chap 1. Introduction

group of N particles, probability to select n electrons when each particles' choosing probability is equal to p

$p = \frac{\Delta t}{T}$

$f(n) = {}_N C_n \, p^n (1-p)^{N-n}$ limit $N \to \infty$, $p \to 0$ but $Np = \lambda$

$$f(n) = \lim_{N\to\infty} \frac{N!}{(N-n)!\,n!} \left(\frac{\lambda}{N}\right)^n \left(1-\frac{\lambda}{N}\right)^{N-n} = \lim_{N\to\infty} \frac{N!}{(N-n)!\,n!} \frac{1}{N^n} \lambda^n \left(1-\frac{\lambda}{N}\right)^{N-n} = \frac{\lambda^n}{n!} e^{-\lambda}$$

$\hookrightarrow$ mean $= \lambda$. var $= \lambda$.

Electronic Noise.

$i = \frac{q}{\Delta t} n$, $I = \frac{\bar{n} q}{\Delta t}$ $\quad \overline{i^2} = \left(\frac{q}{\Delta t}\right)^2 \bar{n} = \frac{q}{\Delta t} I$

chap 2. Probability Distribution

Random Variable ...

characteristic function : FT of the density function

- Continuous : $\varphi(j\omega) = \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$, $\quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(j\omega) e^{j\omega x} d\omega$
- discrete : $\varphi(j\omega) = \sum_{n=-\infty}^{\infty} P(n) e^{-j\omega n}$ $\quad P(n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(j\omega) e^{j\omega n} d\omega$

$$\frac{d\varphi}{d\omega} = -j \int_{-\infty}^{\infty} x f(x) e^{-j\omega x} dx$$

$$\frac{d^2\varphi}{d\omega^2} = -\int_{-\infty}^{\infty} x^2 f(x) e^{-j\omega x} dx$$

$$\frac{d^m\varphi}{d\omega^m} = (-j)^m \int_{-\infty}^{\infty} x^m f(x) e^{-j\omega x} dx$$

Sum distribution $\eta = x_1 + x_2 + \cdots + x_n$, mutually independent.

then $\varphi_\eta(\omega) = \int_{-\infty}^{\infty} f(x_1 + x_2 + \cdots + x_n) e^{-j\omega(x_1+x_2+\cdots)} d\eta$

$= M(e^{j\omega\eta}) = M(e^{j\omega x_1}) M(e^{j\omega x_2}) \cdots M(e^{j\omega x_n})$

$= \varphi_1(\omega) \varphi_2(\omega) \cdots \varphi_n(\omega)$

$\Rightarrow f(x_1 + x_2 + \cdots + x_n) = f(x_1) * f(x_2) * \cdots * f(x_n)$

Central Limit Theorem.

mutually independent random variables $x_1, \ldots, x_n$. each $\mu = 0$, $\sigma = \sigma_i$

$\eta = \sum_{i=1}^{n} x_i$ $\quad M(\eta) = \sum_{i=1}^{n} M(x_i) = 0$

$\sigma^2 = D^2(\eta) = \sum_{i=1}^{n} D^2(x_i) = n D^2(x_i) = n\sigma_i^2$

$\varphi(j\omega) = M(e^{jz\omega}) = \left[\varphi_i\left(\frac{\omega}{\sigma}\right)\right]^n$, $\quad \varphi_i\left(\frac{\omega}{\sigma}\right) = M\left(e^{j\frac{\omega}{\sigma} x_i}\right) = 1 + \underbrace{j\frac{\omega}{\sigma} M(x_i)}_{=0} - \frac{1}{2!}\left(\frac{\omega}{\sigma}\right)^2 D^2(x_i) - \underbrace{\frac{j}{3!}\left(\frac{\omega}{\sigma}\right)^3 \mu_3(x_i)}_{=0} + \cdots$

$z = \frac{\eta}{\sigma} = \frac{x_1 + \cdots + x_n}{\sigma}$

$= 1 - \frac{\omega^2}{2n} + \frac{1}{4!}\frac{\omega^4}{n^2} \cdots$,

$\ln\left(\varphi_i\left(\frac{\omega}{\sigma}\right)\right) \cong \ln\left(1 - \frac{\omega^2}{2n}\right) \cong -\frac{\omega^2}{2n}$ if $n \to \infty$ $\quad \Rightarrow \ln(\varphi(\omega)) = n \ln \varphi_i\left(\frac{\omega}{\sigma}\right) = -\frac{\omega^2}{2}$

$$\Rightarrow f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{w^2}{2}} e^{jwx}\,dw, \qquad e^{-\frac{(w-jx)^2}{2}} e^{-\frac{x^2}{2}}$$

$\uparrow \; \frac{x_1 + \cdots + x_n}{n}$

$$\Rightarrow f(x) = e^{-\frac{x^2}{2}} \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{(w-jx)^2}{2}}\,dw$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

chap 3. Stochastic Signals in the time and freq. domain

Specific function $\xi(t)$. realization of stochastic process

Stochastic function 中 stationary : random variable 로 표현 가능. $\mu_n(\xi(t_1)) = \mu_n(\xi(t_n))$

$$\Gamma_\xi(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} \xi(t)\,\xi(t+\tau)\,dt.$$

