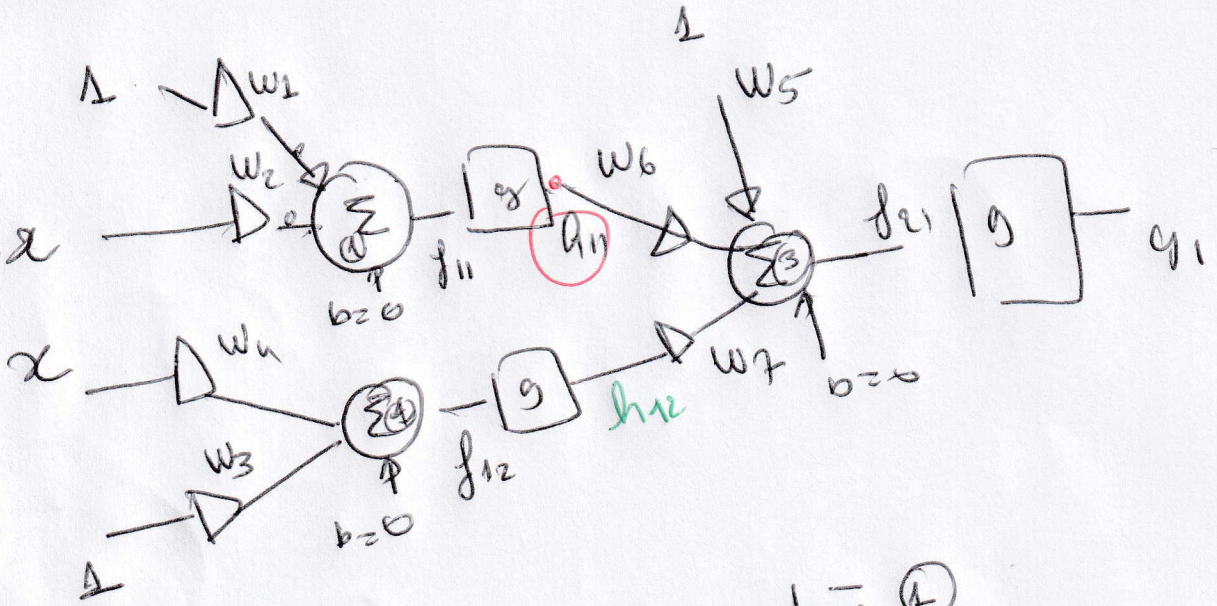
Exercice n°04

Q1 $x_1 = 1, x_2 = x_n = x$



f_{11} est la sortie du sommateur ①

f_{12} est la sortie du sommateur ②

f_{21} est la sortie du sommateur ③

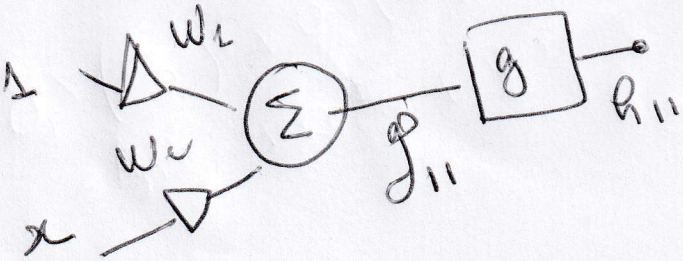
à l'entrée du premier sommateur

$1 \times w_1 + x\, w_2 \rightarrow$ donc la sortie du sommateur ①

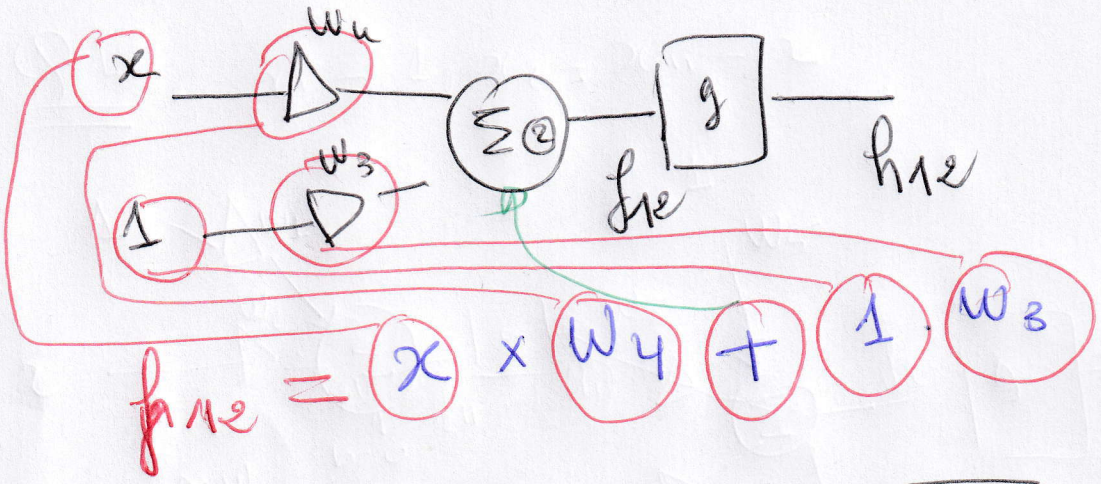
$$\boxed{f_{11} = w_2 x + w_1}$$

$w_1 \rightarrow$ les valeurs dans les petits rongés sont appelés poids

$(w_1, w_2) \rightarrow$ les poids du premier neurone

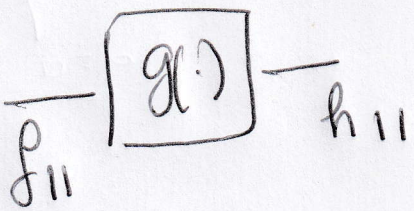


Je refais pareil pour la porte

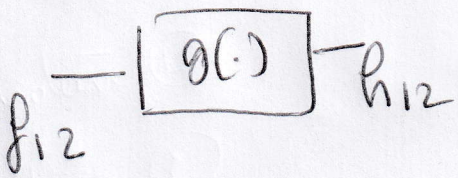


$f_{12} = x \times w_4 + 1 \cdot w_3$

✱ $h_{11} = g(f_{11})$



✱ $h_{12} = g(f_{12})$

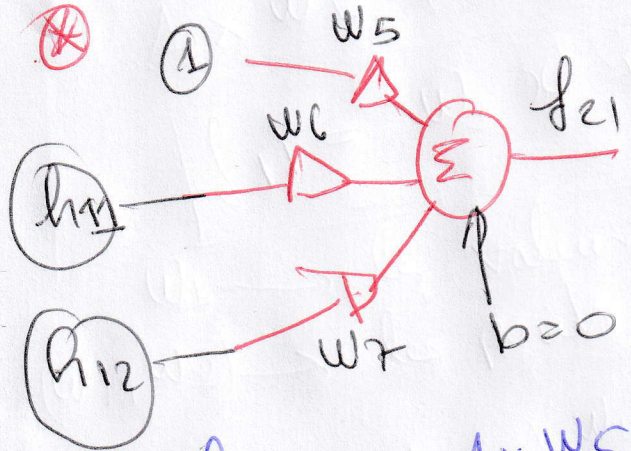


Application :

$$h_{11} = g(f_{11}) = \text{sigmoide}(f_{11}) = \frac{1}{1+e^{-f_{11}}}$$

$$h_{12} = g(f_{12}) = \text{sigmoide}(f_{12}) = \frac{1}{1+e^{-f_{12}}}$$

✱



$f_{21} = 1 \times w_5 + h_{11} \times w_6 + h_{12} \times w_7$

$f_{21} = 1 \times w_5 + h_{11} w_6 + h_{12} \times w_7$

