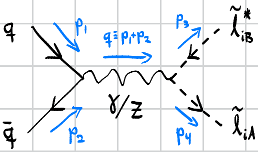
Slepton pair production to leading order

~~Brute-force (no help)~~ ← var planen i starten, men endte opp med å følge Tores metode

Two LO diagrams:



i = flavor

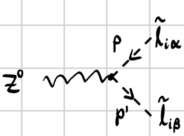
A, B = mass eigenstates

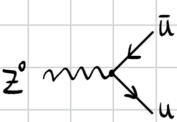
Consider $u\bar{u} \to \tilde{l}_{iA}\tilde{l}^*_{iB}$

Relevant Feynman rules:

$\gamma\tilde{l}_{i\alpha}\tilde{l}_{i\alpha}$ vertex :  $= ie(p+p')^\mu$

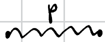
$\gamma u\bar{u}$ vertex :  $= -iee_u\gamma^\mu$

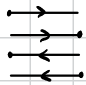
$Z\tilde{l}_{i\alpha}\tilde{l}_{i\beta}$ vertex :  $= \dfrac{ig}{\cos\theta_W} Z^{\alpha\beta}_{e_i}(p+p')^\mu$

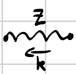
$Zu\bar{u}$ vertex :  $= \dfrac{ig}{\cos\theta_W}\gamma^\mu\left[Z_{u_L}(1-\gamma_5) + Z_{u_R}(1+\gamma_5)\right]$

(Richardson A.5)

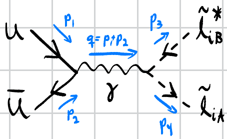
External scalar: $= 1$ (Schwartz 9.2.1)

γ propagator:  $= \frac{-ig_{\mu\nu}}{p^2+i\epsilon}$

External fermions:  $= \begin{matrix}\bar u(p)\\ u(p)\\ v(p)\\ \bar v(p)\end{matrix}$ (QFT-1 formula sheet)

Z propagator:  $= \frac{-i}{k^2-m_Z^2}\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2-\xi m_Z^2}(1-\xi)\right]$ (Peskin & Schroeder (21.54))

Matrix element from photon:

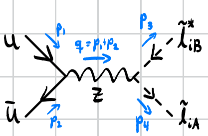
$i\mathcal{M}_\gamma =$ 

$$\cancel{i}\mathcal{M}_\gamma = \bar v^r(p_2)(+\cancel{i}ee_u\gamma^\mu)u^s(p_1)\frac{-\cancel{i}g_{\mu\nu}}{q^2+i\epsilon}\,\cancel{i}e(p_3+p_4)^\nu\,\delta_{AB}$$

$$\mathcal{M}_\gamma = -\frac{e^2e_u}{q^2+i\epsilon}g_{\mu\nu}(p_3+p_4)^\nu\,\bar v^r(p_2)\gamma^\mu u^s(p_1)\,\delta_{AB}$$

corrections

Matrix element from Z-boson:

$i\mathcal{M}_Z =$ 

$g = -\frac{e}{\sin\theta_W}$?

$$\cancel{i}\mathcal{M}_Z = \bar v^r(p_2)\frac{\cancel{i}g}{\cos\theta_W}\gamma^\mu\left[Z_{u_L}(1-\gamma^5)+Z_{u_R}(1+\gamma^5)\right]u^s(p_1)$$

$$\cdot\frac{+\cancel{i}}{q^2-m_Z^2 \underset{+im_Z\Gamma_Z\,?}{}}\left[g^{\mu\nu} - \cancel{\frac{k^\mu k^\nu}{k^2-\xi m_Z^2}(1-\xi)}\,?\right]\frac{\cancel{i}g}{\cos\theta_W}\,Z^{\alpha\beta}_{\cancel{e_i}\,(l_i\,?)}(p_3 \overset{-\,?}{+} p_4)^\nu$$

$g = -\frac{e}{\sin\theta_W}$?

Takes the finite lifetime of the Z-boson into account through decay width Γ_Z - GPT

Figure text A.7 in Richardson! Momenta in Feynman rules are to be taken in direction of arrows!

Corrected by comparison with Tore:

$$i\mathcal{M}_Z = \bar{v}^r(p_2)\,\frac{-ie}{\sin\theta_W\cos\theta_W}\,\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$
$$\times\;\frac{-ig_{\mu\nu}}{Q^2-m_Z^2+im_Z\Gamma_Z}\,\frac{-ie}{\sin\theta_W\cos\theta_W}\,Z^{AB}_{\ell_i}\,(p_3-p_4)^\nu$$

$$i\mathcal{M}_\gamma = \bar{v}^r(p_2)(iQ_q e\gamma^\mu)u^s(p_1)\,\frac{-ig_{\mu\nu}}{Q^2}\,(-ie)(p_3-p_4)^\nu\,\delta_{AB}$$

The differential cross section is given by

Schwartz

$$d\hat{\sigma} = \frac{1}{(2E_1)(2E_2)|\vec{v}_1-\vec{v}_2|}\,|\mathcal{M}|^2\,d\Pi$$

Tore (why?)

$$= \frac{1}{2\hat{s}}\,|\mathcal{M}|^2\,d\Pi$$

Assume $\sqrt{\hat{s}} \gg m_q$ and choose c.o.m. frame:

$$\hat{s} = Q^2 = 2p_1\cdot p_2 = 2(E_1E_2-\vec{p}_1\cdot\vec{p}_2) = 2(E_1E_2+|\vec{p}_1||\vec{p}_2|)$$

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + 2\,\mathrm{Re}\{\mathcal{M}_\gamma^* \mathcal{M}_Z\}$$

$$\cancel{i}\mathcal{M}_\gamma = \bar{v}^r(p_2)(\cancel{i}Q_q e \gamma^\mu) u^s(p_1) \frac{-\cancel{i} g_{\mu\nu}}{Q^2} (+\cancel{i}e)(p_3-p_4)^\nu \delta^{AB}$$

$$\mathcal{M}_\gamma = -\frac{Q_q e^2}{Q^2}(p_3-p_4)_\mu \bar{v}^r(p_2)\gamma^\mu u^s(p_1)\,\delta^{AB}$$

slutt å glemme denne!

Litt for vant til $e^+e^- \to e^+e^-$ og lignende med 4 spinorer, men skal kun summere over s og r her!

$$\langle|\mathcal{M}_\gamma|^2\rangle = \frac{1}{4N}\frac{Q_q^2 e^4}{(Q^2)^2}(p_3-p_4)_\mu (p_3-p_4)_\nu \sum_{s,\cancel{s'},r,\cancel{r'}=1}^{2} \bar{u}^{s}_\alpha(p_1)\gamma^\nu_{\alpha\beta} v^{r}_\beta(p_2)\bar{v}^r_\gamma(p_2)\gamma^\mu_{\gamma\delta} u^s_\delta(p_1)\,\delta^{AB}$$

$$= \frac{1}{4N}\frac{Q_q^2 e^4}{(Q^2)^2}(p_3-p_4)_\mu (p_3-p_4)_\nu \,\mathrm{Tr}\{(\not{p}_1+m_q)\gamma^\nu(\not{p}_2-m_q)\gamma^\mu\}\,\delta^{AB}$$

Her bruker Tore $\frac{1}{4N}$ (dividing by number of spin states (4) and color states (N))

$$= \frac{1}{4N}\frac{Q_q^2 e^4}{(Q^2)^2}\delta^{AB}(p_3-p_4)_\mu (p_3-p_4)_\nu \left[\mathrm{Tr}\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} - \cancel{m_q^2\,\mathrm{Tr}\{\gamma^\mu\gamma^\nu\}}\right]$$

Can be excluded? ($m_q^2 \simeq 0$)

Note: For $m_q^2 \simeq 0$ ligner dette på Tores $L_{\mu\nu(\gamma)} H^{\mu\nu}_{(\gamma)}$

$$\cancel{i}\mathcal{M}_Z = \bar{v}^r(p_2)\frac{+\cancel{i}e}{\sin\theta_W\cos\theta_W}\gamma^\mu\left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$
$$\times \frac{+\cancel{i} g_{\mu\nu}}{Q^2 - m_Z^2 + i m_Z \Gamma_Z}\frac{+\cancel{i}e}{\sin\theta_W\cos\theta_W} Z^{AB}_{l_i}(p_3-p_4)^\nu$$

$$\mathcal{M}_Z = \frac{e^2}{\sin^2\theta_W\cos^2\theta_W}\frac{1}{Q^2-m_Z^2+im_Z\Gamma_Z} Z^{AB}_{l_i}(p_3-p_4)_\mu$$
$$\times \bar{v}^r(p_2)\gamma^\mu\left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$