Power Spectrum

Parseval's relation. $\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(jw)|^2 dw, \qquad S(w) = \lim_{T\to\infty}\frac{1}{4\pi T}|X(jw)|^2$$

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x_T^2(t)\,dt = \lim_{T\to\infty}\frac{1}{4\pi T}\int_{-\infty}^{\infty}|X(jw)|^2 dw = \underbrace{\int_{-\infty}^{\infty} S(w)\,dw}_{\text{power density}}$$

Correlation, Auto Correlation

$$C_{xy} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,(x-M(x))\,(y-M(y))\,dx\,dy$$

independent random variable. $f(x,y) = f(x)f(y)$. $\Rightarrow C_{xy} = \left[\int_{-\infty}^{\infty} f(x)(x-M(x))\,dx\right]\left[\int_{-\infty}^{\infty} f(y)(y-M(y))\,dy\right] = 0$

일반적 경우, $C_{xy} = M(xy) - M(x)M(y)$,

$C_{xx} = M(x^2) - \{M(x)\}^2 = V(x)$.

let $\Gamma_{xy} = M(xy) = C_{xy} + M(x)M(y)$

$$\Gamma_{xy}(\tau) = M(x(t)y(t+\tau)) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)\,y(t+\tau)\,dt.$$

$$\Gamma_x(\tau) = M(x(t)x(t+\tau)) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)\,x(t+\tau)\,dt$$

stationary process, $\lim\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \lim\frac{1}{2T}\int_{-T}^{T} x^2(t+\tau)\,dt$, $\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt \geq \frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)\,dt$

$\tau \to \infty$, $x(t)$ & $x(t+\tau)$ independent. $\Rightarrow \lim_{\tau\to\infty} \Gamma_x(\tau) = 0$.

$$\Gamma_x(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,x(t+\tau)\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-\infty}^{\infty} x_T(t)\,x_T(t+\tau)\,dt$$

$$F\{\Gamma_x(\tau)\} = \int_{-\infty}^{\infty} e^{-j\omega\tau}\left[\frac{1}{2T}\int_{-\infty}^{\infty} x_T(t)\,x_T(t+\tau)\,dt\right]d\tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{2T}\int_{-\infty}^{\infty} x_T(t)\,x_T(t+\tau)\,e^{j\omega t}\,e^{-j\omega(t+\tau)}\,dt\,d\tau$$

$$= \int_{-\infty}^{\infty}\frac{1}{2T}\Bigg[\underbrace{\int_{-\infty}^{\infty} x_T(t+\tau)\,e^{-j\omega(t+\tau)}\,d\tau}_{F_T(\omega)}\Bigg]x_T(t)\,e^{j\omega t}\,dt$$

$$= \frac{1}{2T}F_T(\omega)\,F_T(-\omega) = \frac{1}{2T}|F_T(\omega)|^2 = \lim_{T\to\infty}\frac{1}{2T}|F_T(\omega)|^2 = S(\omega)$$

$$\Rightarrow \boxed{\Gamma_x(\tau) = F^{-1}\{S(\omega)\} = \int_{-\infty}^{\infty} S(\omega)\,e^{j\omega\tau}\,d\omega}$$

Auto Correlation Function Example

$u(t) = U = \text{const} \Rightarrow \Gamma_u(\tau) = U^2$, $\quad S(\omega) = U^2\,\delta(\omega)$

$u(t) = U\sin(\omega_0 t+\varphi) \Rightarrow \Gamma(\tau) = \lim_{T\to\infty}\frac{U^2}{2T}\int_{-T}^{T}\sin(\omega_0 t+\varphi)\sin(\omega_0 t+\omega_0\tau+\varphi)\,dt$

$$= \frac{U^2}{2}\cos\omega_0\tau$$

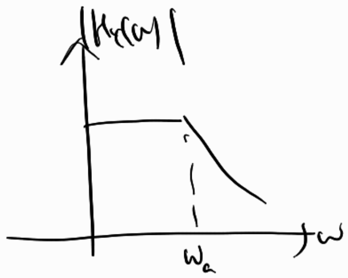
$$S(\omega) = \frac{U^2}{2}\,\frac{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)}{2}$$

$$S(\omega) = \lim_{T\to\infty}\frac{1}{4\pi T}|X(j\omega)|^2 \Rightarrow \text{let } \eta(t) = \int_{-\infty}^{\infty} h(s)\,\xi(t-s)\,ds$$

$$\eta(t)\eta(t+\tau) = \int_{-\infty}^{\infty} h(s)\,\xi(t-s)\,ds\int_{-\infty}^{\infty} h(\theta)\,\xi(t+\tau-\theta)\,d\theta = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(s)\,h(\theta)\,\xi(t-s)\,\xi(t+\tau-\theta)\,d\theta\,ds$$

$$\Rightarrow \Gamma_\eta(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(s)\,h(\theta)\,\Gamma_\xi(\tau-\theta+s)\,d\theta\,ds, \qquad S_\eta(\omega) = |H(j\omega)|^2 S_\xi(\omega)$$

