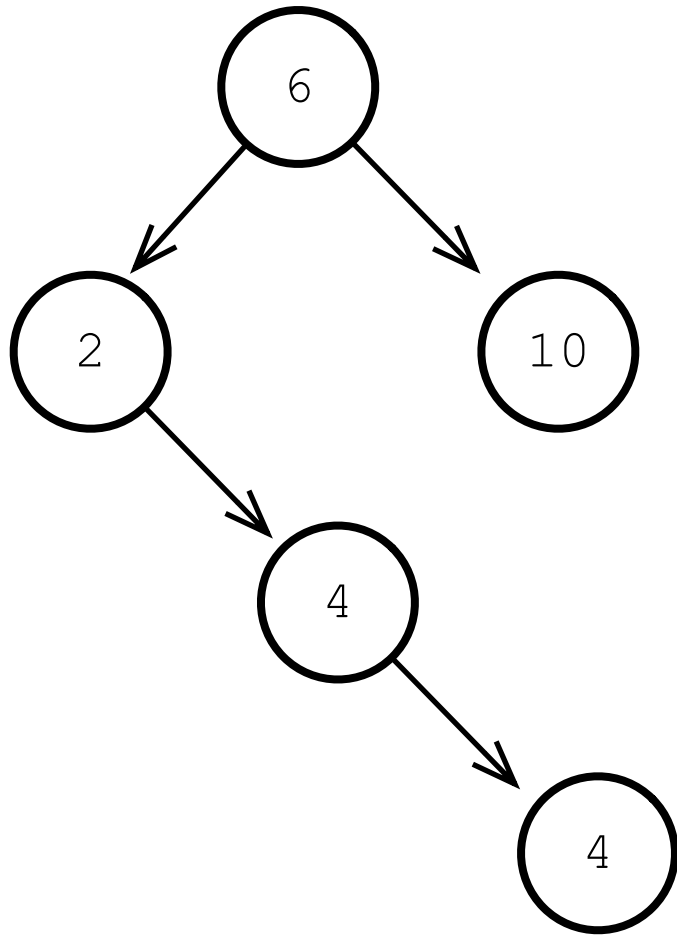
The tree is empty.

The node you are examining has an empty slot where you want to insert the node (shown as the null child).

The node you are examining has two valid child nodes, so you the new node has to be inserted somewhere downstream.

To see if the tree contains a node with a value, find it using a recursive pre-order search:
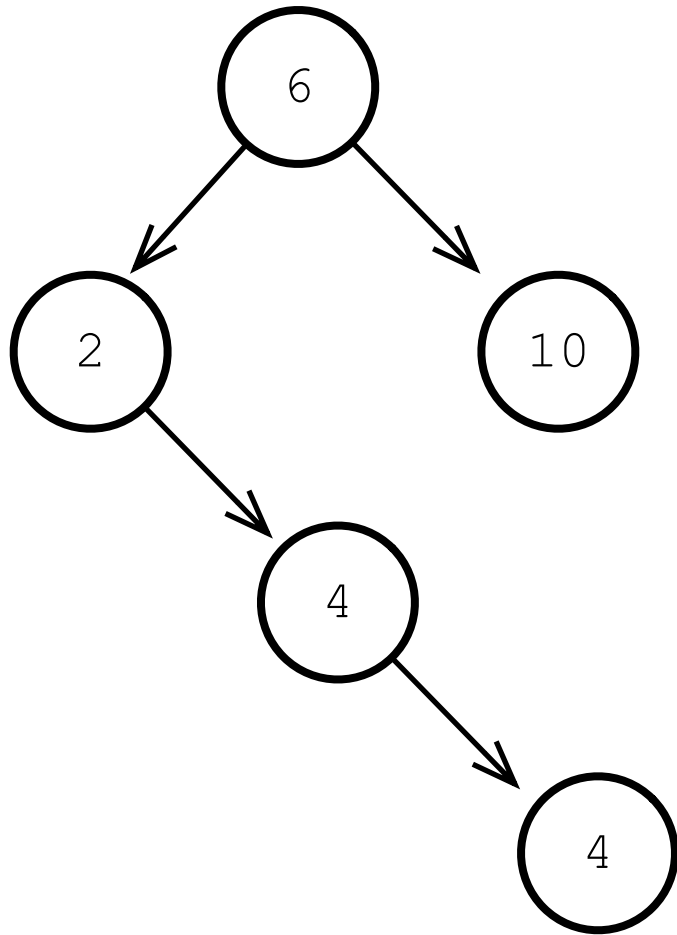
Is this node null? If so, return false.