Richardson: $Z^{AB}_{l_i} = \frac{1}{2}\left(L^{2i-1}_{1\alpha} L^{2i-1}_{1\beta} - 2\sin^2\theta_W \delta_{AB}\right)$ $\rightsquigarrow$ Usikker på hva $L^{2i-1}_{1\alpha}$ er, men GPT sier "mixing matrix"-elemente. Antar ihvertfall matriseelementer i videre utregninger, så behandler $Z^{AB}_{l_i}$ som "bare tall" (muligens komplekse)

↳ sjekk (4.3) til Tore

$$\gamma^{5\dagger} = (i\gamma^0\gamma^1\gamma^2\gamma^3)^\dagger = -i\gamma^{3\dagger}\gamma^{2\dagger}\gamma^{1\dagger}\gamma^{0\dagger}$$
$$= -i(-\gamma^3)(-\gamma^2)(-\gamma^1)\gamma^0$$
$$= i\gamma^3\gamma^2\gamma^1\gamma^0$$
$$= -i\gamma^0\gamma^3\gamma^2\gamma^1$$
$$= -i\gamma^0\gamma^1\gamma^3\gamma^2$$
$$= i\gamma^0\gamma^1\gamma^2\gamma^3$$
$$= \gamma^5$$

$$\left.\begin{aligned}
\gamma^5\gamma^0 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 = -i\gamma^0\gamma^5 \\
\gamma^5\gamma^1 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^1 = -i\gamma^1\gamma^5 \\
\gamma^5\gamma^2 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^2 = -i\gamma^2\gamma^5 \\
\gamma^5\gamma^3 &= i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^3 = -i\gamma^3\gamma^5
\end{aligned}\right\} \quad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$$

$$(\bar{v}\gamma^\mu\gamma^5 u)^\dagger = u^\dagger\gamma^0\gamma^0\gamma^{5\dagger}\gamma^{\mu\dagger}\bar{v}^\dagger$$
$$= \bar{u}\gamma^0\gamma^5\gamma^0\gamma^\mu\cancel{\gamma^0\gamma^0}v$$
$$= -\bar{u}\gamma^5\gamma^\mu v$$
$$= \bar{u}\gamma^\mu\gamma^5 v$$

$Z_{q_L}, Z_{q_R} \in \mathbb{R}$

$$\langle|\mathcal{M}_Z|^2\rangle = \frac{e^4}{\cos^4\theta_W \sin^4\theta_W}\frac{1}{|Q^2-m_Z^2+im_Z\Gamma_Z|^2}\left|Z^{AB}_{\ell_i}\right|^2 (p_3-p_4)_\mu(p_3-p_4)_\nu$$
$$\times \frac{1}{4N}\sum_{s,r=1}^{2}\bar{v}^r_\alpha(p_2)\gamma^\mu_{\alpha\beta}\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]_{\beta\gamma}u^s_\gamma(p_1)$$
$$\times\ \bar{u}^s_\delta(p_1)\gamma^\nu_{\delta\varepsilon}\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]_{\varepsilon\zeta}v^r_\zeta(p_2)$$

Fra Tore.
Fra antall spin-tilstander inn (4)
og antall farge-tilstander inn (N)
Men må se mer på dette!

$$= \frac{e^4}{\cos^4\theta_W \sin^4\theta_W}\frac{1}{|Q^2-m_Z^2+im_Z\Gamma_Z|^2}\left|Z^{AB}_{\ell_i}\right|^2 (p_3-p_4)_\mu(p_3-p_4)_\nu\,\frac{1}{4N}\,(\not{p}_2-m_q)_{\zeta\alpha}\gamma^\mu_{\alpha\beta}$$
$$\times\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]_{\beta\gamma}(\not{p}_1+m_q)_{\gamma\delta}\gamma^\nu_{\delta\varepsilon}\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]_{\varepsilon\zeta}$$

$$= \frac{e^4}{\cos^4\theta_W \sin^4\theta_W}\frac{1}{|Q^2-m_Z^2+im_Z\Gamma_Z|^2}\left|Z^{AB}_{\ell_i}\right|^2 (p_3-p_4)_\mu(p_3-p_4)_\nu\,\frac{1}{4N}$$
$$\times\Big[\mathrm{Tr}\left\{\not{p}_2\gamma^\mu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\not{p}_1\gamma^\nu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\right\}$$
$$\cancel{+\ m_q^2\,\mathrm{Tr}\left\{\gamma^\mu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\gamma^\nu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\right\}}\Big]$$

Can be excluded? $(m_q^2 \simeq 0)$

Note: Med $m_q^2 \simeq 0$ ligner dette veldig på Tores $L_{\mu\nu(Z)}H^{\mu\nu}_{(Z)}$

Tore har $(Z^{AB}_{\ell_i})^2$, ikke $|Z^{AB}_{\ell_i}|^2$, så $Z^{AB}_{\ell_i}$ er kanskje reell?

$$\mathcal{M}_\gamma = -\frac{Q_q e^2}{Q^2}(p_3-p_4)_\mu \bar{v}^r(p_2)\gamma^\mu u^s(p_1)\delta^{AB}$$

$$\mathcal{M}_Z = \frac{e^2}{\sin^2\theta_W\cos^2\theta_W}\frac{1}{Q^2-m_Z^2+im_Z\Gamma_Z} Z_{l_i}^{AB}(p_3-p_4)_\mu$$
$$\times \bar{v}^r(p_2)\gamma^\mu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]u^s(p_1)$$

$$\langle \mathcal{M}_\gamma^\dagger \mathcal{M}_Z\rangle = -\frac{Q_q e^2}{Q^2}\frac{e^2}{\sin^2\theta_W\cos^2\theta_W}\frac{1}{Q^2-m_Z^2+im_Z\Gamma_Z} Z_{l_i}^{AB}\delta^{AB}(p_3-p_4)_\mu(p_3-p_4)_\nu \frac{1}{4N}$$
$$\times \sum_{s,r=1}^{2}\bar{u}^s_\alpha(p_1)\gamma^\mu_{\alpha\beta}v^r_\beta(p_2)\bar{v}^r_\gamma(p_2)\gamma^\nu_{\gamma\delta}\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right]_{\delta\epsilon}u^s_\epsilon(p_1)$$

$$= -\frac{Q_q e^2}{Q^2}\frac{e^2}{\sin^2\theta_W\cos^2\theta_W}\frac{1}{Q^2-m_Z^2+im_Z\Gamma_Z} Z_{l_i}^{AB}\delta^{AB}(p_3-p_4)_\mu(p_3-p_4)_\nu \frac{1}{4N}$$
$$\times \mathrm{Tr}\left\{(\not{p}_2-m_q)\gamma^\nu\left[Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right](\not{p}_1+m_q)\gamma^\mu\right\}$$

Use $(p_3-p_4)_\mu(p_3-p_4)_\nu = (p_3-p_4)_\nu(p_3-p_4)_\mu$, then relabel $\mu\leftrightarrow\nu$

$$= -\frac{Q_q e^2}{Q^2}\frac{e^2}{\sin^2\theta_W\cos^2\theta_W}\frac{1}{Q^2-m_Z^2+im_Z\Gamma_Z} Z_{l_i}^{AB}\delta^{AB}(p_3-p_4)_\mu(p_3-p_4)_\nu \frac{1}{4N}$$
$$\times\Big[\mathrm{Tr}\left\{\not{p}_2\gamma^\mu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\not{p}_1\gamma^\nu\right\}$$
$$\cancel{-m_q^2\,\mathrm{Tr}\left\{\gamma^\mu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\gamma^\nu\right\}}\Big]$$

Again, excluded from $m_q^2\simeq 0$?

