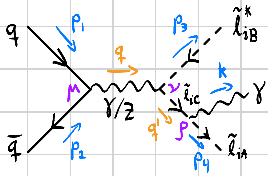
Photon radiated from $\tilde{l}_{iA}$ "from scratch"



$$q = p_1+p_2 = p_3+p_4+k = q'+p_3$$

$$q' = p_1+p_2-p_3 = p_4+k = q-p_3$$

$$\cancel{i}\mathcal{M}_\gamma = \bar{v}^r(p_2)(+\cancel{i}eQ_q\gamma^\mu)u^s(p_1)\,\frac{+\cancel{i}g_{\mu\nu}}{q^2}\,\cancel{i}e(q'-p_3)^\nu\,\frac{\cancel{i}}{q'^2-m^2_{\tilde{l}_{iC}}}\,\cancel{i}e(q'+p_4)^\rho\epsilon^*_\rho(k)\,\overbrace{\delta^{AC}\delta^{CB}}^{\delta^{AB}}$$

$$\mathcal{M}_\gamma = \underbrace{\frac{Q_q e^3\delta^{AB}}{(p_3+p_4+k)^2((p_4+k)^2-m^2_{\tilde{l}_{iA}})}}_{=C^{AB}_{\gamma,q\tilde{l}_i}}\left[\cancel{k^\rho\epsilon^*_\rho(k)}+2p_4^\rho\epsilon^*_\rho(k)\right]\bar{v}^r(p_2)(\underbrace{\not{k}+\not{p}_4-\not{p}_3}_{\cancel{\not{p}_1}+\cancel{\not{p}_2}-2\not{p}_3})u^s(p_1)$$

← Coincides with Mathematica ✓

γ massless $\Rightarrow k\cdot\epsilon=0$

$(\not{p}-m)u(p)=\bar{v}(p)(\not{p}+m)=0$, $m_q\approx 0 \Rightarrow \bar{v}(p_2)\not{p}_2=\not{p}_1u(p_1)=0$

$$= -4\,C^{AB}_{\gamma,q\tilde{l}_i}\,p_4^\mu\epsilon^*_\mu(k)\,\bar{v}^r(p_2)\not{p}_3u^s(p_1)\quad,\quad C^{AB}_{\gamma,q\tilde{l}_i}\equiv\frac{Q_q e^3\delta^{AB}}{(p_3+p_4+k)^2((p_4+k)^2-m^2_{\tilde{l}_{iA}})}$$

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4N}\cdot 16\left(C^{AB}_{\gamma,q\tilde{l}_i}\right)^2\sum_{pol.}p_4^\mu\epsilon^*_\mu(k)\,p_4^\nu\epsilon_\nu(k)\,Tr\{\not{p}_2\not{p}_3\not{p}_1\not{p}_3\}$$

$\sum_{pol.}\epsilon^*_\mu\epsilon_\nu=-g_{\mu\nu}$

$p_4^2=m^2_{\tilde{l}_{iA}}$

$$= -\frac{4}{N}\left(C^{AB}_{\gamma,q\tilde{l}_i}\right)^2 m^2_{\tilde{l}_{iA}}\,Tr\{\not{p}_2\underbrace{\not{p}_3\not{p}_1\not{p}_3}\}$$

$$\not{p}_2(2p_1\cdot p_3-\not{p}_1\not{p}_3)\not{p}_3 = 2(p_1\cdot p_3)\not{p}_2\not{p}_3-p_3^2\not{p}_2\not{p}_1$$

$$= [2(p_1\cdot p_3)p_{3\nu}-m^2_{\tilde{l}_{iB}}p_{1\nu}]p_{2\mu}\gamma^\mu\gamma^\nu\}$$

$$(2(p_1\cdot p_3)p_{3\nu}p_{2\mu}-m^2_{\tilde{l}_{iB}}p_{1\nu}p_{2\mu})\underbrace{Tr\{\gamma^\mu\gamma^\nu\}}_{=4g^{\mu\nu}}$$

$$= 4\left[2(p_1\cdot p_3)(p_2\cdot p_3)-m^2_{\tilde{l}_{iB}}(p_1\cdot p_2)\right]$$