Does this node have the right value? If so, return true!

If this node is larger than the target node, recurse left and return whatever it gives you. If not, recurse right.

If this node has no children, return false.
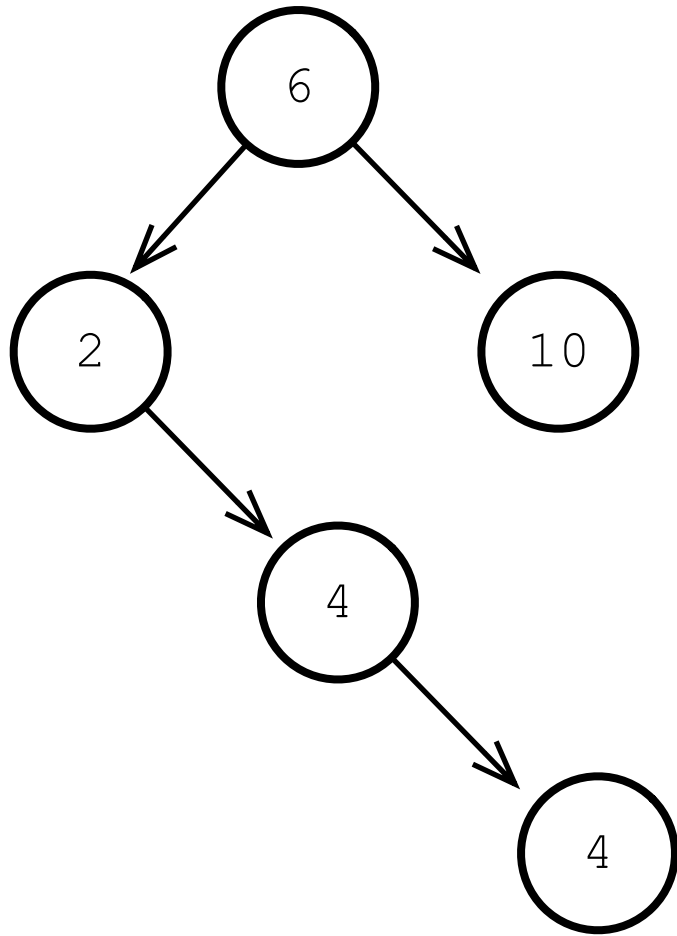
Example: say we are looking for '4'.

Is 6 it? No. Recurse left, because 4 < 6!

Is 2 it? No. Recurse right, because 4 <= 2!

Is it 4? Yep. Return true.

Return true.

Return true.

```
   6
  ↙ ↘
 2    10
  ↘
   4
    ↘
     4
```
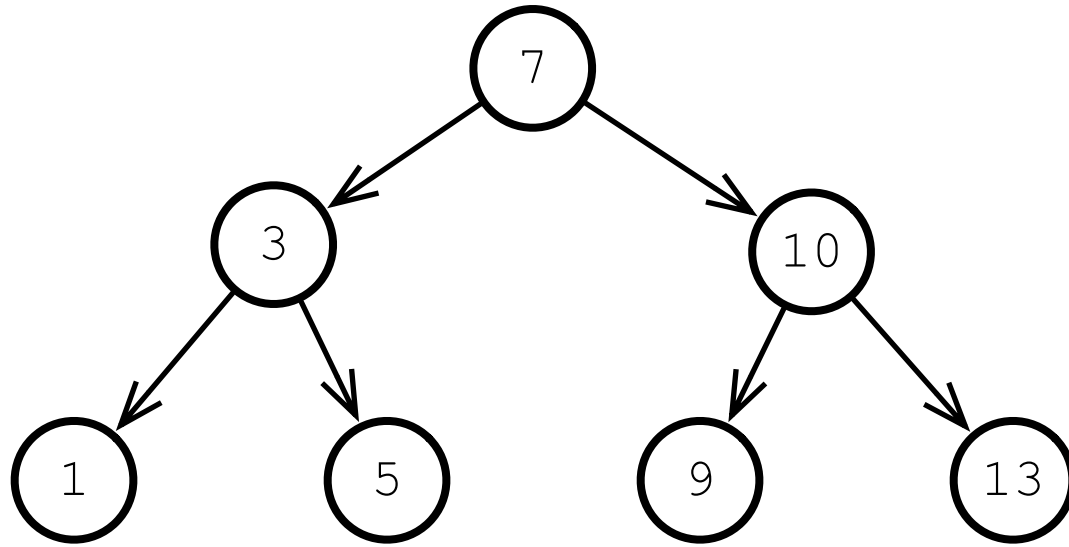
To find the **size** of the tree, simply perform a walk (of any kind) and increment a counter when you visit a node.

