XY drive for superconducting qubits

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In this note, I derive and summarize the relation between the experimental knobs and the XY drive Hamiltonian applied to a system of superconducting qubits.

I. SYNTHESIS OF MICROWAVE SIGNALS

In this section, I describe how the microwave signal for driving superconducting qubits is synthesized with electronic hardware at room temperature and the resulting voltage signal arriving to the qubits. For simplicity, multiplicative factors added during the process (due to e.g., attenuation, amplification, or conversion loss) are ignored.

A. Generation of IF signals from baseband waveforms

The first stage of synthesis of microwave signals happens inside the controller as illustratetd in Fig. 1(a). We specify the real-valued baseband waveforms $A_{\rm I}(t)$ and $A_{\rm Q}(t)$ with 1-ns resolution, which act as envelope functions for real-valued intermediate-frequency (IF) voltage signals $S_{\rm I}(t)$ and $S_{\rm Q}(t)$ that are analog outputs from the instrument. The relation between the specified baseband waveforms and the outputs from digital-to-analog converter (DAC) are given by

$$\begin{bmatrix} S_{\rm I}(t) \\ S_{\rm Q}(t) \end{bmatrix} = \begin{bmatrix} \cos(\omega_{\rm IF}t+\phi) & -\sin(\omega_{\rm IF}t+\phi) \\ \sin(\omega_{\rm IF}t+\phi) & \cos(\omega_{\rm IF}t+\phi) \end{bmatrix} \begin{bmatrix} A_{\rm I}(t) \\ A_{\rm Q}(t) \end{bmatrix}, \tag{1}$$

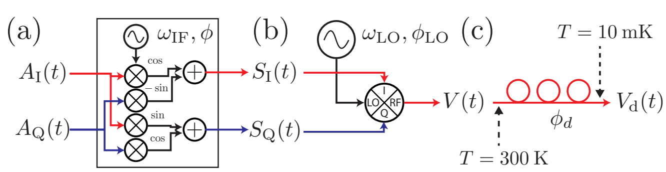

FIG. 1. Synthesis of microwave signals for XY drive of qubits. (a) The baseband waveforms $A_{\rm I}(t)$ and $A_{\rm Q}(t)$ are multiplied to signals from IF oscillator inside DAC instrument and output as IF voltage signals $S_{\rm I}(t)$ and $S_{\rm Q}(t)$. (b) The IF voltage signals are mixed with LO signal at the IQ mixer and are upconverted to a RF signal $V(t)$ at frequency $\omega = \omega_{\rm LO}+\omega_{\rm IF}$. (c) The RF signal synthesized at room temperature propagates to the qubit inside the dilution refrigerator (base temperature $\sim 10\,$mK), collecting phase ϕ_d associated with response from components along the coaxial cables. The signals corresponding to the real (imaginary) part of phasor variables $\boldsymbol{A}(t)$, $\boldsymbol{S}(t)$, $\boldsymbol{V}(t)$, and $\boldsymbol{V}_{\rm d}(t)$ are colored red (blue).

where $\omega_{\rm IF}$ is the angular frequency of the IF signal and ϕ is the angle of frame rotation that we can specify and sweep over in real time (zero if not specified). Note that t is the time of all the DAC output channels (continuously evolving in time unless reset). An equivalent representation of Eq. (1) using complex variables $\boldsymbol{S}(t) \equiv S_{\rm I}(t) + iS_{\rm Q}(t)$ and $\boldsymbol{A}(t) \equiv A_{\rm I}(t) + iA_{\rm Q}(t)$ is given by

$$\boldsymbol{S}(t) = e^{i(\omega_{\rm IF}t+\phi)}\boldsymbol{A}(t). \tag{2}$$

B. Upconversion of signals from IF to RF

The IF signals from DAC are multiplied with local oscillator (LO) signal at the IQ mixer and are upconverted into radio frequency (RF) signal as shown in Fig. 1(b). The upconverted real-valued voltage signal $V(t)$ from the IQ mixer can be written as

$$\begin{aligned} V(t) &= S_{\rm I}(t)\cos(\omega_{\rm LO}t+\phi_{\rm LO}) - S_{\rm Q}(t)\sin(\omega_{\rm LO}t+\phi_{\rm LO}) \\ &= A_{\rm I}(t)\cos(\omega t+\phi+\phi_{\rm LO}) - A_{\rm Q}(t)\sin(\omega t+\phi+\phi_{\rm LO}) \end{aligned} \tag{3}$$

