1. Qubit not coupled with Resonator : driving

$$H = -\frac{\omega_q}{2}\sigma_z + A\sigma_x \cos(\omega_{rot} t + \phi), \quad \omega_{rot} = \omega_{rf}$$

$$R = e^{-i\frac{\omega_{rot}}{2} t \sigma_z} = e^{-i\frac{\omega_{rot}}{2} t (|0\rangle\langle 0| - |1\rangle\langle 1|)} = e^{-i\frac{\omega_{rot}}{2} t}|0\rangle\langle 0| + e^{+i\frac{\omega_{rot}}{2} t}|1\rangle\langle 1|$$

$$\dot{R} = -i\frac{\omega_{rot}}{2} e^{-i\frac{\omega_{rot}}{2} t}|0\rangle\langle 0| + i\frac{\omega_{rot}}{2} e^{i\frac{\omega_{rot}}{2} t}|1\rangle\langle 1|$$

$$i\dot{R}R^\dagger = \frac{\omega_{rot}}{2}(|0\rangle\langle 0| - |1\rangle\langle 1|) = \frac{\omega_{rot}}{2}\sigma_z$$

$$R\sigma_x R^\dagger = e^{-i\frac{\omega_{rot}}{2} t\sigma_z}\sigma_x e^{i\frac{\omega_{rot}}{2} t\sigma_z} = \left(e^{-i\frac{\omega_{rot}}{2} t}|0\rangle\langle 0| + e^{i\frac{\omega_{rot}}{2} t}|1\rangle\langle 1|\right)\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)\left(e^{i\frac{\omega_{rot}}{2} t}|0\rangle\langle 0| + e^{-i\frac{\omega_{rot}}{2} t}|1\rangle\langle 1|\right)$$

$$= e^{-i\omega_{rot} t}|0\rangle\langle 1| + e^{i\omega_{rot} t}|1\rangle\langle 0|$$

$$\tilde{H} = RHR^\dagger + i\dot{R}R^\dagger = \frac{\omega_{rot} - \omega_q}{2}\sigma_z + A\frac{e^{i(\omega_{rot} t + \phi)} + e^{-i(\omega_{rot} t + \phi)}}{2}\left(e^{-i\omega_{rot} t}|0\rangle\langle 1| + e^{i\omega_{rot} t}|1\rangle\langle 0|\right)$$

$$= \frac{\omega_{rot} - \omega_q}{2}\sigma_z + \frac{A}{2}\left(e^{i\phi}|0\rangle\langle 1| + e^{-i\phi}|1\rangle\langle 0|\right)$$

$\omega_{rot} = \omega_q$

$$\Rightarrow \tilde{H} = \frac{A}{2}(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y)$$

$$\phi = 0 \Rightarrow \tilde{H} = \frac{A}{2}\sigma_x$$

$$\phi = 90^\circ \Rightarrow \tilde{H} = \frac{A}{2}\sigma_y$$

$$e^{-i\frac{At}{2}X} = e^{-i\frac{At}{2}}|+\rangle\langle +| + e^{i\frac{At}{2}}|-\rangle\langle -| = \frac{1}{2}\begin{bmatrix} e^{i\frac{At}{2}} + e^{-i\frac{At}{2}} & e^{-i\frac{At}{2}} + e^{i\frac{At}{2}} \\ e^{-i\frac{At}{2}} + e^{i\frac{At}{2}} & e^{i\frac{At}{2}} + e^{-i\frac{At}{2}} \end{bmatrix}$$

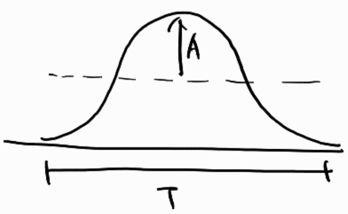
$$= \begin{bmatrix} \cos\left(\frac{At}{2}\right) & -i\sin\left(\frac{At}{2}\right) \\ -i\sin\left(\frac{At}{2}\right) & \cos\left(\frac{At}{2}\right) \end{bmatrix}$$

2. shaping

① square pulse

$AT = \pi \Rightarrow T = \frac{\pi}{A}$, $\left[-\frac{\pi}{2A}, \frac{\pi}{2A}\right]$ 동안 X pulse

② cosine pulse



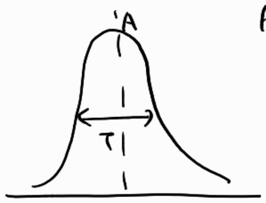
$AT = \pi \Rightarrow T = \frac{\pi}{A}$

$A(t) = A + A\cos\left(\frac{2\pi}{T}t\right)$

$= A + A\cos(2At)$,

$-\frac{\pi}{2A} \le t \le \frac{\pi}{2A}$ 동안 X pulse

③ Gaussian pulse



$$A(t) = \frac{\pi}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2}$$

$\frac{1}{\sigma}\sqrt{\frac{\pi}{2}} = A$,

$\sigma = \frac{1}{A}\sqrt{\frac{\pi}{2}}$ $\quad T = \frac{\sqrt{2\pi}}{A}$

$2\sigma^2 = \frac{\pi}{A^2}$

$$\Rightarrow A(t) = A e^{-\frac{(At)^2}{\pi}}$$

3. CP, CPMG, UDD sequence

CPMG

CP

UDD

$$T = C\cdot\left(\sin\left(\frac{\pi}{N+2}\right) + \sin\left(\frac{2\pi}{N+2}\right) + \cdots + \sin\left(\frac{N+1}{N+2}\pi\right)\right)$$

$$= \frac{C}{\sin\frac{\theta}{2}}\left(\sin\theta\sin\frac{\theta}{2} + \sin 2\theta\sin\frac{\theta}{2} + \cdots + \sin(n+1)\theta\sin\frac{\theta}{2}\right) = C\,\frac{\cos\frac{\theta}{2} - \cos\left(\left(n-\frac{1}{2}\right)\theta\right)}{2\sin\frac{\theta}{2}} = C\,\frac{2\cos\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right)}$$

$\Rightarrow$ time interval : $T\dfrac{\sin\left(\frac{\pi}{2(N+2)}\right)}{\cos\left(\frac{\pi}{2(N+2)}\right)}\sin\left(\frac{\pi}{N+2}\right), \ \ldots$