Note: Med $m_q^2\simeq 0$ ligner dette veldig på Tores $L_{\mu\nu(\gamma Z)}H^{\mu\nu}_{(\gamma Z)}$

We consider the c.o.m. frame, specifically:

$$p_1 = \left(\frac{\sqrt{\hat{s}}}{2}, 0, 0, \frac{\sqrt{\hat{s}}}{2}\right)$$
$$p_2 = \left(\frac{\sqrt{\hat{s}}}{2}, 0, 0, -\frac{\sqrt{\hat{s}}}{2}\right)$$

$$p_3 = \left(E_B, p\sin\theta, 0, p\cos\theta\right)$$
$$p_4 = \left(E_A, -p\sin\theta, 0, -p\cos\theta\right)$$

Conservation of 4-mom. gives

$$E_B + E_A = \sqrt{m^2_{\tilde{l}_{iB}} + p^2} + \sqrt{m^2_{\tilde{l}_{iA}} + p^2} = \sqrt{\hat{s}} = Q$$

$$\sqrt{m^2_{\tilde{l}_{iA}} + p^2} = Q - \sqrt{m^2_{\tilde{l}_{iB}} + p^2}$$

$$m^2_{\tilde{l}_{iA}} + \cancel{p^2} = Q^2 + m^2_{\tilde{l}_{iB}} + \cancel{p^2} - 2Q\sqrt{m^2_{\tilde{l}_{iB}} + p^2}$$

$$\sqrt{m^2_{\tilde{l}_{iB}} + p^2} = \frac{Q}{2} + \frac{m^2_{\tilde{l}_{iB}} - m^2_{\tilde{l}_{iA}}}{2Q}$$

$$m^2_{\tilde{l}_{iB}} + p^2 = \frac{Q^2}{4} + \frac{\left(m^2_{\tilde{l}_{iB}} - m^2_{\tilde{l}_{iA}}\right)^2}{4Q^2} + \frac{1}{2}\left(m^2_{\tilde{l}_{iB}} - m^2_{\tilde{l}_{iA}}\right)$$

$$p^2 = \frac{Q^2}{4} - \frac{m^2_{\tilde{l}_{iA}} + m^2_{\tilde{l}_{iB}}}{2} + \frac{\left(m^2_{\tilde{l}_{iA}} - m^2_{\tilde{l}_{iB}}\right)^2}{4Q^2}$$

$$\Longrightarrow \quad E_{B,A} = \sqrt{m^2_{\tilde{l}_{iB,A}} + p^2}$$
$$= \sqrt{m^2_{\tilde{l}_{iB,A}} + \frac{Q^2}{4} - \frac{m^2_{\tilde{l}_{iA}} + m^2_{\tilde{l}_{iB}}}{2} + \frac{\left(m^2_{\tilde{l}_{iA}} - m^2_{\tilde{l}_{iB}}\right)^2}{4Q^2}}$$

$$= \sqrt{\frac{Q^2}{4} + \frac{m^2_{\tilde{\ell}_i B,A} - m^2_{\tilde{\ell}_i A,B}}{2} + \frac{(m^2_{\tilde{\ell}_i A} - m^2_{\tilde{\ell}_i B})^2}{4Q^2}}$$

$$= \frac{1}{2Q}\sqrt{Q^4 + 2Q^2(m^2_{\tilde{\ell}_i B,A} - m^2_{\tilde{\ell}_i A,B}) + (m^2_{\tilde{\ell}_i A} - m^2_{\tilde{\ell}_i B})^2}$$

$$= \frac{1}{2Q}\left(Q^2 + (m^2_{\tilde{\ell}_i B,A} - m^2_{\tilde{\ell}_i A,B})\right)$$

$$\boxed{E_{B,A} = \frac{Q^2 + m^2_{\tilde{\ell}_i B,A} - m^2_{\tilde{\ell}_i A,B}}{2Q}}$$

specifically:
$p_1=\left(\frac{\sqrt{\hat{s}}}{2},0,0,\frac{\sqrt{\hat{s}}}{2}\right)$, $p_2=\left(\frac{\sqrt{\hat{s}}}{2},0,0,-\frac{\sqrt{\hat{s}}}{2}\right)$
$p_3=(E_B, p\sin\theta, 0, p\cos\theta)$,
$p_4=(E_A, -p\sin\theta, 0, -p\cos\theta)$
with $p=|\vec{p}_3|=|\vec{p}_4|$

Consider c.o.m. frame with $m_q^2 \simeq 0$. Then

$$\hat{s}=(p_1+p_2)^2 = 2\,p_1\cdot p_2 = 2(E_1E_2+|\vec{p}_1||\vec{p}_2|) = 4|\vec{p}_1|^2$$

$$\Rightarrow \quad 2E_1 2E_2 |\vec{v}_1-\vec{v}_2| = 4|\vec{p}_1|^2\left(\frac{|\vec{p}_1|}{E_1}+\frac{|\vec{p}_2|}{E_2}\right) = 4|\vec{p}_1|^2\cdot 2 = 8|\vec{p}_1|^2 = 2\hat{s}$$

$$\Rightarrow \quad d\hat{\sigma} = \frac{1}{2E_1 2E_2|\vec{v}_1-\vec{v}_2|}\langle|\mathcal{M}|^2\rangle\, d\Pi$$
$$= \underline{\frac{1}{2\hat{s}}\langle|\mathcal{M}|^2\rangle\, d\Pi}$$

with

$$d\Pi = (2\pi)^4\delta^{(4)}(p_3+p_4-p_1-p_2)\frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_B}\frac{d^3p_4}{(2\pi)^4}\frac{1}{2E_A}$$

If we generalize to the case where we can have a third particle with momentum $k^\mu=(\omega,\vec{k})$ (massless, so $k^2=0$) which radiates off the interaction (thus appearing in the final state), the phase space is extended to

$$d\Pi = (2\pi)^4\delta^{(4)}(k+p_3+p_4-p_1-p_2)\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}\frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_B}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2E_A}$$

In d dimensions this generalizes to

$$d\Pi = (2\pi)^d \, \delta^{(d)}(k+p_3+p_4-p_1-p_2) \, \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\omega} \frac{d^{d-1}p_3}{(2\pi)^{d-1}} \frac{1}{2E_B} \frac{d^{d-1}p_4}{(2\pi)^{d-1}} \frac{1}{2E_A}$$

Fra Tore, men gir mening

We introduce two factors that integrate to unity:

$$\frac{d^d q}{(2\pi)^d} (2\pi)^d \, \delta^{(d)}(k+q-p_1-p_2)$$

~~Disse er vel kun "unity" når vi inkluderer begge?~~

nevermind!

$$\delta(q^2-Q^2)\, dQ^2$$

we get:

$$d\Pi = (2\pi)^d \, \delta^d(k+p_3+p_4-p_1-p_2) \, \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\omega} \frac{d^{d-1}p_3}{(2\pi)^{d-1}} \frac{1}{2E_B} \frac{d^{d-1}p_4}{(2\pi)^{d-1}} \frac{1}{2E_A}$$
$$\times \frac{d^d q}{(2\pi)^d} (2\pi)^d \, \delta^d(k+q-p_1-p_2)\, \delta(q^2-Q^2)\, dQ^2$$

$\delta(q+k-p_1-p_2)$

$\Rightarrow q = p_1+p_2-k$

$\Rightarrow \delta(p_3+p_4-p_1-p_2+k)$

$= \delta(p_3+p_4-q)$

$$= (2\pi)^d \, \delta^d(q+k-p_1-p_2) \, \frac{d^d q}{(2\pi)^d} \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\omega} \, \delta(q^2-Q^2)\, dQ^2$$
$$\times (2\pi)^d \, \delta^d(p_3+p_4-q) \, \frac{d^{d-1}p_3}{(2\pi)^{d-1}} \frac{1}{2E_B} \frac{d^{d-1}p_4}{(2\pi)^{d-1}} \frac{1}{2E_A}$$

We now make use of the following identity:

$$\delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|}$$

where the sum is over all zero-points of f, i.e., $f(x_i)=0$.

Using the identity on $\delta(q^2-Q^2)$ we can rewrite it as:

$$\delta(q^2-Q^2) = \delta(q^{0^2} - q^i q_i - Q^2)$$
$$= \frac{\delta\left(q^0 - \sqrt{q^i q_i + Q^2}\right)}{2|q^0|} + \frac{\delta\left(q^0 + \sqrt{q^i q_i + Q^2}\right)}{2|q^0|}$$

In our case we consider q to be the 4-momentum of a massive on-shell boson with mass Q, so the $\delta\left(q^0 + \sqrt{q^i q_i + Q^2}\right)$ term is unphysical (q^0 is the energy of the boson, so we must have $q^0 > 0$). We are thus left with only the first term. Also, since $q^0 > 0$ we can set $|q^0| = q^0$.