Only tricks here are to be careful about making sure you're not off by one, and having a way to pass along your count using a recursive function.

e.g.,
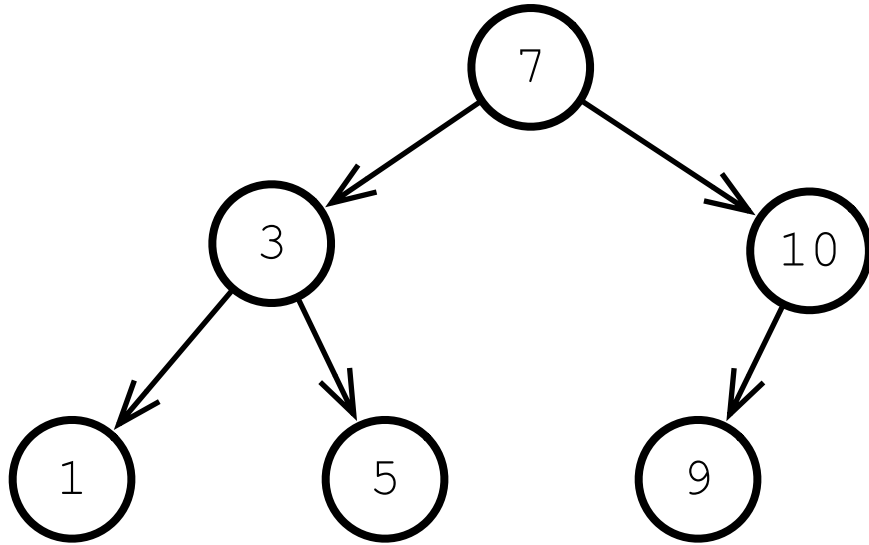
return 1 + left_size() + right_size()
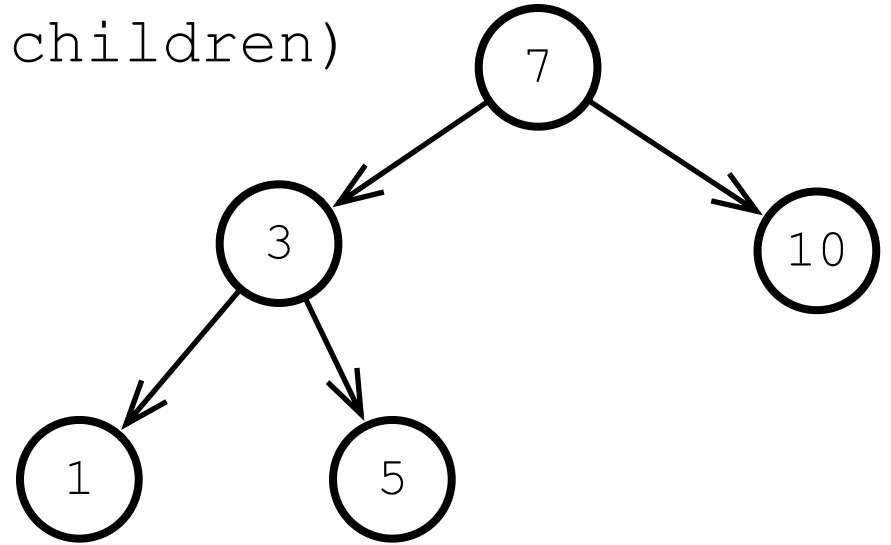
Removing a node is not completely obvious.