$= w_5 + h_{11} w_6 + h_{12} w_7$

$$f_{21} = w_5 + \frac{w_6}{1+e^{-f_{11}}} + \frac{w_7}{1+e^{-f_{12}}}$$

$$y_i = g(f_{21}) = g'(f_{21}) = f_{21} = w_5 + w_6 h_{11} + w_7 h_{12}$$

$$y_i = w_5 + \frac{w_6}{1+e^{-f_{11}}} + \frac{w_7}{1+e^{-f_{12}}}$$

③ La fonction cutère $J(w) = (y_i - d)^2$ pour calculer les w_i, $i = 1, \dots 7$. nous utilisons l'algo de back-propagation.

$$\frac{\partial J(w)}{\partial w} = -2(y_i - d)\frac{\partial y_i}{\partial w_j}$$

l'algo de forward-propagation :

$$y_i = f_{21} = w_6 h_{11} + w_7 h_{12} + w_5$$

les dérivés :

$$\frac{\partial y_i}{\partial w_j} \longrightarrow \frac{\partial y_i}{\partial w_5} = 1$$

$$\frac{\partial y_i}{\partial w_6} = h_{11}$$

$$\frac{\partial y_i}{\partial w_7} = h_{12}$$

$$\frac{\partial y_i}{\partial w_1} = \frac{\partial y_i}{\partial h_{11}} \frac{\partial h_{11}}{\partial f_{11}} \frac{\partial f_{11}}{\partial w_1}$$

$$y_i = f_{21} = w_6 h_{11} + w_7 h_{12} + w_5$$

$$= w_6 \cdot \underbrace{g(w_2 x + w_1)}_{h_{11}(w_1)} + w_7 \, g(w_4 x + w_3) + w_5$$