From this we find

riktig argumentasjon?

$$d^d q\, \delta(q^2-Q^2) = d^{d-1}q\, dq^0 \frac{1}{2q^0}\, \delta\left(q^0 - \sqrt{q^i q_i + Q^2}\right)$$
$$= d^{d-1} q\, \frac{1}{2q^0}$$

where we use that $dq^0\, \delta\left(q^0 - \sqrt{q^i q_i + Q^2}\right)$ integrates to unity when we interpret q the way we do $\left(q^{0^2} = E_q^2 = \vec{q}^{\,2} + m_q^2 = q^i q_i + Q^2\right)$.

We can then rewrite our phase space:

$$d\Pi = (2\pi)^d \delta^d(q+k-p_1-p_2)\,\frac{d^d q}{(2\pi)^d}\,\frac{d^{d-1}k}{(2\pi)^{d-1}}\,\frac{1}{2\omega}\,\delta(q^2-Q^2)\,dQ^2$$
$$\times\ (2\pi)^d \delta^d(p_3+p_4-q)\,\frac{d^{d-1}p_3}{(2\pi)^{d-1}}\,\frac{1}{2E_B}\,\frac{d^{d-1}p_4}{(2\pi)^{d-1}}\,\frac{1}{2E_A}$$

$$= \frac{1}{2\pi}(2\pi)^d \delta^d(q+k-p_1-p_2)\,\frac{d^{d-1}q}{(2\pi)^{d-1}}\,\frac{1}{2q^0}\,\frac{d^{d-1}k}{(2\pi)^{d-1}}\,\frac{1}{2\omega}$$
$$\times\ (2\pi)^d \delta^d(p_3+p_4-q)\,\frac{d^{d-1}p_3}{(2\pi)^{d-1}}\,\frac{1}{2E_B}\,\frac{d^{d-1}p_4}{(2\pi)^{d-1}}\,\frac{1}{2E_A}\,dQ^2$$

$$= \frac{1}{2\pi}\,d\Pi_L\,d\Pi_H\,dQ^2$$

with

$$d\Pi_L = (2\pi)^d \delta^d(p_3+p_4-q)\,\frac{d^{d-1}p_3}{(2\pi)^{d-1}}\,\frac{1}{2E_B}\,\frac{d^{d-1}p_4}{(2\pi)^{d-1}}\,\frac{1}{2E_A}$$

$$d\Pi_H = (2\pi)^d \delta^d(q+k-p_1-p_2)\,\frac{d^{d-1}q}{(2\pi)^{d-1}}\,\frac{1}{2q^0}\,\frac{d^{d-1}k}{(2\pi)^{d-1}}\,\frac{1}{2\omega}$$

Remembering the amplitude contributions (with $m_q^2 \approx 0$):

$$\langle|\mathcal{M}_\gamma|^2\rangle = \frac{1}{4N}\,\frac{Q_q^2 e^4}{(Q^2)^2}\,\delta^{AB}(p_3-p_4)_\mu(p_3-p_4)_\nu\left[\mathrm{Tr}\left\{\not p_2\gamma^\mu \not p_1\gamma^\nu\right\}\right]$$

$$\langle|\mathcal{M}_Z|^2\rangle = \frac{e^4}{\cos^4\theta_W\sin^4\theta_W}\,\frac{1}{|Q^2-m_Z^2+im_Z\Gamma_Z|^2}\,|Z_{\ell_i}^{AB}|^2(p_3-p_4)_\mu(p_3-p_4)_\nu\,\frac{1}{4N}$$
$$\times\left[\mathrm{Tr}\left\{\not p_2\gamma^\mu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\not p_1\gamma^\nu\left(Z_{q_L}(1-\gamma^5)+Z_{q_R}(1+\gamma^5)\right)\right\}\right]$$

$\mathcal{M}$ scalar!

$$\langle \mathcal{M}_\gamma^{\dagger *} \mathcal{M}_Z \rangle = -\frac{Q_q e^2}{Q^2} \frac{e^2}{\sin^2\theta_w \cos^2\theta_w} \frac{1}{Q^2 - m_Z^2 + i m_Z \Gamma_Z} Z_{\ell_i}^{AB} \delta^{AB} (p_3 - p_4)_\mu (p_3 - p_4)_\nu \frac{1}{4N}$$
$$\times \left[Tr \left\{ \not{p}_2 \gamma^\mu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \not{p}_1 \gamma^\nu \right\} \right]$$

We find the three contributions to the differential cross section:

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \langle |\mathcal{M}|^2 \rangle d\Pi$$
$$= \underbrace{\frac{1}{2\hat{s}} \langle |\mathcal{M}_\gamma|^2 \rangle d\Pi}_{d\sigma_\gamma} + \underbrace{\frac{1}{2\hat{s}} \langle |\mathcal{M}_Z|^2 \rangle d\Pi}_{d\sigma_Z} + \underbrace{\frac{1}{\hat{s}} Re \left\{ \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle \right\} d\Pi}_{d\sigma_{\gamma Z}}$$

$$d\sigma_\gamma = \frac{1}{\cancel{2}\hat{s}} \frac{1}{4N} \frac{Q_q^2 \overset{4\pi\alpha^2}{\cancel{e^4}}}{(Q^2)^2} \delta^{AB} (p_3 - p_4)_\mu (p_3 - p_4)_\nu \left[Tr \left\{ \not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \right\} \right] \frac{1}{\cancel{2\pi}} d\Pi_L d\Pi_H dQ^2$$

$$\frac{d\sigma_\gamma}{dQ^2} = \frac{4\pi\alpha^2}{\hat{s}} \frac{Q_q^2}{(Q^2)^2} \underbrace{\left[\frac{1}{4N} \int d\Pi_H \, Tr \left\{ \not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \right\} \right]}_{\equiv H_{(\gamma)}^{\mu\nu}} \delta^{AB} \underbrace{\left[\int d\Pi_L (p_3 - p_4)_\mu (p_3 - p_4)_\nu \right]}_{\equiv L_{\mu\nu}}$$

$$= \frac{4\pi\alpha^2}{\hat{s}} \frac{Q_q^2}{(Q^2)^2} H_{(\gamma)}^{\mu\nu} L_{(\gamma)\mu\nu} \quad , \quad L_{(\gamma)}^{\mu\nu} \equiv \delta^{AB} L^{\mu\nu}$$

$$= \frac{4\pi\alpha^2}{\hat{s}} P_\gamma H_{(\gamma)}^{\mu\nu} L_{(\gamma)\mu\nu} \quad , \quad P_\gamma = \frac{Q_q^2}{(Q^2)^2}$$

Flytter dQ^2 til venstresiden, men integrerer over $d\Pi_H$ og $d\Pi_L$.
Hvorfor er dette lov?

$$d\sigma_Z = \frac{1}{\cancel{2}\hat{s}} \frac{\cancel{e^4}\ 4\pi\alpha^2}{\cos^4\theta_W \sin^4\theta_W} \frac{1}{|Q^2 - m_Z^2 + i m_Z \Gamma_Z|^2} |Z_{\ell_i}^{AB}|^2 (p_3 - p_4)_\mu (p_3 - p_4)_\nu \frac{1}{4N}$$

$$\times \left[\mathrm{Tr}\left\{ \not{p}_2 \gamma^\mu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \not{p}_1 \gamma^\nu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \right\} \right] \frac{1}{\cancel{2\pi}} d\Pi_H \, d\Pi_L \, dQ^2$$

$$\frac{d\sigma_Z}{dQ^2} = \frac{4\pi\alpha^2}{\hat{s}} \frac{1}{\cos^4\theta_W \sin^4\theta_W} \frac{1}{|Q^2 - m_Z^2 + i m_Z \Gamma_Z|^2} (Z_{\ell_i}^{AB})^2 \left[\int d\Pi_L (p_3 - p_4)_\mu (p_3 - p_4)_\nu \right]$$

$$\times \underbrace{\frac{1}{4N} \int d\Pi_H \, \mathrm{Tr}\left\{ \not{p}_2 \gamma^\mu \left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right] \not{p}_1 \gamma^\nu \left[Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right] \right\}}_{H^{\mu\nu}_{(Z)}}$$

$$\frac{d\sigma_Z}{dQ^2} = \frac{4\pi\alpha^2}{\hat{s}} P_Z H^{\mu\nu}_{(Z)} L_{(Z)\mu\nu} \quad , \quad P_Z = \frac{1}{\cos^4\theta_W \sin^4\theta_W} \frac{1}{|Q^2 - m^2 + i m_Z \Gamma_Z|^2}$$

$$L^{\mu\nu}_{(Z)} = (Z_{\ell_i}^{AB})^2 L^{\mu\nu}$$

$$2\,\mathrm{Re}\{\langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle\} = \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle + \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle^*$$