Here, $\omega = \omega_{\rm LO} + \omega_{\rm IF}$ is the upconverted frequency. An equivalent complex variable representation with $V(t) = {\rm Re}\,[\boldsymbol{V}(t)]$ is given by

$$\boldsymbol{V}(t) = e^{i(\omega_{\rm LO}t+\phi_{\rm LO})}\boldsymbol{S}(t) = e^{i(\omega t+\phi+\phi_{\rm LO})}\boldsymbol{A}(t). \tag{4}$$

C. Propagation to the qubit

The RF signal $V(t)$ synthesized at room temperature propagates to the XY drive line of the qubit inside the dilution refrigerator after passing through multiple coaxial cables and various microwave components such as amplifiers, attenuators, and filters as illustrated in Fig. 1(c). The ideal net effect of propagation is frequency-dependent group delay t_d resulting in a phase shift. Then, the qubit drive voltage $V_{\rm d}(t)$ can be written in the form

$$\begin{aligned} V_{\rm d}(t) &= V_{\rm RF}(t-t_d) \\ &= A_{\rm I}(t)\cos(\omega t+\phi+\phi_d) - A_{\rm Q}(t)\sin(\omega t+\phi+\phi_d). \end{aligned}$$

where ϕ_d is the sum of phases that we don't have control over.

Equivalent complex variable representation with $V_\mathrm{d}(t) = \mathrm{Re}[\boldsymbol{V}_\mathrm{d}(t)]$ is given by

$$\boldsymbol{V}_\mathrm{d}(t) = e^{i\phi_d}\boldsymbol{V}(t) = e^{i(\omega t+\phi+\phi_\mathrm{LO}+\phi_d)}\boldsymbol{A}(t).$$

D. Example: amplitude- and phase-modulated baseband waveforms

Let us consider the most general case of baseband waveforms $A_\mathrm{I}(t) = a(t)\cos[\theta(t)]$ and $A_\mathrm{Q}(t) = a(t)\sin[\theta(t)]$ [complex representation $\boldsymbol{A}(t) = a(t)e^{i\theta(t)}$], which represents amplitude modulation with $a(t)$ and phase modulation with $\theta(t)$. The resulting IF outputs from DAC are given by

$$\begin{aligned} S_\mathrm{I}(t) &= a(t)\cos[\omega_\mathrm{IF}t+\phi+\theta(t)], \\ S_\mathrm{Q}(t) &= a(t)\sin[\omega_\mathrm{IF}t+\phi+\theta(t)]. \end{aligned}$$

or

$$\boldsymbol{S}(t) = a(t)e^{i[\omega_\mathrm{IF}t+\phi+\theta(t)]}.$$

After upconversion to RF, the signal becomes

$$V(t) = a(t)\cos[\omega t+\phi+\theta(t)+\phi_\mathrm{LO}].$$

or

$$\boldsymbol{V}(t) = a(t)e^{i[\omega_\mathrm{IF}t+\phi+\theta(t)+\phi_\mathrm{LO}]}.$$

After propagation to the qubit, the drive voltage is given by

$$V_\mathrm{d}(t) = a(t)\cos[\omega t+\varphi(t)], \tag{5}$$

where $\varphi(t) = \phi+\theta(t)+\phi_\mathrm{LO}+\phi_d$ is the sum of all phases.

II. DRIVE HAMILTONIAN OF SUPERCONDUCTING QUBITS

The Hamiltonian of a superconducting transmon qubit coupled to a local drive line with time-dependent voltage $V_\mathrm{d}(t)$ is given by [1]

$$\hat{H} \approx \frac{\hat{Q}^2}{2C_\Sigma} + \frac{\hat{\Phi}^2}{2L_J} - \frac{E_J}{24}\left(\frac{2e}{\hbar}\right)^4\hat{\Phi}^4 + \frac{C_\mathrm{d}}{C_\Sigma}V_\mathrm{d}(t)\hat{Q} \tag{6}$$

where C_Σ is the effective capacitance of qubit, C_d is the capacitance between qubit and the drive line, E_J is the Josephson energy ($L_J = [E_J(2e/\hbar)^2]^{-1}$ is the corresponding Josephson inductance), and $\Phi_0 = h/2e$ is the magnetic flux quantum. Here, $\hat{Q}$ is the charge operator canonically conjugate to the flux operator $\hat{\Phi}$, satisfying the commutation relation $[\hat{\Phi},\hat{Q}] = i\hbar$.