Constant pour repérer à w_1

$$\frac{\partial y_i}{\partial w_1} = \frac{\partial y_i}{\partial h_{11}} \frac{\partial h_{11}}{\partial f_{11}} \cdot \frac{\partial f_{11}}{\partial w_1}$$

$$\frac{\partial y_i}{\partial h_{11}} = w_6$$

$$\frac{\partial h_{11}}{\partial f_{11}} = h_{11}(1 - h_{11})$$

h_{11} → fonction sigmoïde

$$h_{11} = \frac{1}{1 + e^{-f_{11}}}$$

$$\frac{\partial f_{11}}{\partial w_1} = \frac{\partial (w_2 x + w_1)}{\partial w_1} = 1$$

$$\frac{\partial h_{11}}{\partial f_{11}} = \frac{1}{1 + e^{-f_{11}}} \left(1 - \frac{1}{1 + e^{-f_{11}}}\right)$$

$$\frac{\partial y_i}{\partial w_1} = w_6 \cdot h_{11}(1 - h_{11}) \cdot x 1$$

$$= w_6 \cdot h_{11}(1 - h_{11})$$

$$\frac{\partial y_i}{\partial w_2} = \frac{\partial y_i}{\partial h_{11}} \cdot \frac{\partial h_{11}}{\partial f_{11}} \cdot \frac{\partial f_{11}}{\partial w_2}$$

$$= w_6 h_{11}(1-h_{11}) x$$

$$\frac{\partial y_i}{\partial h_{11}} = w_6$$

$$\frac{\partial h_{11}}{\partial f_{11}} = h_{11}(1-h_{11})$$ dérivée de sigmoïde

$$\frac{\partial f_{11}}{\partial w_2} = x$$

$$\frac{\partial y_i}{\partial w_3} = \frac{\partial y_i}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial f_{12}} \cdot \frac{\partial f_{12}}{\partial w_3} = w_7 h_{12}(1-h_{12}) x$$

$$\frac{\partial y_i}{\partial h_{12}} = w_7, \quad \frac{\partial h_{12}}{\partial f_{12}} = h_{12}(1-h_{12}), \quad \frac{\partial f_{12}}{\partial w_3} = 1$$

dérivée de sigmoïde

$$\frac{\partial y_i}{\partial w_4} = \frac{\partial y_i}{\partial h_{12}} \cdot \frac{\partial h_{12}}{\partial f_{12}} \cdot \frac{\partial f_{12}}{\partial w_4} = w_7 h_{12}(1-h_{12}) x$$

dérivée de sigmoïde

$$\frac{\partial y_i}{\partial h_{12}} = w_7, \quad \frac{\partial h_{12}}{\partial f_{12}} = h_{12}(1-h_{12})$$

$$\frac{\partial f_{12}}{\partial w_4} = x$$

$$\left.\begin{array}{l} \frac{\Delta w_j}{\Delta t} = \frac{\Delta w_j}{\alpha} \end{array}\right\}$$

$$\frac{\partial w_j}{\partial t} \simeq \longrightarrow \frac{\Delta w_j}{\Delta t} = \frac{\Delta w_j}{\alpha'} = \frac{\Delta w_j}{\alpha'}$$

$$\frac{\partial w_j}{\partial t} = \frac{\partial J}{\partial w_j} = \frac{\partial}{\partial w_j}(y_i - d)^2$$

$$= \frac{\partial (y_i - d)}{\partial w_j} \times 2$$

$$\frac{\Delta w_j}{\Delta t} \simeq (y_i - d) \times 2 \frac{\partial (y_i - d)}{\partial w_j}$$

$$= 2(y_i - d) \frac{\partial y_i}{\partial w_j}$$

$$\frac{\Delta w_j}{2\Delta t} = \frac{\Delta w_j}{2\alpha'} = \frac{\Delta w_j}{\alpha} = (y_i - d) \frac{\partial y_i}{\partial w_j}$$

$$y_i = f(w_1, \ldots w_j)$$

↑ fonction

donc $$\frac{\Delta w_j}{\alpha} = (y_i - d) \frac{\partial y_i}{\partial w_j}$$

d'où

① $\frac{\Delta w_1}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_1} = (y_i - d)\, w_6 h_{11}(1 - h_{11})$

② $\frac{\Delta w_2}{\Delta x} = (y_i - d)\frac{\partial y_i}{\partial w_2} = (y_i - d)\, w_6 h_{11}(1 - h_{11})\, x$

③ $\frac{\Delta w_3}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_3} = (y_i - d)\, w_7 h_{12}(1 - h_{12})$

$\frac{\Delta w_3}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_4} = (y_i - d) \cdot w_7 h_{12}(1 - h_{12})\, x$