Need to find $\langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle^*$, so need to find the complex conjugate of the trace appearing in $\langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle$:

$$\mathrm{Tr}\left\{ \left[\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \right]^\dagger \right\}$$

$$= \mathrm{Tr}\left\{ \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \gamma^0 \gamma^\nu \cancel{\gamma^0 \gamma^0} \not{p}_1 \cancel{\gamma^0 \gamma^0} \gamma^\mu \cancel{\gamma^0 \gamma^0} \not{p}_2 \gamma^0 \right\}$$

$$= \mathrm{Tr}\left\{ \gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2 \gamma^0 \left(Z_{q_L}(1+\gamma^5) + Z_{q_R}(1\mp\gamma^5) \right) \gamma^0 \right\}$$

$$= \mathrm{Tr}\left\{ \gamma^\nu \not{p}_1 \gamma^\mu \not{p}_2 \left(Z_{q_L}(1+\gamma^5) + Z_{q_R}(1-\gamma^5) \right) \right\}$$

+ cyclic

$$= \mathrm{Tr}\left\{ \not{p}_2 \gamma^\nu \not{p}_1 \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \gamma^\mu \right\}$$

$$= \mathrm{Tr}\left\{ \not{p}_2 \gamma^\nu \not{p}_1 \gamma^\mu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \right\}$$

Using this we find

$$2\,\mathrm{Re}\left\{ \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle \right\} = \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle + \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle^*$$

$$= -\frac{Q_q e^2}{Q^2} \frac{e^2}{\sin^2\theta_W \cos^2\theta_W} \frac{1}{Q^2 - m_Z^2 + i m_Z \Gamma_Z} Z_{\ell_i}^{AB} \delta^{AB} (p_3 - p_4)_\mu (p_3 - p_4)_\nu \frac{1}{4N}$$

$$\times \mathrm{Tr}\left\{ \not{p}_2 \gamma^\mu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \not{p}_1 \gamma^\nu \right\}$$

$$- \frac{Q_q e^2}{Q^2} \frac{e^2}{\sin^2\theta_W \cos^2\theta_W} \frac{1}{Q^2 - m_Z^2 - i m_Z \Gamma_Z} Z_{\ell_i}^{AB} \delta^{AB} (p_3 - p_4)_{\cancel{\mu}\,\nu} (p_3 - p_4)_{\cancel{\nu}\,\mu} \frac{1}{4N}$$

$$\times \mathrm{Tr}\left\{ \not{p}_2 \gamma^{\cancel{\nu}\,\mu} \not{p}_1 \gamma^{\cancel{\mu}\,\nu} \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \right\}$$

$$= -\frac{Q_q e^4}{Q^2} \frac{1}{\sin^2\theta_W \cos^2\theta_W} \left(\frac{1}{Q^2 - m_Z^2 + i m_Z \Gamma_Z} + \frac{1}{Q^2 - m_Z^2 - i m_Z \Gamma_Z} \right) Z_{\ell_i}^{AB} \delta^{AB} (p_3 - p_4)_\mu (p_3 - p_4)_\nu \frac{1}{4N}$$

$$\times \mathrm{Tr}\left\{ \not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \right\}$$

$$\Rightarrow \quad \frac{d\sigma_{\gamma Z}}{dQ^2} = \frac{1}{2\hat{s}} 2\,\mathrm{Re}\left\{ \langle \mathcal{M}_\gamma^* \mathcal{M}_Z \rangle \right\} d\Pi$$

$$= -\frac{1}{\cancel{2}\hat{s}} \frac{Q_q \cancel{e^4}^{\,4\pi\alpha^2}}{Q^2} \frac{1}{\sin^2\theta_W \cos^2\theta_W} \left(\frac{1}{Q^2 - m_Z^2 + i m_Z \Gamma_Z} + \frac{1}{Q^2 - m_Z^2 - i m_Z \Gamma_Z} \right) Z_{\ell_i}^{AB} \delta^{AB} \int d\Pi_L (p_3 - p_4)_\mu (p_3 - p_4)_\nu$$

$$\times \frac{1}{4N} \int d\Pi_H \, \mathrm{Tr}\left\{ \not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5) \right) \right\} \frac{1}{\cancel{2\pi}}$$

$$= \frac{4\pi\alpha^2}{\hat{s}} P_{\gamma Z} H^{\mu\nu}_{(\gamma Z)} L_{(\gamma Z)\mu\nu}$$

with $P_{\gamma Z} = -\frac{Q_q}{Q^2} \frac{1}{\sin^2\theta_W \cos^2\theta_W} \frac{1}{2}\left(\frac{1}{Q^2-m_Z^2+im_Z\Gamma_Z} + \frac{1}{Q^2-m_Z^2-im_Z\Gamma_Z}\right)$

$$L^{\mu\nu}_{(\gamma Z)} = Z^{AB}_{l_i} \delta^{AB} L^{\mu\nu}$$

$$H^{\mu\nu}_{(\gamma Z)} = \frac{1}{2N} \int d\Pi_H \, Tr\left\{ \not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right)\right\}$$

Putting all expressions together we get

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{\hat{s}} \left(P_\gamma H^{\mu\nu}_{(\gamma)} L_{(\gamma)\mu\nu} + P_Z H^{\mu\nu}_{(Z)} L_{(Z)\mu\nu} + P_{\gamma Z} H^{\mu\nu}_{(\gamma Z)} L_{(\gamma Z)\mu\nu}\right)$$

Will now rewrite $H^{\mu\nu}_{(i)}$ by noting that they are all proportional to $H^{\mu\nu} \equiv H^{\mu\nu}_{(\gamma)}$. We must rewrite the traces:

$$Tr\left\{ \not{p}_2 \gamma^\mu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right) \not{p}_1 \gamma^\nu \left(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5)\right)\right\}$$

$$= Tr\left\{ \not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu \left(Z_{q_L} + Z_{q_R} + (Z_{q_R} - Z_{q_L})\gamma^5\right)\left(Z_{q_L} + Z_{q_R} + (Z_{q_R} - Z_{q_L})\gamma^5\right)\right\}$$

$$= (Z_{q_L}+Z_{q_R})^2 \, Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} + 2(Z_{q_L}+Z_{q_R})(Z_{q_R}-Z_{q_L}) \, Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\gamma^5\}$$

$$+ (Z_{q_R}-Z_{q_L})^2 \, Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu \cancel{\gamma^5\gamma^5}\}$$

$(\gamma^5)^2 = 1$

$Tr\{\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^5\} = -4i\epsilon^{\rho\mu\sigma\nu}$

$$= \left[(Z_{q_L}+Z_{q_R})^2 + (Z_{q_R}-Z_{q_L})^2\right] Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} - 8i(Z_{q_R}^2 - Z_{q_L}^2)\, p_{2\rho} p_{1\sigma} \epsilon^{\rho\mu\sigma\nu}$$

$$= 2(Z_{q_L}^2 + Z_{q_R}^2) Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} + \cancel{8i(Z_{q_L}^2 - Z_{q_R}^2)\, p_{2\rho} p_{1\sigma} \epsilon^{\rho\mu\sigma\nu}}$$

Will have $H^{\mu\nu}L_{\mu\nu}$, where $L_{\mu\nu}$ sym. and $\epsilon^{\rho\mu\sigma\nu}$ anti sym.