We can introduce annihilation operator $\hat{a}$ satisfying the commutation relation $[\hat{a},\hat{a}^\dagger] = 1$, allowing us to write the flux and the charge operators as

$$\hat{\Phi} = \Phi_\mathrm{zpf}(\hat{a}+\hat{a}^\dagger), \quad \hat{Q} = -iQ_\mathrm{zpf}(\hat{a}-\hat{a}^\dagger) \tag{7}$$

where $\Phi_\mathrm{zpf} = \sqrt{\hbar Z_q/2}$ and $Q_\mathrm{zpf} = \sqrt{\hbar/2Z_q}$ are zero-point fluctuations of the flux and the charge operators defined with impedance

$$Z_q = \sqrt{\frac{L_J}{C_\Sigma}} = \frac{\hbar}{e^2}\sqrt{\frac{E_C}{2E_J}}$$

of the qubit. Here, $E_C = e^2/2C_\Sigma$ is the charging energy of the qubit. In this case, unperturbed part of the Hamiltonian is simplified as

$$\hat{H}_0/\hbar = \omega_q\hat{a}^\dagger\hat{a} + \frac{\alpha}{2}\hat{a}^\dagger\hat{a}(\hat{a}^\dagger\hat{a}-1), \tag{8}$$

where $\omega_q = (\sqrt{8E_JE_C}-E_C)/\hbar$ is the 0–1 transition frequency of the qubit, $\alpha = -E_C/\hbar$ is the anharmonicity [2].

The driving Hamiltonian $\hat{H}_\mathrm{d}(t)$ under drive voltage $V_\mathrm{d}(t) = a_\mathrm{d}(t)\cos[\omega_\mathrm{d}t+\varphi_\mathrm{d}(t)]$ with amplitude modulation $a_\mathrm{d}(t)$ and phase modulation $\varphi_\mathrm{d}(t)$ can be written as

$$\hat{H}_\mathrm{d}(t)/\hbar = -i\Omega_\mathrm{d}(t)\cos[\omega_\mathrm{d}t+\varphi_\mathrm{d}(t)](\hat{a}-\hat{a}^\dagger),$$

where the magnitude of the drive is given by

$$\hbar\Omega_\mathrm{d}(t) = e\frac{C_\mathrm{d}}{C_\Sigma}\left(\frac{E_J}{2E_C}\right)^{1/4}a_\mathrm{d}(t). \tag{9}$$

Moving to a frame rotating with the drive frequency ω_d, the Hamiltonian transforms into $\tilde{H} = UHU^\dagger - i\hbar U\partial_tU^\dagger$ with $U(t) = \exp(i\omega_\mathrm{d}t\hat{a}^\dagger\hat{a})$, resulting in unperturbed qubit Hamiltonian

$$\tilde{H}_0/\hbar = (\omega_q-\omega_\mathrm{d})\hat{a}^\dagger\hat{a} + \frac{\alpha}{2}\hat{a}^\dagger\hat{a}(\hat{a}^\dagger\hat{a}-1) \tag{10}$$

and drive Hamiltonian

$$\begin{aligned} \tilde{H}_\mathrm{d}(t)/\hbar &\approx -i\frac{\Omega_\mathrm{d}(t)}{2}\left[\hat{a}e^{i\varphi_\mathrm{d}(t)} - \hat{a}^\dagger e^{-i\varphi_\mathrm{d}(t)}\right] \\ &= \Omega_\mathrm{d}(t)\left[\hat{x}\sin\varphi_\mathrm{d}(t) + \hat{p}\cos\varphi_\mathrm{d}(t)\right], \end{aligned} \tag{11}$$

where $\hat{x} = (\hat{a}+\hat{a}^\dagger)/2$ and $\hat{p} = -i(\hat{a}-\hat{a}^\dagger)/2$ are canonical coordinates of the phase space. Defining the zero- and one-photon Fock states as the ground and excited states of qubit manifold, $|g\rangle \equiv |0\rangle$, $|e\rangle \equiv |1\rangle$, the Pauli operators for atomic pseudospin-1/2 are defined as [3]

$$\hat{\sigma}_x = |e\rangle\langle g| + |g\rangle\langle e|, \tag{12a}$$
$$\hat{\sigma}_y = -i|e\rangle\langle g| + i|g\rangle\langle e|, \tag{12b}$$
$$\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|. \tag{12c}$$