Whatever we do, we have to maintain three rules:

1. On exit, the doomed node is not in the tree.
2. On exit, it is still one tree—meaning we didn't orphan anything.
3. On exit, the sort order is still intact.

initial tree

7

3   10

1   5   9

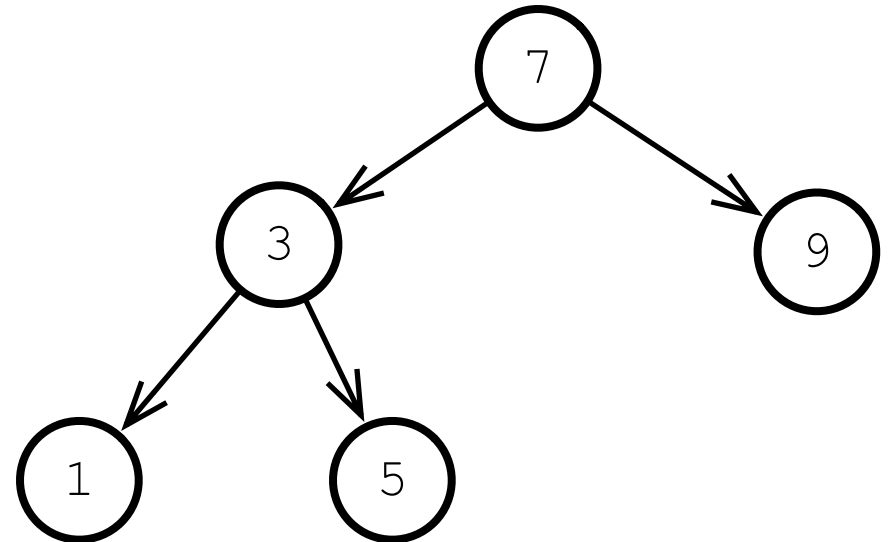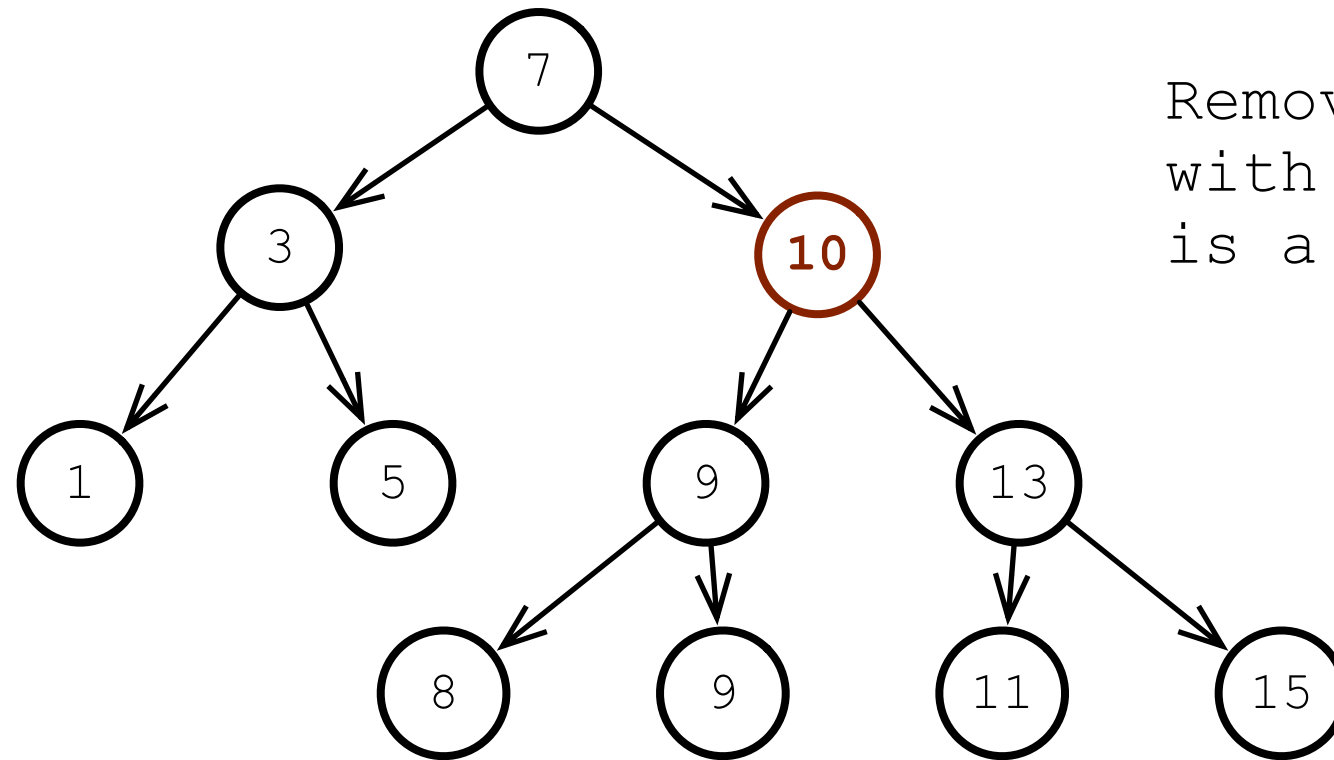delete 9 (no
children)

7

3   10

1   5

Removed nodes could have
0, 1, or 2 children.