Thus

$$H^{\mu\nu}_{(Z)} = 2(Z_{q_L}^2 + Z_{q_R}^2) H^{\mu\nu}$$

Similarly:

$$Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu(Z_{q_L}(1-\gamma^5) + Z_{q_R}(1+\gamma^5))\}$$

$$= (Z_{q_L}+Z_{q_R}) Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} + (Z_{q_R}-Z_{q_L}) Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\gamma^5\}$$

$$= (Z_{q_L}+Z_{q_R}) Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} + (Z_{q_R}-Z_{q_L})\, p_{2\rho} p_{1\sigma} Tr\{\gamma^5\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\}$$

$Tr\{\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^5\} = -4i\epsilon^{\rho\mu\sigma\nu}$

$$= (Z_{q_L}+Z_{q_R}) Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\} \cancel{- (Z_{q_R}-Z_{q_L})\, p_{2\rho} p_{1\sigma} 4i\epsilon^{\rho\mu\sigma\nu}}$$

$\epsilon^{\rho\mu\sigma\nu}$ anti sym. under $\mu\leftrightarrow\nu$ while $L_{\mu\nu}$ sym. under $\mu\leftrightarrow\nu$

$$= (Z_{q_L}+Z_{q_R}) Tr\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\}$$

Thus (remember $\frac{1}{2N} = 2\,\frac{1}{4N}$):

$$H^{\mu\nu}_{(\gamma Z)} = 2(Z_{q_L}+Z_{q_R}) H^{\mu\nu}$$

Using these new $L^{\mu\nu}_{(i)}$ and $H^{\mu\nu}_{(i)}$ we find

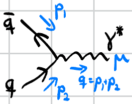
$$
\begin{aligned}
\frac{d\sigma}{dQ^2} &= \frac{4\pi\alpha^2}{\hat{s}}\left(P_\gamma H^{\mu\nu}_{(\gamma)} L_{(\gamma)\mu\nu} + P_Z H^{\mu\nu}_{(Z)} L_{(Z)\mu\nu} + P_{\gamma Z} H^{\mu\nu}_{(\gamma Z)} L_{(\gamma Z)\mu\nu}\right) \\
&= \frac{4\pi\alpha^2}{\hat{s}}\left[P_\gamma \delta^{AB} H^{\mu\nu} L_{\mu\nu} + P_Z \left(Z^{AB}_{l_i}\right)^2 2\left(Z^2_{q_L}+Z^2_{q_R}\right) H^{\mu\nu} L_{\mu\nu} + P_{\gamma Z} Z^{AB}_{l_i} \delta^{AB} 2\left(Z_{q_L}+Z_{q_R}\right) H^{\mu\nu} L_{\mu\nu}\right] \\
&= \frac{4\pi\alpha^2}{\hat{s}}\left[\frac{Q_q^2}{Q^4}\delta^{AB} - \frac{Q_q}{Q^2}\frac{1}{\sin^2\theta_W\cos^2\theta_W}\frac{Q^2-m_Z^2}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2} Z^{AB}_{l_i}\delta^{AB} 2\left(Z_{q_L}+Z_{q_R}\right)\right. \\
&\qquad \left. + \frac{1}{\cos^4\theta_W\sin^4\theta_W}\frac{1}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left(Z^{AB}_{l_i}\right)^2 2\left(Z^2_{q_L}+Z^2_{q_R}\right)\right] H^{\mu\nu} L_{\mu\nu} \\
&= \frac{4\pi\alpha^2}{\hat{s}Q^4} F^{AB}_{ql_i}(Q) H^{\mu\nu} L_{\mu\nu}
\end{aligned}
$$

with

$$
\begin{aligned}
F^{AB}_{ql_i} \equiv\ & Q_q^2\delta^{AB} - 2Q_q\delta^{AB}\frac{Z^{AB}_{l_i}}{\sin^2\theta_W\cos^2\theta_W}\left(Z_{q_L}+Z_{q_R}\right)\frac{Q^2(Q^2-m_Z^2)}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2} \\
& + 2\frac{\left(Z^{AB}_{l_i}\right)^2}{\sin^4\theta_W\cos^4\theta_W}\left(Z^2_{q_L}+Z^2_{q_R}\right)\frac{Q^4}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}
\end{aligned}
$$

We define the matrix element $\mathcal{M}(q\bar{q}\to\gamma^*+X)$ such that the $q\bar{q}\gamma^*$ vertex is given only by γ^μ, i.e.:

$$i\mathcal{M}(q\bar{q}\to\gamma^*+X) =$$



$$= \bar{v}^s(p_1)\gamma^\mu u^r(p_2)\,\epsilon^*_\mu(q)$$

$$\Rightarrow \sum_X \langle|\mathcal{M}(q\bar{q}\to\gamma^*+X)|^2\rangle = \frac{1}{4N}\sum_{s,r}\epsilon_\mu(q)\epsilon^*_\nu(q)\,\bar{v}^s(p_1)\gamma^\nu u^r(p_2)\bar{u}^r(p_2)\gamma^\mu v^s(p_1)$$

$$= \epsilon_\mu(q)\epsilon^*_\nu(q)\frac{1}{4N}\operatorname{Tr}\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\}$$

We must sum over all X since we want the resulting contribution independent of other remnants than γ^*.

$$\epsilon_\mu\epsilon^*_\nu H^{\mu\nu} = \int d\Pi_H\,\epsilon_\mu\epsilon^*_\nu\frac{1}{4N}\operatorname{Tr}\{\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\}$$

$$= \int d\Pi_H \sum_X \langle|\mathcal{M}(q\bar{q}\to\gamma^*+X)|^2\rangle$$

Skjønner ikke egentlig dette! Hvorfor $\sum_X$? Hvor blir det av q-avhengigheten til ϵ_μ og ϵ^*_ν? Hvordan ta hensyn til X generelt?

Expression for $L_{\mu\nu}$:

$$L^{\mu\nu} = \int d\Pi_L \, (p_3-p_4)^\mu (p_3-p_4)^\nu$$

p_3, p_4 integrated over, so $L^{\mu\nu}$ can only depend on q^μ:

$$L^{\mu\nu} = L_1(Q)\, q^\mu q^\nu + L_2(Q)\, g^{\mu\nu} Q^2$$

By contracting this we get two coupled equations:

$$(1) \quad q_\mu q_\nu L^{\mu\nu} = Q^4 (L_1 + L_2)$$

$$(2) \quad g_{\mu\nu} L^{\mu\nu} = Q^2 (L_1 + d L_2)$$

d = dimension, comes from $g_{\mu\nu} g^{\mu\nu} = \delta^\mu_\mu = d$

Taking $(1) - \frac{Q^2}{d}(2)$ gives

$$q_\mu q_\nu L^{\mu\nu} - \frac{Q^2}{d} g_{\mu\nu} L^{\mu\nu} = Q^4(L_1 + \cancel{L_2}) - Q^4\left(\frac{1}{d}L_1 + \cancel{L_2}\right)$$

$$= Q^4\left(1 - \frac{1}{d}\right) L_1$$

$$q_\mu q_\nu L^{\mu\nu} - \frac{Q^2}{d} g_{\mu\nu} L^{\mu\nu} = Q^4 \frac{d-1}{d} L_1$$

$$L_1 = \frac{1}{1-d} \frac{1}{Q^2} g_{\mu\nu} L^{\mu\nu} - \frac{d}{1-d} \frac{1}{Q^4} q_\mu q_\nu L^{\mu\nu}$$

Taking (1) $- Q^2$ (2) gives

$$q_\mu q_\nu L^{\mu\nu} - Q^2 g_{\mu\nu} L^{\mu\nu} = Q^4(\cancel{L_1} + L_2) - Q^4(\cancel{L_1} + dL_2)$$

$$q_\mu q_\nu L^{\mu\nu} - Q^2 g_{\mu\nu} L^{\mu\nu} = Q^4(1-d)L_2$$

$$\underline{L_2 = \frac{1}{1-d}\frac{1}{Q^4} q_\mu q_\nu L^{\mu\nu} - \frac{1}{1-d}\frac{1}{Q^2} g_{\mu\nu} L^{\mu\nu}}$$