$$\langle|\mathcal{M}_\gamma|^2\rangle = \frac{16}{N}\left(C^{AB}_{\gamma,q\tilde{l}_i}\right)^2 m^2_{\tilde{l}_{iA}}\left[m^2_{\tilde{l}_{iB}}(p_1\cdot p_2)-2(p_1\cdot p_3)(p_2\cdot p_3)\right]$$

$$\cancel{i}\mathcal{M}_Z = \bar{v}^r(p_2)\frac{\cancel{i}ge}{s_W c_W}\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)\frac{+\cancel{i}g_{\mu\nu}}{q^2-m_Z^2+im_Z\Gamma_Z}\frac{\cancel{i}ge}{s_W c_W}Z_{\ell_i}^{B\cancel{C}A}\big(\underbrace{-q'}_{q'=p_1+p_2-p_3}+p_3\big)^\nu\frac{\cancel{i}}{q'^2-m^2_{\tilde{\ell}_{iA}}}\cancel{i}e\big(\underbrace{q'}_{q'=p_4+k}+p_4\big)^\rho\epsilon^*_\rho(k)\cancel{\delta^{AC}}$$

$$\mathcal{M}_Z = \underbrace{\frac{e^3Z_{\ell_i}^{AB}}{s_W^2c_W^2}\frac{1}{q'^2-m^2_{\tilde{\ell}_{iA}}}\frac{1}{q^2-m_Z^2+im_Z\Gamma_Z}}_{\equiv C^{AB}_{Z,\tilde{\ell}_i}}\big(\underbrace{\cancel{k^\rho}}_{k\cdot\epsilon=0}+2p_4^\rho\big)\epsilon^*_\rho(k)\,\bar{v}^r(p_2)\big(2\not{p}_3-\cancel{\not{p}_1}-\underbrace{\cancel{\not{p}_2}}_{\bar{v}(p_2)\not{p}_2=0}\big)\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$

$\not{p}_1(1\pm\gamma^5)u(p_1)=(1\mp\gamma^5)\not{p}_1u(p_1)=0$

$$= C^{AB}_{Z,\tilde{\ell}_i}\,2p_4^\rho\epsilon^*_\rho(k)\,2\,\bar{v}^r(p_2)\not{p}_3\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)\quad,\quad C^{AB}_{Z,\tilde{\ell}_i}\equiv\frac{e^3Z^{AB}_{\ell_i}}{s_W^2c_W^2}\frac{1}{\left[(p_4+k)^2-m^2_{\tilde{\ell}_{iA}}\right]\left[(p_3+p_4+k)^2-m_Z^2+im_Z\Gamma_Z\right]}$$

$$\langle|\mathcal{M}_Z|^2\rangle = \left|C^{AB}_{Z,\tilde{\ell}_i}\right|^2 16\,\frac{1}{4N}\sum_{\text{pol.}}p_4^\rho p_4^\sigma\epsilon^*_\rho(k)\epsilon_\sigma(k)\,\mathrm{Tr}\left\{\not{p}_2\not{p}_3\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]\not{p}_1\left[Z_{q_L}(1+\gamma^5)+Z_{q_R}(1-\gamma^5)\right]\not{p}_3\right\}$$

$$= -\left|C^{AB}_{Z,\tilde{\ell}_i}\right|^2\frac{4}{N}m^2_{\tilde{\ell}_{iA}}\,\mathrm{Tr}\Big\{\not{p}_1\not{p}_3\not{p}_2\not{p}_3\underbrace{\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]}\Big\}$$

$$\left[Z_{q_L}+Z_{q_R}+(Z_{q_R}-Z_{q_L})\gamma^5\right]^2=(Z_{q_L}+Z_{q_R})^2+(Z_{q_L}-Z_{q_R})^2+2(Z_{q_L}+Z_{q_R})(Z_{q_R}-Z_{q_L})\gamma^5$$