Zero: set the parent
pointer to null.

One: replace the doomed
node with its child.

or delete 10
(one child)

7

3   9

1   5

Removing nodes
with two children
is a bit tougher.

The committee has determined that node 10 must go.
There are a number of ways to perform this
operation, and we won't require that you use any
specific one. All that matters is that the tree
obey the rules enumerated earlier.

Study this tree and imagine what it could look
like with the 10 node removed.

Sort-order
predecessor

Sort-order
successor
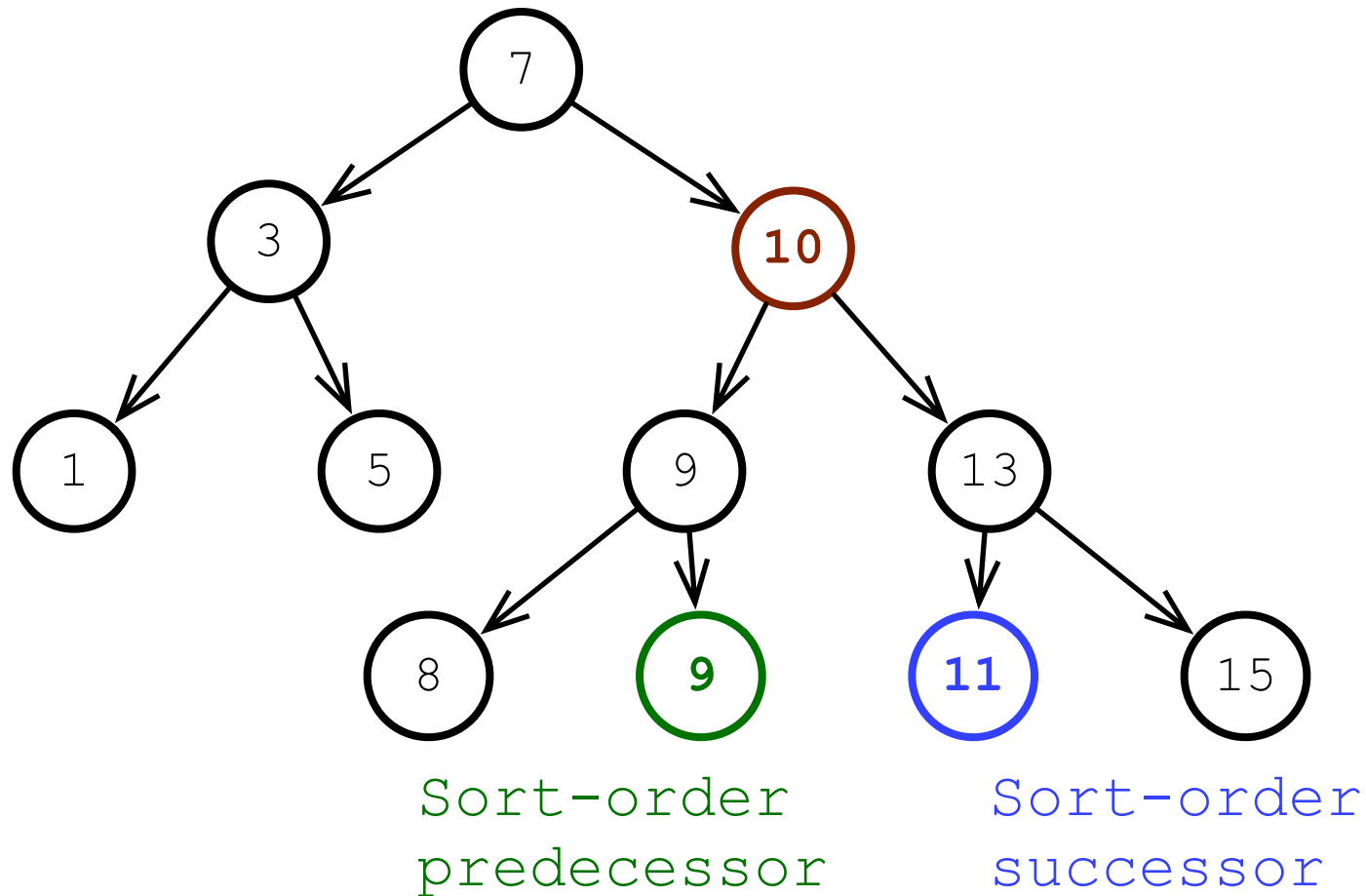
One way: if the doomed node has two children,
find its in-order predecessor ('buddy'),
and *replace* the doomed value with buddy's value,
then *delete buddy*.