Consider the integrands of the LHS of both equations:

$$g_{\mu\nu}(p_3-p_4)^\mu(p_3-p_4)^\nu = (p_3-p_4)^2$$

$$= (E_B - E_A, 2p\sin\theta, 0, 2p\cos\theta)^2$$

$$= (E_B - E_A)^2 - 4p^2$$

$$= \cancel{\left(\frac{\cancel{Q^2} + m^2_{\tilde{l}_iB} - m^2_{\tilde{l}_iA} - \cancel{Q^2} - \cancel{m^2_{\tilde{l}_iA}} + \cancel{m^2_{\tilde{l}_iB}}}{\cancel{2}Q}\right)^2} - Q^2 + 2(m^2_{\tilde{l}_iA} + m^2_{\tilde{l}_iB}) - \cancel{\frac{(m^2_{\tilde{l}_iA} - m^2_{\tilde{l}_iB})^2}{Q^2}}$$

$$\underline{= 2(m^2_{\tilde{l}_iA} + m^2_{\tilde{l}_iB}) - Q^2}$$

$$q_\mu q_\nu (p_3-p_4)^\mu (p_3-p_4)^\nu = \left[(p_1+p_2)\cdot(p_3-p_4)\right]^2$$

$$= \left[(Q,0,0,0)\cdot(E_B-E_A, 2p\sin\theta, 0, 2p\cos\theta)\right]^2$$

$$= \left[Q(E_B-E_A)\right]^2$$

$$= Q^2(E_B-E_A)^2 = \frac{1}{\cancel{4}}\left(\cancel{Q^2} + m^2_{\tilde{l}_iB} - m^2_{\tilde{l}_iA} - \cancel{Q^2} - \cancel{m^2_{\tilde{l}_iA}} + \cancel{m^2_{\tilde{l}_iB}}\right)^2$$

$$\underline{= (m^2_{\tilde{l}_iB} - m^2_{\tilde{l}_iA})^2}$$

Inserting this into L_1 and L_2 gives

$$L_1 = \frac{1}{1-d}\frac{1}{Q^2} g_{\mu\nu} L^{\mu\nu} - \frac{d}{1-d}\frac{1}{Q^4} q_\mu q_\nu L^{\mu\nu}$$

$$= \frac{1}{1-d}\frac{1}{Q^2}\left[2\left(m^2_{\tilde{\ell}iA} + m^2_{\tilde{\ell}iB}\right) - Q^2\right]\int d\Pi_L - \frac{d}{1-d}\frac{1}{Q^4}\left(m^2_{\tilde{\ell}iB} - m^2_{\tilde{\ell}iA}\right)^2 \int d\Pi_L$$

$$L_1 = \frac{1}{1-d}\left[\frac{2\left(m^2_{\tilde{\ell}iA} + m^2_{\tilde{\ell}iB}\right) - Q^2}{Q^2} - d\,\frac{\left(m^2_{\tilde{\ell}iB} - m^2_{\tilde{\ell}iA}\right)^2}{Q^4}\right]\int d\Pi_L$$

$$L_2 = \frac{1}{1-d}\frac{1}{Q^4} q_\mu q_\nu L^{\mu\nu} - \frac{1}{1-d}\frac{1}{Q^2} g_{\mu\nu} L^{\mu\nu}$$

$$= \frac{1}{1-d}\left[\frac{\left(m^2_{\tilde{\ell}iB} - m^2_{\tilde{\ell}iA}\right)^2}{Q^4} - \frac{2\left(m^2_{\tilde{\ell}iA} + m^2_{\tilde{\ell}iB}\right) - Q^2}{Q^2}\right]\int d\Pi_L$$

$$= \frac{1}{1-d}\frac{4}{Q^2}\left[\frac{Q^2}{4} - \frac{\left(m^2_{\tilde{\ell}iA} + m^2_{\tilde{\ell}iB}\right)}{2} + \frac{\left(m^2_{\tilde{\ell}iB} - m^2_{\tilde{\ell}iA}\right)^2}{4Q^2}\right]\int d\Pi_L$$

$$L_2 = \frac{4}{1-d}\frac{p^2}{Q^2}\int d\Pi_L$$

Need to evaluate phase-space integral $\int d\Pi_L$:

$$\int d\Pi_L = \int (2\pi)^d \delta^d(p_3 + p_4 - q)\frac{d^{d-1}p_3}{(2\pi)^{d-1}}\frac{1}{2E_B}\frac{d^{d-1}p_4}{(2\pi)^{d-1}}\frac{1}{2E_A}$$

See "proofs & derivations" note with $|\vec{p}_3| = p$, $E_{cm} = Q$ and omitting Heaviside

$$= \int d\Omega_{d-1}\frac{1}{2^d \pi^{d-2}}\frac{p^{d-3}}{Q}$$

$$= \frac{p^{d-3}}{2^d \pi^{d-2} Q} \int d\Omega_{d-1} \qquad \Big| \quad \Omega_{d-1} = \int d\Omega_{d-1} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}$$

$$= \frac{p^{d-3}}{2^d \pi^{d-2} Q} \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}$$

$$= \underline{\frac{p^{d-3}}{2^{d-1} \pi^{(d-3)/2} \Gamma(\frac{d-1}{2}) Q}}$$

In Tore's master thesis he only considers QCD corrections, which don't appear in the slepton part, so it's safe to set $d=4$, giving

$$\int d\Pi_L = \frac{p}{8\sqrt{\pi}\,\Gamma(\frac{3}{2}) Q} \qquad \Big| \quad \Gamma(\tfrac{3}{2}) = \frac{\sqrt{\pi}}{2}$$

$$= \underline{\frac{p}{4\pi Q}}$$

Note: $L^{\mu\nu}$ as written now does not satisfy the Ward identity. Including the δ^{AB} from $F^{AB}_{q\tilde{\ell}_i}(Q)$ gives $A=B$, thus

$$L_1 = \frac{1}{1-d} \frac{2(m^2_{\tilde{l}_iA} + m^2_{\tilde{l}_iA}) - Q^2}{Q^2} \int d\Pi_L$$

But δ^{AB} is not included in all terms in $F^{AB}_{q\tilde{\ell}_i}(Q)$?

$$= -\frac{1}{1-d} \frac{4}{Q^2} \left[\frac{Q^2}{4} - \frac{m^2_{\tilde{l}_iA} + m^2_{\tilde{l}_iA}}{2} \right] \int d\Pi_L$$

$$= -\frac{4}{1-d} \frac{p^2}{Q^2} \int d\Pi_L$$

$$L_1 = -L_2$$

and so the Ward identity is satisfied

$$
\begin{aligned}
q_\mu L^{\mu\nu} &= q_\mu q^\mu q^\nu L_1 - q_\mu g^{\mu\nu} Q^2 L_1 \\
&= Q^2 q^\nu L_1 - Q^2 q^\nu L_1 \\
&= 0
\end{aligned}
$$

Since $q_\mu \mathcal{M}^\mu(q\bar{q} \to \gamma^* + X) = 0$ by the Ward identity, where $\epsilon^*_\mu(q)\mathcal{M}^\mu = \mathcal{M}$, we have that the first term in $L^{\mu\nu}$ does not contribute when contracted with $H^{\mu\nu}$:

$$
\begin{aligned}
H^{\mu\nu} L_1(Q)\, q_\mu q_\nu &= L_1(Q) \int d\Pi_H \sum_X q_\mu q_\nu \left\langle \mathcal{M}^\mu(q\bar{q} \to \gamma^* + X)\, \mathcal{M}^\nu(q\bar{q} \to \gamma^* + X)^* \right\rangle \\
&= L_1(Q) \int d\Pi_H \sum_X \left\langle q_\mu \mathcal{M}^\mu (q_\nu \mathcal{M}^\nu)^* \right\rangle \\
&= 0
\end{aligned}
$$

Riktig? Hvor kommer gauge symmetri inn?

Thus we can write

$$
L_{\mu\nu} = L_2(Q)\, g_{\mu\nu} Q^2
$$

$$= \frac{4}{1-d} \frac{p^2}{Q^2} g_{\mu\nu} Q^2 \int d\Pi_L \qquad \Big| d=4$$

$$= -\frac{4}{3} p^2 g_{\mu\nu} \frac{p}{4\pi Q}$$

$$= -\frac{p^3}{3\pi Q} g_{\mu\nu}$$

sum included in this notation?