$$=2(Z_{q_L}^2+Z_{q_R}^2)+\cancel{2(Z_{q_R}^2-Z_{q_L}^2)\gamma^5}$$

$\mathrm{Tr}\{\gamma^{\mu\nu\rho\sigma}\gamma^5\}\propto\varepsilon^{\mu\nu\rho\sigma}=0$ since $p_{3\mu}p_{3\nu}$ commute in μ,ν

$$= -\left|C^{AB}_{Z,\tilde{\ell}_i}\right|^2\frac{8}{N}m^2_{\tilde{\ell}_{iA}}\left(Z_{q_L}^2+Z_{q_R}^2\right)\mathrm{Tr}\Big\{\underbrace{\not{p}_1\not{p}_3\underbrace{\not{p}_2\not{p}_3}_{2p_2\cdot p_3-\not{p}_3\not{p}_2}}_{\not{p}_1(2p_2\cdot p_3\not{p}_3-m^2_{\tilde{\ell}_{iB}}\not{p}_2)}\Big\}$$

$$= -\left|C^{AB}_{Z,\tilde{\ell}_i}\right|^2\frac{8m^2_{\tilde{\ell}_{iA}}\left(Z_{q_L}^2+Z_{q_R}^2\right)}{N}\,p_{1\mu}\left[2(p_2\cdot p_3)p_{3\nu}-m^2_{\tilde{\ell}_{iB}}p_{2\nu}\right]\mathrm{Tr}\{\gamma^\mu\gamma^\nu\}$$

$$= \frac{32}{N}\left|C^{AB}_{Z,\tilde{\ell}_i}\right|^2\left(Z_{q_L}^2+Z_{q_R}^2\right)m^2_{\tilde{\ell}_{iA}}\left[m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2)-2(p_1\cdot p_3)(p_2\cdot p_3)\right]$$

$$\langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle = + 4 C^{AB}_{\gamma,q\tilde{\ell}_i} \cancel{4}\, C^{AB}_{Z,\tilde{\ell}_i} \frac{1}{\cancel{4}N} \cancel{\sum_{\text{pol.}}} \sum_{\text{spin}} \cancel{p_4^\mu} \overset{-g_{\mu\nu}}{\cancel{\epsilon_\mu(k)}}\, \bar{u}^s(p_1) \not{p}_3 v^r(p_2)\, \overset{=-p_4^2=+m^2_{\tilde{\ell}_{iA}}}{\cancel{p_4^\nu}\, \cancel{\epsilon_\nu^*(k)}}\, \bar{v}^r(p_2) \not{p}_3 \left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right] u^s(p_1)$$

$Tr\{\gamma^{\mu\nu\rho\sigma 5}\} \propto \epsilon^{\mu\nu\rho\sigma} = 0$ since $p_1^\mu p_3^\nu p_2^\rho p_3^\sigma$ symmetric in $\nu \leftrightarrow \sigma$

$$= \frac{4}{N} C^{AB}_{\gamma,q\tilde{\ell}_i} C^{AB}_{Z,\tilde{\ell}_i} m^2_{\tilde{\ell}_{iA}} \underbrace{Tr\{ \not{p}_1 \not{p}_3 \not{p}_2 \not{p}_3}_{=4\left[2(p_1\cdot p_3)(p_2\cdot p_3) - m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2)\right]} \left[Z_{q_L}(1-\cancel{\gamma^5}) + Z_{q_R}(1+\cancel{\gamma^5}) \right] \}$$

from $|\mathcal{M}_\gamma|^2$ calculation

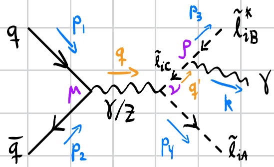
$$= \frac{16}{N} C^{AB}_{\gamma,q\tilde{\ell}_i} C^{AB}_{Z,\tilde{\ell}_i} (Z_{q_L} + Z_{q_R})\, m^2_{\tilde{\ell}_{iA}} \left[2(p_1\cdot p_3)(p_2\cdot p_3) - m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2) \right]$$

