

Canonical Ensemble

Closed Quantum System.  $\hat{H}$  is time independent  $\Rightarrow$  energy conserved.

Eigen states of Quantum Many Body.

Local subsystem thermal equilibrium  $\Rightarrow$  thermodynamics

Local subsystem thermal equilibrium in steady state, but not necessarily in equilibrium.

$$G = \sum_{\alpha} E_{\alpha} \sigma_{\alpha}^{\dagger} \sigma_{\alpha}$$

MBL system: closed system  $\Rightarrow$  decoherence critical.

$$S(\omega) = k \left( \frac{\omega_c}{\omega} + 1 \right) \quad \Gamma(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k \left( \frac{\omega_c}{\omega} + 1 \right) e^{i\omega\tau} d\omega$$

$$= \underbrace{\delta(\tau)}_{\text{white noise}} + \underbrace{\frac{k\omega_c}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega} e^{i\omega\tau} d\omega}_{?}$$

$$A(\tau) \int_{-\infty}^{\infty} \frac{1}{\omega} e^{i\omega\tau} d\omega, \quad \frac{dA}{d\tau} = \int_{-\infty}^{\infty} e^{i\omega\tau} d\omega = 2\pi \delta(\tau)$$

$\Rightarrow A(\tau) = 2\pi U(\tau)$ , step function.

$$\Rightarrow \Gamma(\tau) = \delta(\tau) + k\omega_c U(\tau) \dots (X)$$

$$S(\omega) = k \left( \frac{\omega_c}{\omega} + 1 \right) \Rightarrow k \sqrt{\left( \frac{\omega_c}{\omega} \right)^2 + 1} \quad / \quad k \left( \frac{\omega_c}{|\omega|} + 1 \right)$$

$$\Rightarrow \Gamma(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k \sqrt{\left( \frac{\omega_c}{\omega} \right)^2 + 1} e^{i\omega\tau} d\omega \dots \text{analytic \& solution of}$$

$$\Gamma(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k \left( \frac{\omega_c}{|\omega|} + 1 \right) e^{i\omega\tau} d\omega = \delta(\tau) + \frac{k\omega_c}{2\pi} \int_{-\infty}^{\infty} \frac{1}{|\omega|} e^{i\omega\tau} d\omega$$

$$\underbrace{\int_{-\infty}^{\infty} \frac{1}{|\omega|} e^{j\omega\tau} d\omega}_{\text{let } A(\tau)} = \int_0^{\infty} \frac{1}{\omega} e^{j\omega\tau} d\omega + \int_{-\infty}^0 -\frac{1}{\omega} e^{j\omega\tau} d\omega$$

let  $A(\tau)$

$$\frac{dA}{d\tau} = \int_0^{\infty} e^{j\omega\tau} d\omega + \int_{-\infty}^0 -e^{j\omega\tau} d\omega = \frac{1}{j\tau} \left( [e^{j\omega\tau}]_0^{\infty} \right) - \frac{1}{j\tau} \left( [e^{j\omega\tau}]_0^{-\infty} \right)$$

$$= \frac{1}{j\tau} [e^{j\omega\tau}]_0^{\infty} + \frac{1}{j\tau} [e^{j\omega\tau}]_0^{-\infty} = \frac{1}{j\tau} \left( \lim_{T \rightarrow \infty} (\cos(\omega\tau) + j\sin(\omega\tau)) \right)$$