④ $\frac{\Delta w_4}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_4} = (y_i - d) \cdot 1$

⑤ $\frac{\Delta w_5}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_5}$

⑥ $\frac{\Delta w_6}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_6} = (y_i - d) \cdot h_{11}$

⑦ $\frac{\Delta w_7}{\alpha} = (y_i - d)\frac{\partial y_i}{\partial w_7} = (y_i - d) \cdot h_{12}$

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④ calcul des sorties intermédiaires

pour $w_1 = 0,5$, $w_2 = -1$, $w_3 = -2$

$w_4 = 1,5$, $w_5 = 1$, $w_6 = 1$, $w_7 = -1$

$(x, d) = (2, 1)$

(4)

$f_{12} = 1,5 \times 2 - 2 = 1$

$h_{11} = g(f_{11}) = \frac{1}{1 + e^{+1,5}} = 0,1824$

$h_{12} = g(f_{12}) = \frac{1}{1 + e^{-1}} = 0,7311$

$f_{21} = w_5 + h_{11} w_6 + w_7 \cdot h_{12}$

$= 1 + w_6 (0,1824) + w_7 (0,7311)$

$= 1 + 0,1824 - 1,0 \times 0,7311$

$= 1 + 0,1824 - 0,7311 = 0,4513$

$y = f_{21} = w_6 h_{11} + w_7 h_{12} + w_5 = 0,4513 =$

(*) $\Delta w_1 = \alpha (y_i - d) \frac{\partial y_i}{\partial w_1} = \alpha (y_i - d) \cdot w_6 h_{11} (1 - h_{11})$

$= 0,2 (2 - 1) \times 1 \cdot (0,1824) (1 -$

$\simeq 0,2 (2 - 1) (0,182) (1 - 0,4513) \times 1$

$\simeq 0,2 \times 0,1824 \times (1 - y_i) \times w_1$

$\simeq 0,2 \times 0,1824 (1 - 0,4513) (1 - 0,1824)$

$= 0,01636 \simeq \boxed{0,0164}$

(7)

$$\Delta w_2 = \alpha (y - d) \frac{\partial y_i}{\partial w_2} = 0.2 (1 - 0,4513) \times 2\, h_{11}(1 - h_{11})$$

$$= 0,2 (1 - 0,4513) \times 2 \times 0,1824 \times (1 - 0,1824)$$

$$= 0,2 \times 0,5487 \times 2 \times 0,1824 \times 0,8176$$

$$= 0,03273 \simeq \boxed{0,0328}$$

$$\Delta w_3 = \alpha (y - d)\, w_7 \cdot h_{12}(1 - h_{12})$$

$$= \alpha (1 - 0,4513) 1 \times 0,7311 \times (1 - 0,7311)$$

$$= 0,2 \times 0,548 \times 0,7311 \times 0,2689$$

$$= 0,021546 \simeq \boxed{0,0216.}$$

$$\Delta w_4 = \alpha (y - d)\, w_7\, h_{12}(1 - h_{12}) \times x$$

$$= 0,2 (1 - 0,4513) \times 0,7311 (1 - 0,7311)$$

$$= 0,2 \times 0,5487 \times 0,7311 \times 0,2689 \times 2$$

$$= 0,0216 \times 2 = \boxed{0,0432}$$

$$\Delta w_5 = \alpha (y - d)\, 1 = 0,2 \times (1 - 0,4513) \times 1$$

$$= 0,2 \times 0,5487 \times 1$$

$$= \boxed{0,10974}$$

$$\Delta w_6 = \alpha (y - d) \cdot h_{11}$$

$$= 0,2 (1 - 0,4513) \cdot 0,1824 =$$

$$0,2 \times 0,5487 \times 0,1824 \simeq \boxed{0,02}$$

$$DW_7 = \alpha(y_i - d)\, f_{12}$$

$$= 0,2(1 - 0,4513) \times 0,7311$$

$$= 0,2 \times 0,5487 \times 0,7311$$

$$= 0,08023 \simeq \boxed{0,0802}$$

$$w_1 = w_1 + DW_1 = 0,5 + 0,0164 = 0,5164$$

$$w_2 = w_2 + DW_2 = -1 + 0,0328 = -0,972$$

$$w_3 = w_3 + DW_3 = -2 + 0,0216 = -1,9784$$

$$w_4 = w_4 + DW_4 = 1,5 + 0,0432 = 1.5432$$

$$w_5 = w_5 + DW_5 = 1 + 0,1097 = 1,1097$$

$$w_6 = w_6 + DW_6 = 1 + 0,02 = 1,02$$

$$w_7 = w_7 + DW_7 = 1 + 0,0802 = 1,0802$$