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma|^2 + 2\,\mathrm{Re}\{\mathcal{M}_\gamma^* \mathcal{M}_Z\} + |\mathcal{M}_Z|^2$$

$$= \frac{16}{N} \left(C^{AB}_{\gamma,q\tilde{\ell}_i}\right)^2 m^2_{\tilde{\ell}_{iA}} \left[m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2) - 2(p_1\cdot p_3)(p_2\cdot p_3) \right]$$
$$- 2\,\mathrm{Re}\left\{ \frac{16}{N} C^{AB}_{\gamma,q\tilde{\ell}_i} C^{AB}_{Z,\tilde{\ell}_i} (Z_{q_L} + Z_{q_R})\, m^2_{\tilde{\ell}_{iA}} \left[m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2) - 2(p_1\cdot p_3)(p_2\cdot p_3) \right] \right\}$$
$$+ \frac{32}{N} \left| C^{AB}_{Z,\tilde{\ell}_i} \right|^2 (Z_{q_L}^2 + Z_{q_R}^2)\, m^2_{\tilde{\ell}_{iA}} \left[m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2) - 2(p_1\cdot p_3)(p_2\cdot p_3) \right]$$

$$= \frac{4}{N} \left[\left(C^{AB}_{\gamma,q\tilde{\ell}_i}\right)^2 + 2\left| C^{AB}_{Z,\tilde{\ell}_i} \right|^2 (Z_{q_L}^2 + Z_{q_R}^2) - 2 C^{AB}_{\gamma,q\tilde{\ell}_i} \mathrm{Re}\{ C^{AB}_{Z,\tilde{\ell}_i} \} (Z_{q_L} + Z_{q_R}) \right]$$
$$\times 4 m^2_{\tilde{\ell}_{iA}} \left[m^2_{\tilde{\ell}_{iB}}(p_1\cdot p_2) - 2(p_1\cdot p_3)(p_2\cdot p_3) \right]$$

Photon radiated from $\tilde{l}_{iB}$ "from scratch"



$$q = p_1 + p_2 = p_3 + p_4 + k = q' + p_4$$

$$q' = p_1 + p_2 - p_4 = p_3 + k = q - p_4$$

since both p_3, q' flows against scalar charge flow

$$\cancel{i}\mathcal{M}_\gamma = \bar{v}^r(p_2)\left(\cancel{-}\cancel{i} e Q_q \gamma^\mu\right) u^s(p_1) \frac{-\cancel{i} g_{\mu\nu}}{q^2} \cancel{i} e \left(p_4 - q'\right)^\nu \frac{\cancel{i}}{q'^2 - m^2_{\tilde{l}_{iC}}} \cancel{(-1)} \cancel{i} e \left(p_3 + q'\right)^\rho \epsilon^*_\rho(k)\, \delta^{AC} \delta^{BC}$$

$$\mathcal{M}_\gamma = -\frac{Q_q e^3}{q^2 (q'^2 - m^2_{\tilde{l}_{iA}})} \bar{v}^r(p_2)\left(\not{p}_4 - \not{q}'\right) u^s(p_1) \left(p_3 + q'\right)^\rho \epsilon^*_\rho(k)\, \delta^{AB}$$

$$\cancel{i}\mathcal{M}_Z = \bar{v}^r(p_2) \frac{\cancel{i} g}{c_W} \gamma^\mu \left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right] u^s(p_1) \frac{\cancel{+}\cancel{i} g_{\mu\nu}}{q^2 - m_Z^2 + i m_Z \Gamma_Z} \frac{\cancel{i} g}{c_W} Z^{AC}_{l_i} \left(p_4 - q'\right)^\nu$$

$$\times \frac{\cancel{i}}{q'^2 - m^2_{\tilde{l}_{iC}}} \cancel{(-1)}\, i e \left(p_3 + q'\right)^\rho \epsilon^*_\rho(k)\, \delta^{BC}$$

