# POD-RBF

POD is used to build the model that calculates the boundary displacement field caused by unknown cracks within a limited range of sizes and orientations. Using a set of boundary displacements fields issued from finite element simulations, called the snapshot matrix U:

$$\mathbf{U} = \begin{bmatrix} u_1^1 & u_1^2 & \cdots & u_1^S \\ u_2^1 & u_2^2 & & u_2^S \\ \vdots & \vdots & \ddots & \vdots \\ u_N^1 & u_N^2 & \cdots & u_N^S \end{bmatrix} \tag{1}$$

Where N is the total number of sensor points, which are the points on the boundary where the displacement is measured. And S is the number of snapshot vectors $\mathbf{u}_i$, each representing the field caused by a crack with different length and orientation. On the other hand, the matrix $\mathbf{P}$ stores parameters of these known cracks i.e. Lengths and orientations.

The main purpose of POD is to extract a set of orthogonal vectors $\mathbf{\Phi}$, called POD basis vectors. Where $\mathbf{\Phi}$ is related to the matrix $\mathbf{U}$ by the following linear relationship:

$$\mathbf{U} = \mathbf{\Phi} \cdot \mathbf{A} \tag{2}$$

Thus, according to the orthogonally of $\mathbf{\Phi}$, the amplitude matrix A can be computed from:

$$\mathbf{A} = \mathbf{\Phi}^{\mathrm{T}} \cdot \mathbf{U} \tag{3}$$

These vectors can also be calculated through the POD operation [1, 2], by extracting the eigenvectors of the covariance matrix $\mathbf{C}$

$$\mathbf{C} = \mathbf{U}^{\mathrm{T}} \cdot \mathbf{U} \tag{4}$$

$$\mathbf{\Phi} = \mathbf{U} \cdot \mathbf{V} \cdot \mathbf{\Lambda}^{-1/2}, \tag{5}$$

Where $\mathbf{V}$ is the matrix storing the normalized eigenvectors of the covariance matrix $\mathbf{C}$, and $\mathbf{\Lambda}$ the diagonal matrix storing its eigenvalues. This method gives eigenvalues that are stored in a descending order. Also called POD directions. And they represent the original data.

Because lower values POD directions hold very little representation, they can be sacrificed in order to reduce the size of the model. With negligible influence the accuracy of the representation. This step is called the truncation. Consequently, the amplitude matrix $\widehat{\mathbf{A}}$ is given by:

$$\widehat{\mathbf{A}} = \widehat{\mathbf{\Phi}}^{\mathrm{T}} \cdot \mathbf{U} \tag{6}$$

Where $\widehat{\mathbf{\Phi}}$ is the sub-space extracted out from $\mathbf{\Phi}$ by preserving the first K (K $\ll$ S) columns of $\mathbf{\Phi}$ that correspond to the largest eigenvalues. We can then reconstruct the original data using the reduced model.

$$\mathbf{U} = \widehat{\mathbf{\Phi}} \cdot \widehat{\mathbf{A}} \tag{7}$$

On the other hand, Radial Basis Function (RBF) allows the generation of the responses corresponding to new sets crack parameters by interpolation between values of the initial selection. This is done using the matrix $\mathbf{B}$ containing the coefficients calculated by the following equation:

$$\mathbf{B} = \mathbf{A} \cdot \mathbf{G}^{-1} \tag{8}$$

Where $\mathbf{A}$ is already calculated by equation (3) and **G** is defined as the matrix of interpolation functions $\mathrm{g_i(p)}$. In this work, the identity RBF been employed [3-5]:

$$\mathbf{g}_\mathrm{i} = |\mathbf{P} - \mathbf{P}_\mathrm{i}| \tag{9}$$

In Equation (9), the set of interpolation functions $\mathbf{g}_\mathrm{i}(\mathbf{P})$ is arbitrary chosen from the matrix **G** and $\mathbf{P}_\mathrm{i}$ is the parameter corresponding to $\mathbf{U}_\mathrm{i}$ (i=1,2,…,S). The argument of the i-th RBF is the distance$|\mathbf{P} - \mathbf{P}_\mathrm{i}|$, $\mathbf{P}$ and $\mathbf{P}_\mathrm{i}$ being respectively current and reference parameters.

After the evaluation of the coefficient matrix $\mathbf{B}$, a low-dimensional model of (8) can be put in vector form, as follows:

$$\mathbf{a}(\mathbf{P}) = \mathbf{B} \cdot \mathbf{g}(\mathbf{P}) \tag{10}$$

By defining the amplitude vector $\mathbf{a}(\mathbf{P})$ as a function of parameters **P**, the Equation (7) can be rewritten as the approximation of the snapshot $\mathbf{u}$ corresponding to a new crack parameter vector $\mathrm{P}$:

$$\mathbf{u}(\mathbf{P}) = \widehat{\mathbf{\Phi}} \cdot \mathbf{a}(\mathbf{P}) \tag{11}$$

The reduced model given by Equation (11) is defined as the POD-RBF network. It is able to reproduce unknown displacement fields corresponding to any set of crack parameter $\mathbf{P}$ However, it should be noted that this model can lead to results with weak precision when making extrapolation outside the range of known crack parameters **P**.

## References


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