Using that the sum of photon polarizations is $\epsilon_\mu^* \epsilon_\nu = -g_{\mu\nu}$:

$$H^{\mu\nu} L_{\mu\nu} = -\frac{p^3}{3\pi Q} g_{\mu\nu} H^{\mu\nu}$$

$$= \frac{p^3}{3\pi Q} \epsilon_\mu \epsilon_\nu^* H^{\mu\nu}$$

$$= \frac{p^3}{3\pi Q} \int d\Pi_H \sum_X \langle |\mathcal{M}(q\bar{q} \to \gamma^* + X)|^2 \rangle$$

and so the differential cross section becomes

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{4\pi\alpha^2}{\hat{s} Q^4} F^{AB}_{q_{k_i}}(Q) H^{\mu\nu} L_{\mu\nu}$$

$$= \frac{4\cancel{\pi}\alpha^2}{\hat{s} Q^4} F^{AB}_{q_{k_i}}(Q) \frac{p^3}{3\cancel{\pi} Q} \int d\Pi_H \sum_X \langle |\mathcal{M}(q\bar{q} \to \gamma^* + X)|^2 \rangle$$

$$\boxed{\frac{d\hat{\sigma}}{dQ^2} = \frac{4\alpha^2 p^3}{3\hat{s} Q^5} F^{AB}_{q_{k_i}}(Q) \int d\Pi_H \sum_X \langle |\mathcal{M}(q\bar{q} \to \gamma^* + X)|^2 \rangle}$$

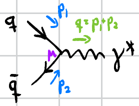
Leading order cross section

No divergences at leading order, so $d=4$. Tree-level diagram for

not $+X$?

$q\bar{q} \to \gamma^*$:

color conservation

$$i\mathcal{M}_0 = $$ 

$$= \bar{v}^r(p_2)\gamma^\mu u^s(p_1)\,\epsilon^*_\mu(q)\,\delta^{AB}$$

Note from future: A,B brukes allerede for masse-egentilstande! Bruke andre indekser for å unngå forvirring!

Riktig? eller setter man bare $\frac{1}{N_c}$ i starten og ignorerer δ-ene?

spinsum, color sum, $(\not{p}_1+m)\gamma^\nu(\not{p}_2-m)\gamma^\mu\,(-g_{\mu\nu})$, $-g_{\mu\nu}$, $\delta^{AA}=\cancel{N_c}$

$$\Rightarrow \langle|\mathcal{M}_0|^2\rangle = \frac{1}{4N_c^{\cancel{2}}}\sum_{s,r}\cancel{\sum_{\lambda,\lambda'}}\cancel{\sum_{A,B}}\bar{v}^r(p_2)\gamma^\mu u^s(p_1)\bar{u}^s(p_1)\gamma^\nu v^r(p_2)\,\cancel{\epsilon^{(\lambda)*}_\mu(q)\,\epsilon^{(\lambda')*}_\nu(q)}\,\cancel{\delta^{AB}\delta^{AB}}$$

initial state

polarization sum

$$= -\frac{1}{4N_c}\,\mathrm{Tr}\{\not{p}_2\gamma^\mu\not{p}_1\gamma_\mu\}$$

$$= -\frac{1}{4N_c}\,p_{2\rho}p_{1\sigma}\,\mathrm{Tr}\{\gamma^\rho\underbrace{\gamma^\mu\gamma^\sigma\gamma_\mu}_{-(d-2)\gamma^\sigma}\}$$

$$= \frac{d-2}{4N_c}\,p_{2\rho}p_{1\sigma}\,\underbrace{\mathrm{Tr}\{\gamma^\rho\gamma^\sigma\}}_{4g^{\rho\sigma}}$$

$$= \frac{d-2}{N_c}\,(p_1\cdot p_2) \qquad \Big|\ s=(p_1+p_2)^2=2p_1\cdot p_2$$

$$= \frac{d-2}{2N_c}\,\hat{s}$$

no radiating particle

$$\Rightarrow \int d\Pi_H\,\langle|\mathcal{M}_0|^2\rangle = \frac{d-2}{2N_c}\,\hat{s}\int\frac{d^{d-1}q}{(2\pi)^{d-1}}\frac{1}{2q^0}\,\cancel{\frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{2\omega}}\,(2\pi)^d\delta^d(q+\cancel{k}-p_1-p_2)\ \Big|\ q^0=\sqrt{s}$$

$$= \frac{d-2}{2N_c}\,\frac{q^0}{2}\int\frac{d^{d-1}q}{\cancel{(2\pi)^{d-1}}}\,(2\pi)\,\delta(q^0\underbrace{-p_1^0-p_2^0}_{-\sqrt{s}})\,\cancel{(2\pi)^{d-1}}\,\delta^{d-1}(\vec{q}-\vec{p}_1-\vec{p}_2)$$

$$= \frac{d-2}{2N_c}\, q^0 \tilde{\Pi}\, \delta\left(q^0 - \sqrt{\hat{s}}\right)$$

We define $z \equiv \frac{Q^2}{\hat{s}}$ and use the composition identity of $\delta(f(z))$ with $f(z) = q^0 - \sqrt{\hat{s}} = \sqrt{\hat{s}}\left(\frac{q^0}{\sqrt{\hat{s}}} - 1\right) = \frac{Q}{\sqrt{z}}\left(\sqrt{z} - 1\right)$, giving

$$f(z) \overset{!}{=} 0 \Rightarrow \sqrt{z} = 1 \Rightarrow z = 1$$

$$f'(z) = -\frac{Q}{2}\frac{1}{z\sqrt{z}}\left(\sqrt{z}-1\right) + \frac{Q}{\sqrt{z}}\frac{1}{2\sqrt{z}} = \cancel{-\frac{1}{2}\frac{Q}{z}} + \frac{Q}{2z\sqrt{z}} \cancel{+\frac{1}{2}\frac{Q}{z}} = \frac{Q}{2z\sqrt{z}}$$

$$\Rightarrow \delta\left(q^0 - \sqrt{\hat{s}}\right) = \frac{\delta(z-1)}{\frac{Q}{2z\sqrt{z}}} = \frac{2z\sqrt{z}}{Q}\,\delta(z-1) = 2\frac{Q^2}{\hat{s}}\frac{\cancel{Q}}{\sqrt{\hat{s}}}\frac{1}{\cancel{Q}}\,\delta(z-1) = \frac{2Q^2}{\hat{s}\sqrt{\hat{s}}}\,\delta(z-1)$$

Inserting this back into the phase space integral gives

$$\int d\Pi_H \left\langle |\mathcal{M}_0|^2 \right\rangle = \frac{d-2}{2N_c}\, q^0 \tilde{\Pi}\, \frac{2Q^2}{\hat{s}\sqrt{\hat{s}}}\,\delta(z-1)$$

| $q^0 = Q$, and $Q = \sqrt{\hat{s}}$, by requirement of the δ-function.

$$= \frac{d-2}{\cancel{2}N_c}\, \cancel{Q}\, \tilde{\Pi}\, \frac{\cancel{2}\cancel{Q^2}}{\cancel{Q^2}\cancel{Q}}\,\delta(z-1)$$

$$= \frac{d-2}{N_c}\, \tilde{\Pi}\, \delta(z-1)$$

The differential cross section then becomes

$$\frac{d\hat{\sigma}_0^d}{dQ^2} = \frac{4\alpha^2\rho^3}{3\hat{s}Q^5} F^{AB}_{q\bar{q}i}(Q) \int d\Pi_H \sum_X \left\langle \left|\mathcal{M}(q\bar{q} \to \gamma^* + X)\right|^2 \right\rangle$$

$$= \frac{4\alpha^2 p^3}{3\hat{s}Q^5} F^{AB}_{q_{\ell i}}(Q) \frac{d-2}{N_c} \tilde{\pi} \delta(1-z)$$

$$= (d-2) \frac{4\pi\alpha^2 p^3}{3N_c\hat{s}Q^5} F^{AB}_{q_{\ell i}}(Q) \delta(1-z)$$

Tore mangler α^2, men den skal vel være der?

Setting $d=4$ gives

$$\frac{d\hat{\sigma}_0}{dQ^2} = \frac{8\pi\alpha^2 p^3}{3N_c\hat{s}Q^5} F^{AB}_{q_{\ell i}}(Q)\delta(1-z)$$

$$\equiv \sigma_0 \delta(1-z)$$

Can also rewrite the general case in terms of σ_0:

$$\frac{d\hat{\sigma}_0^d}{dQ^2} = \frac{d-2}{2} \sigma_0 \delta(1-z)$$

$$\equiv \sigma_0^d \delta(1-z)$$

Er det ikke rart med δ-funksjon? Er det greit fordi den uansett forsvinner når vi integrerer over Q^2?

Hvorfor hatt på LHS?