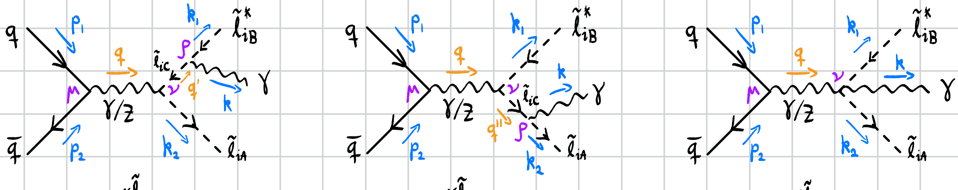
$g = \frac{e}{s_W}$

$$\mathcal{M}_Z = \frac{e^3}{s_W^2 c_W^2 (q'^2 - m^2_{\tilde{l}_{iB}})(q^2 - m_Z^2 + i m_Z \Gamma_Z)} \bar{v}^r(p_2)\left(\not{p}_4 - \not{q}'\right)\left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right] u^s(p_1) \left(p_3 + q'\right)^\rho \epsilon^*_\rho(k)\, Z^{AB}_{l_i}$$

Slepton-side photon radiation

Doing all diagrams with radiation from slepton-side this time.

Matrix elements:



$\mathcal{M}^{r\tilde{l}_1}_{\gamma/Z}$ $\qquad$ $\mathcal{M}^{r\tilde{l}_2}_{\gamma/Z}$ $\qquad$ $\mathcal{M}^{r\tilde{l}_{1,2}}_{\gamma/Z}$

$$q = p_1+p_2 = k_1+k_2+k \ , \quad q' = p_1+p_2-k_2 = k_1+k \ , \quad q'' = p_1+p_2-k_1 = k_2+k$$

since both p_3, q' flows against scalar charge flow

$$\cancel{i}\mathcal{M}^{r\tilde{l}_1}_{\gamma} = \bar{v}^r(p_2)(+\cancel{i}eQ_q\gamma^\mu)u^s(p_1)\frac{-\cancel{i}g_{\mu\nu}}{q^2}\cancel{i}e(k_2-q')^\nu\frac{\cancel{i}}{q'^2-m^2_{\tilde{l}_{iC\,A}}}\cancel{(-1)}\cancel{i}e(k_1+q')^\rho\,\epsilon^*_\rho(k)\,\underbrace{\delta^{AC}\delta^{BC}}_{\delta^{AB}}$$

$$\mathcal{M}^{r\tilde{l}_1}_{\gamma} = \frac{Q_q e^3\delta^{AB}}{q^2(q'^2-m^2_{\tilde{l}_{iA}})}(k_1+q')^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)(\not{q}'-\not{k}_2)u^s(p_1)$$

FeynCalc ✓

$$= C^{AB}_{\gamma,q\tilde{l}_i}(q,q')(k_1+q')^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)(\not{q}'-\not{k}_2)u^s(p_1) \ , \qquad C^{AB}_{\gamma,q\tilde{l}_i}(q,q') = \frac{Q_q e^3\delta^{AB}}{q^2(q'^2-m^2_{\tilde{l}_{iA}})}$$

quark type (for Q_q), not momentum

$$\cancel{i}\mathcal{M}^{r\tilde{l}_1}_{Z} = \bar{v}^r(p_2)\frac{\cancel{i}g}{c_W}\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)\frac{+\cancel{i}g_{\mu\nu}}{q^2-m_Z^2+im_Z\Gamma_Z}\frac{\cancel{i}g}{c_W}Z^{AC}_{l_i}(k_2-q')^\nu$$

$$\times\frac{\cancel{i}}{q'^2-m^2_{\tilde{l}_{iC}}}\cancel{(-1)}\cancel{i}e(k_1+q')^\rho\epsilon^*_\rho(k)\,\delta^{BC}$$

$g = \frac{e}{s_W}$

$$\mathcal{M}^{r\tilde{l}_1}_{Z} = \frac{e^3 Z^{AB}_{l_i}}{s_W^2c_W^2(q'^2-m^2_{\tilde{l}_{iB}})(q^2-m_Z^2+im_Z\Gamma_Z)}(k_1+q')^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)(\not{k}_2-\not{q}')\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$

$$= C^{AB}_{Z,\tilde{l}_i}(q,q',m_{\tilde{l}_{iB}})(k_1+q')^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)(\not{k}_2-\not{q}')\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1) \ ,$$

$$C^{AB}_{Z,\tilde{l}_i}(q,q',m_{\tilde{l}_{iB}}) = \frac{e^3 Z^{AB}_{l_i}}{s_W^2c_W^2(q^2-m_Z^2+im_Z\Gamma_Z)(q'^2-m^2_{\tilde{l}_{iB}})}$$

