Color of noise
- white : 1
- pink : $1/f$
- red : $1/f^2$
- purple : f^2

1. Spectral Density approach

$$S(\omega) = \frac{1}{|\omega|} \leftrightarrow \quad r(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)\,dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}\,dt \leftrightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}\,d\omega$$

$$S(\omega) = \int_{-\infty}^{\infty} r(\tau)e^{-j\omega\tau}\,d\tau = \lim_{T\to\infty}\frac{1}{2T}\int_{-\infty}^{\infty}\int_{-T}^{T} x(t)x(t+\tau)e^{-j\omega\tau}\,dt\,d\tau = \lim_{T\to\infty}\frac{1}{2T}\int_{-\infty}^{\infty} x(t+\tau)e^{-j\omega(t+\tau)}\,d\tau\int_{-\infty}^{\infty} x(t)e^{j\omega t}\,dt$$

$$\fallingdotseq \frac{1}{2T}X(\omega)X(-\omega) = \frac{1}{2T}|X(\omega)|^2 \Rightarrow X(\omega) \propto \frac{1}{\sqrt{|\omega|}}$$

2. Fourier Series Method

for long time T, Gaussian random fcn $y(t) = \sum_{k=1}^{\infty} a_k \cos 2\pi f_k t + b_k \sin 2\pi f_k t$

where $f_k = \frac{k}{T}$, a_k, b_k Gaussian random variable whose $\sigma = \sigma_k$

$$x(t) = y(t) * h(t) \quad \text{when} \quad H(\omega) = \frac{1}{\sqrt{|\omega|}}, \Rightarrow X(\omega) = Y(\omega)\underbrace{H(\omega)}_{\frac{1}{\sqrt{|\omega|}}}$$

$$\Rightarrow x(t) = \sum_{k=1}^{\infty} \frac{a_k}{\sqrt{|f_k|}}\cos 2\pi f_k t + \frac{b_k}{\sqrt{|f_k|}}\sin 2\pi f_k t$$

$$= \sum_{k=1}^{\infty} \frac{x_k}{\sqrt{|f_k|}}\sin(2\pi f_k t + \phi_k) \quad \text{when} \quad x_k = \sqrt{a_k^2 + b_k^2} \sim \chi(2), \quad \phi_k \sim U(0, 2\pi)$$

white noise 때서 더하기

$$x(t) = A\left(\sum_{k=1}^{\infty}\frac{x_k}{\sqrt{|f_k|}}\sin(2\pi f_k t + \phi_k)\right) + B\,n_k, \quad n_k \sim N(0,\sigma).$$

Power Spectrum of chsn ensemble average / $x(t)$ trace - ensemble average

time evolution.