$S_x(\omega) = A$ $\quad S_r(\omega) = \dfrac{A}{H\left(\frac{\omega}{\omega_a}\right)^2} \Rightarrow H_r(\omega) = \dfrac{1}{1+j\frac{\omega}{\omega_a}}$



$\leftarrow \Gamma_n(\tau) = A\omega_a \frac{\pi}{2} e^{-|\omega_a \tau|}$

$h(t) = C e^{-\omega_a t}$

Chap4. The physical Aspects of Noise

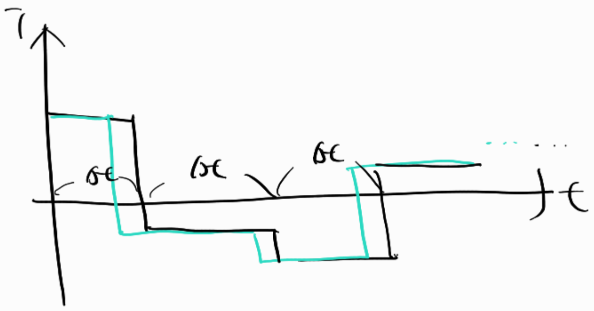
Shot Noise

generated when the charge carriers supplying the current pass through an energy gap.

each carrier band gap 통과 확률 : $\frac{\Delta t}{T} = p$.

$i(t) = \frac{q}{\Delta t} n(t)$, $\quad I = M(n(t)) \frac{q}{\Delta t}$,

$$\Gamma_i = M\left[\Delta n(t)\, \Delta n(t+\tau)\right] = \lim_{T\to\infty} \frac{1}{2T} \left(\frac{q}{\Delta t}\right)^2 \int_{-T}^{T} \Delta n(t)\, \Delta n(t+\tau)\, dt.$$



$\Delta n(t), \Delta n(t+\Delta t), \cdots$ are independent to each other

$\Rightarrow \Gamma_i(\Delta t) = \Gamma_i(-\Delta t) = 0,$

$\Gamma_i(0) = \left(\frac{q}{\Delta t}\right)^2 M(\Delta n^2)$

$$\Rightarrow \Gamma_i(\tau) = \begin{cases} \left(\frac{q}{\Delta t}\right)^2 M(\Delta n^2)\left(1 - \frac{|\tau|}{\Delta t}\right) & (|\tau| \le \Delta t) \\ 0 & (\text{else}) \end{cases},$$

$M(\Delta n^2) = D^2(n) = M(n) = I.$

$$\Rightarrow \Gamma_i(\tau) = \frac{q}{\Delta t} I \left(1 - \frac{|\tau|}{\Delta t}\right),$$

$$S(\omega) = \int_{-\infty}^{\infty} \Gamma_i(\tau) e^{-j\omega\tau} d\tau = \frac{qI}{\Delta t} \int_{-\Delta t}^{\Delta t} e^{-j\omega\tau} d\tau - \frac{qI}{(\Delta t)^2} \left[\underbrace{\int_{-\Delta t}^{0} -\tau e^{-j\omega\tau} d\tau + \int_{0}^{\Delta t} \tau e^{-j\omega\tau} d\tau} \right]$$

$$\frac{qI}{\Delta t} \cdot \frac{e^{j\omega\Delta t} - e^{-j\omega\Delta t}}{-j\omega}$$

$$= -j\frac{d}{d\omega} \int_{-\Delta t}^{0} e^{-j\omega\tau} d\tau + j\frac{d}{d\omega} \int_{0}^{\Delta t} e^{-j\omega\tau} d\tau$$

$$= -j\frac{d}{d\omega} \left[\frac{1}{-j\omega} e^{-j\omega\tau}\right]_{-\Delta t}^{0} + j\frac{d}{d\omega} \left[\frac{1}{-j\omega} e^{-j\omega\tau}\right]_{0}^{\Delta t}$$

$$= \frac{d}{d\omega}\left(\frac{1 - e^{j\omega\Delta t}}{\omega}\right) - \frac{d}{d\omega}\left(\frac{e^{-j\omega\Delta t} - 1}{\omega}\right)$$

$$= \frac{-j\Delta t e^{j\omega\Delta t} - (1 - e^{j\omega\Delta t})}{\omega^2} - \frac{-j\Delta t e^{-j\omega\Delta t} - (e^{-j\omega\Delta t} - 1)}{\omega^2}$$

$$= \frac{e^{j\omega\delta t} - 2 + e^{-j\omega\delta t}}{\omega^2} = 4\,\frac{\sin^2\left(\frac{\omega\delta t}{2}\right)}{\omega^2}$$

$$\Rightarrow S(\omega) = \frac{qI}{\pi}\,\frac{\sin^2\left(\frac{\omega\delta t}{2}\right)}{\left(\frac{\omega\delta t}{2}\right)^2} + \ldots$$