FeynCalc ✓

$$\cancel{i}\mathcal{M}_\gamma^{r\tilde{\ell}_2} = \bar{v}^r(p_2)\left(+\cancel{i}eQ_q\gamma^\mu\right)u^s(p_1)\,\frac{+\cancel{i}g_{\mu\nu}}{q^2}\,\cancel{i}e\left(q''-k_1\right)^\nu\,\frac{\cancel{i}}{q''^2-m^2_{\tilde{\ell}_{iC}}}\,\cancel{i}e\left(q''+k_2\right)^\rho\,\epsilon^*_\rho(k)\,\overbrace{\delta^{AC}\delta^{BC}}^{\delta^{AB}}$$

$$\mathcal{M}_\gamma^{r\tilde{\ell}_2} = \frac{Q_q e^3\delta^{AB}}{q^2\left(q''^2-m^2_{\tilde{\ell}_{iA}}\right)}\left(q''+k_2\right)^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)\left(\not{q}''-\not{k}_1\right)u^s(p_1)$$

$$= \underline{C^{AB}_{\gamma,q\tilde{\ell}_i}(q,q'')\left(q''+k_2\right)^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)\left(\not{q}''-\not{k}_1\right)u^s(p_1)}$$ ← FeynCalc ✓

$$\cancel{i}\mathcal{M}_Z^{r\tilde{\ell}_2} = \bar{v}^r(p_2)\,\frac{\cancel{i}\cancel{g}e}{s_Wc_W}\,\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)\,\frac{+\cancel{i}g_{\mu\nu}}{q^2-m_Z^2+im_Z\Gamma_Z}\,\frac{\cancel{i}\cancel{g}e}{s_Wc_W}\,Z^{B\overset{A}{\cancel{C}}}_{\ell_i}\left(-q''+k_1\right)^\nu\,\frac{\cancel{i}}{q''^2-m^2_{\tilde{\ell}_{i\cancel{C}A}}}\,\cancel{i}e\left(q''+k_2\right)^\rho\epsilon^*_\rho(k)\,\cancel{\delta^{AC}}$$

$$\mathcal{M}_Z^{r\tilde{\ell}_2} = \frac{e^3 Z^{AB}_{\ell_i}}{s_W^2c_W^2}\,\frac{1}{q''^2-m^2_{\tilde{\ell}_{iA}}}\,\frac{1}{q^2-m_Z^2+im_Z\Gamma_Z}\left(q''+k_2\right)^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)\left(\not{k}_1-\not{q}''\right)\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$

$$= \underline{C^{AB}_{Z,\tilde{\ell}_i}(q,q'',m_{\tilde{\ell}_{iA}})\left(q''+k_2\right)^\rho\epsilon^*_\rho(k)\,\bar{v}^r(p_2)\left(\not{k}_1-\not{q}''\right)\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)}$$ ← FeynCalc ✓

(note: FC seemingly has an extra factor 2, but this is because $\gamma^{6,7}=\frac{1}{2}(1\pm\gamma^5)$. I spent way too long realising this!)

$$\cancel{i}\mathcal{M}_\gamma^{r\tilde{\ell}_{1,2}} = \bar{v}^r(p_2)\left(-\cancel{i}eQ_q\gamma^\mu\right)u^s(p_1)\,\frac{+\cancel{i}g_{\mu\nu}}{q^2}\,2\cancel{i}e^2g^{\nu\rho}\,\epsilon^*_\rho(k)$$

$$\mathcal{M}_\gamma^{r\tilde{\ell}_{1,2}} = -\frac{2Q_qe^3\delta^{AB}}{q^2}\,\epsilon^*_\mu(k)\,\bar{v}^r(p_2)\,\gamma^\mu u^s(p_1)$$

$$= \underline{-C^{12}_{\gamma,q}(q)\,\epsilon^*_\mu(k)\,\bar{v}^r(p_2)\,\gamma^\mu u^s(p_1)}\,,\qquad C^{12}_{\gamma,q}(q)=\frac{2Q_qe^3\delta^{AB}}{q^2}$$