Here, 'buddy' is either the green or blue one.
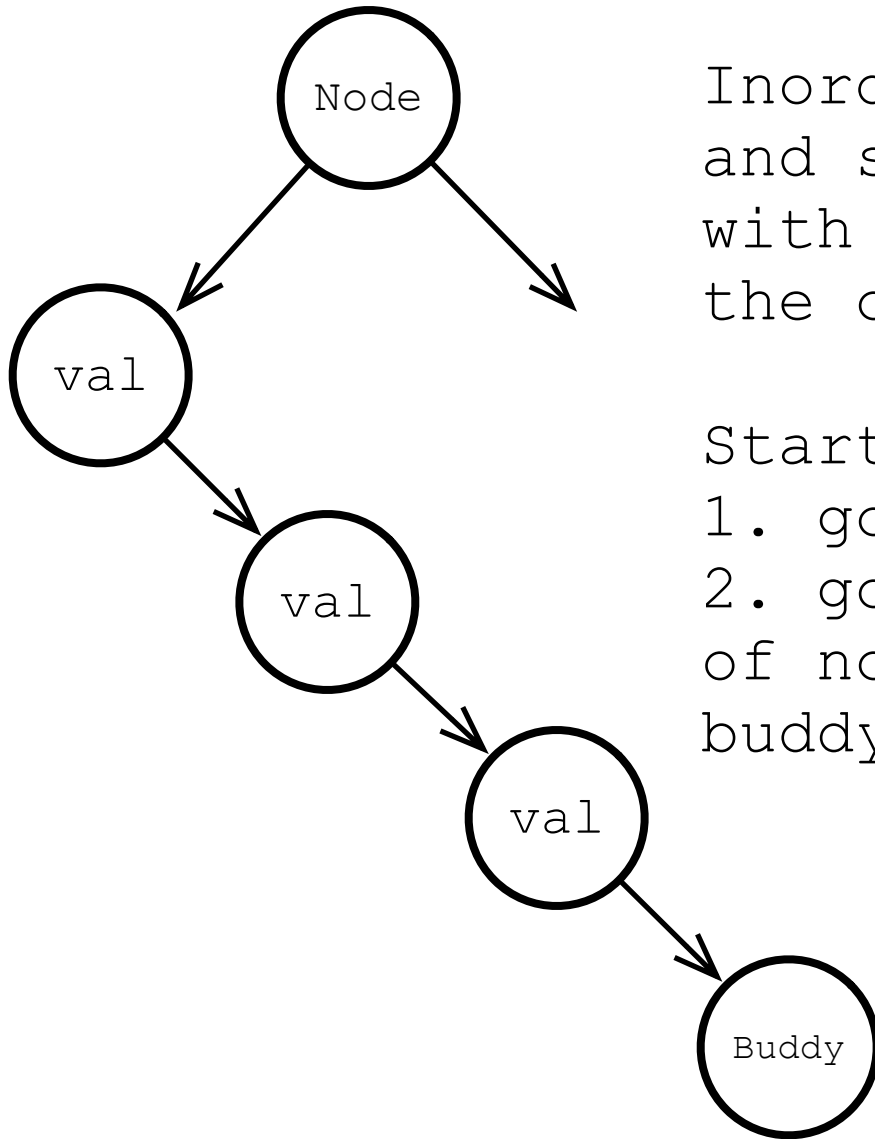
Inorder neighbors (predecessors
and successors) can be found
with a short algorithm. Here's
the one for predecessors:

Starting from Node,
1. go left one time
2. go right until you run out
of nodes. the last one is your
buddy.

The procedure for finding
the in-order successor is
similar; just switch left
and right.