↑ FeynCalc ✓

$$\cancel{i}\mathcal{M}_Z^{r\tilde{\ell}_{1,2}} = \bar{v}^r(p_2)\,\frac{\cancel{i}\cancel{g}e}{s_Wc_W}\,\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)\,\frac{+\cancel{i}g_{\mu\nu}}{q^2-m_Z^2+im_Z\Gamma_Z}\,\frac{2\cancel{i}e^2}{s_Wc_W}\,Z^{AB}_{\ell_i}\,g^{\nu\rho}\,\epsilon^*_\rho(k)$$

$$\mathcal{M}_Z^{r\tilde{\ell}_{1,2}} = \frac{2e^3Z^{AB}_{\ell_i}}{s_W^2c_W^2\left(q^2-m_Z^2+im_Z\Gamma_Z\right)}\,\epsilon^*_\mu(k)\,\bar{v}^r(p_2)\,\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$

$$= \underline{C^{12,AB}_{Z,\ell_i}(q)\,\epsilon^*_\mu(k)\,\bar{v}^r(p_2)\,\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)}\,,\qquad C^{12,AB}_{Z,\ell_i}(q)=\frac{2e^3Z^{AB}_{\ell_i}}{s_W^2c_W^2\left(q^2-m_Z^2+im_Z\Gamma_Z\right)}$$

↑ FeynCalc ✓

List of all matrix elements:

$$\mathcal{M}_{\gamma}^{r\tilde{l}_1} = C^{AB}_{\gamma, q\tilde{l}_i}(q, q')(k_1 + q')^{\rho}\, \epsilon^{*}_{\rho}(k)\, \bar{v}^{r}(p_2)(\not{q}' - \not{k}_2)\, u^{s}(p_1)$$

$$\mathcal{M}_{\gamma}^{r\tilde{l}_2} = C^{AB}_{\gamma, q\tilde{l}_i}(q, q'')(q'' + k_2)^{\rho}\, \epsilon^{*}_{\rho}(k)\, \bar{v}^{r}(p_2)(\not{q}'' - \not{k}_1)\, u^{s}(p_1)$$

$$\mathcal{M}_{\gamma}^{r\tilde{l}_{1,2}} = - C^{12}_{\gamma, q}(q)\, \epsilon^{*}_{\mu}(k)\, \bar{v}^{r}(p_2)\, \gamma^{\mu}\, u^{s}(p_1)$$

$$\mathcal{M}_{Z}^{r\tilde{l}_1} = C^{AB}_{Z, \tilde{l}_i}(q, q', m_{\tilde{l}_{iB}})(k_1 + q')^{\rho}\, \epsilon^{*}_{\rho}(k)\, \bar{v}^{r}(p_2)(\not{k}_2 - \not{q}')\left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right] u^{s}(p_1)$$

$$\mathcal{M}_{Z}^{r\tilde{l}_2} = C^{AB}_{Z, \tilde{l}_i}(q, q'', m_{\tilde{l}_{iA}})(q'' + k_2)^{\rho}\, \epsilon^{*}_{\rho}(k)\, \bar{v}^{r}(p_2)(\not{k}_1 - \not{q}'')\left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right] u^{s}(p_1)$$

$$\mathcal{M}_{Z}^{r\tilde{l}_{1,2}} = C^{12,AB}_{Z, l_i}(q)\, \epsilon^{*}_{\mu}(k)\, \bar{v}^{r}(p_2)\, \gamma^{\mu}\left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right] u^{s}(p_1)$$

List of all prefactors:

$$C^{AB}_{\gamma, q\tilde{l}_i}(q, q') = \frac{Q_q e^3 \delta^{AB}}{q^2 (q'^2 - m^2_{\tilde{l}_{iA}})}$$

$$C^{AB}_{\gamma, q\tilde{l}_i}(q, q'') = \frac{Q_q e^3 \delta^{AB}}{q^2 (q'' - m^2_{\tilde{l}_{iA}})}$$

$$C^{12}_{\gamma, q}(q) = \frac{2 Q_q e^3 \delta^{AB}}{q^2}$$

$$C^{AB}_{Z, \tilde{l}_i}(q, q', m_{\tilde{l}_{iB}}) = \frac{e^3 Z^{AB}_{l_i}}{s_W^2 c_W^2 (q^2 - m_Z^2 + i m_Z \Gamma_Z)(q'^2 - m^2_{\tilde{l}_{iB}})}$$

$$C^{AB}_{Z, \tilde{l}_i}(q, q'', m_{\tilde{l}_{iA}}) = \frac{e^3 Z^{AB}_{l_i}}{s_W^2 c_W^2 (q^2 - m_Z^2 + i m_Z \Gamma_Z)(q'' - m^2_{\tilde{l}_{iA}})}$$

$$C^{12,AB}_{Z, l_i}(q) = \frac{2 e^3 Z^{AB}_{l_i}}{s_W^2 c_W^2 (q^2 - m_Z^2 + i m_Z \Gamma_Z)}$$

Rewrite γ-prefactors in terms of $C^{AB}_{\gamma,q}(q) = \dfrac{Q_q e^3 \delta^{AB}}{q^2}$:

$$C^{AB}_{\gamma,q\tilde{\ell}_i}(q,q') = \frac{1}{q'^2 - m^2_{\tilde{\ell}_{iB}}} C^{AB}_{\gamma,q}(q)$$

(Changed $m^2_{\tilde{\ell}_{iA}}$ to $m^2_{\tilde{\ell}_{iB}}$ to match Z-prefactors. Can do this since δ^{AB})

$$C^{AB}_{\gamma,q\tilde{\ell}_i}(q,q'') = \frac{1}{q''^2 - m^2_{\tilde{\ell}_{iA}}} C^{AB}_{\gamma,q}(q)$$

$$C^{12}_{\gamma,q}(q) = 2\,C^{AB}_{\gamma,q}(q)$$

Rewrite Z-prefactors in terms of $C^{AB}_{Z,l_i}(q) = \dfrac{e^3 Z^{AB}_{\ell_i}}{s^2_W c^2_W (q^2 - m^2_Z + i m_Z \Gamma_Z)}$:

$$C^{AB}_{Z,\tilde{\ell}_i}(q,q',m_{\tilde{\ell}_{iB}}) = \frac{1}{q'^2 - m^2_{\tilde{\ell}_{iB}}} C^{AB}_{Z,\ell_i}(q)$$

$$C^{AB}_{Z,\tilde{\ell}_i}(q,q'',m_{\tilde{\ell}_{iA}}) = \frac{1}{q''^2 - m^2_{\tilde{\ell}_{iA}}} C^{AB}_{Z,\ell_i}(q)$$

$$C^{12,AB}_{Z,l_i}(q) = 2\,C^{AB}_{Z,\ell_i}(q)$$

Combine the matrix elements

$$\mathcal{M}^{r\tilde{\ell}}_\gamma = \mathcal{M}^{r\tilde{\ell}_1}_\gamma + \mathcal{M}^{r\tilde{\ell}_2}_\gamma + \mathcal{M}^{r\tilde{\ell}_3}_\gamma$$

$$= C^{AB}_{\gamma,q\tilde{\ell}_i}(q,q')(k_1+q')^\rho \epsilon^*_\rho(k)\, \bar{v}^r(p_2)(\not{q}'-\not{k}_2)\, u^s(p_1)$$

$$C^{AB}_{\gamma,q\tilde{\ell}_i}(q,q'')(q''+k_2)^\rho \epsilon^*_\rho(k)\, \bar{v}^r(p_2)(\not{q}''-\not{k}_1)\, u^s(p_1)$$

$$- C^{12}_{\gamma,q}(q)\, \epsilon^*_\mu(k)\, \bar{v}^r(p_2)\, \gamma^\mu u^s(p_1)$$