According to this definition, the ±1 eigenstates of $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ are given by

$$|\pm\rangle_{\sigma_x} = \frac{|g\rangle \pm |e\rangle}{\sqrt{2}}, \quad |\pm\rangle_{\sigma_y} = \frac{|g\rangle \mp i|e\rangle}{\sqrt{2}} \tag{13a}$$
$$|+\rangle_{\sigma_z} = |e\rangle, \quad |-\rangle_{\sigma_z} = |g\rangle. \tag{13b}$$

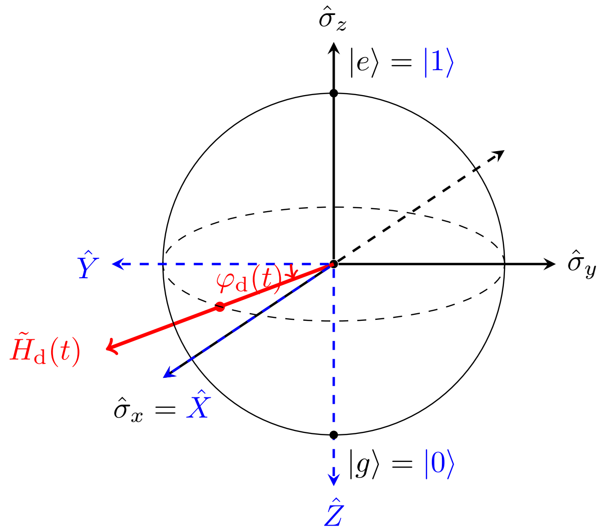

FIG. 2. Bloch sphere representation of local XY drive Hamiltonian $\tilde{H}_\mathrm{d}(t)$ of a qubit defined in Eq. (14b). The solid black (blue dashed) lines with arrows represent coordinate axes of the Bloch sphere using the convention of Pauli operators of atomic pseudospin-1/2 (quantum computation), whose north and south poles correspond to states $|e\rangle$ and $|g\rangle$ ($|0\rangle$ and $|1\rangle$), respectively.

Under this definition, the Hamiltonian in the frame rotating with the drive frequency can be written in the qubit manifold as

$$\tilde{H}_0/\hbar = \frac{\omega_q - \omega_\mathrm{d}}{2}\hat{\sigma}_z \tag{14a}$$

$$\tilde{H}_\mathrm{d}(t)/\hbar = \frac{\Omega_\mathrm{d}(t)}{2}\left[\hat{\sigma}_x \sin\varphi_\mathrm{d}(t) - \hat{\sigma}_y \cos\varphi_\mathrm{d}(t)\right]. \tag{14b}$$

Note that defining the same Fock basis states $\{|0\rangle, |1\rangle\}$ as quantum computational basis lead to a *different* definition of Pauli operators [4] written as

$$\hat{X} = |0\rangle\langle 1| + |1\rangle\langle 0|, \tag{15a}$$

$$\hat{Y} = -i|0\rangle\langle 1| + i|1\rangle\langle 0|, \tag{15b}$$

$$\hat{Z} = |0\rangle\langle 0| - |1\rangle\langle 1|, \tag{15c}$$

where the y and z axes are flipped from the definition in Eqs. (12a)-(12c), giving $\hat{X} = \hat{\sigma}_x$, $\hat{Y} = -\hat{\sigma}_y$, and $\hat{Z} = -\hat{\sigma}_z$. According to this computational definition, the ± 1 eigenstates of $\hat{X}$, $\hat{Y}$, and $\hat{Z}$ are given by

$$|\pm\rangle_X = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}, \quad |\pm\rangle_Y = \frac{|0\rangle \pm i|1\rangle}{\sqrt{2}} \tag{16a}$$

$$|+\rangle_Z = |0\rangle, \quad |-\rangle_Z = |1\rangle. \tag{16b}$$

Both conventions are being widely used, so care must be taken in describing what we mean by Pauli operators and gates. The action of drive Hamiltonian in the rotating frame is illustrated in the Bloch sphere illustrated in Fig. 2.

III. PHASE COHERENCE OF MULTI-QUBIT DRIVE

In this section, we discuss the case of simultaneously driving multiple qubits utilizing individual drive lines separately synthesized at room temperature and how phase coherence between the multiple XY lines could be achieved.

In the experiment, we specify the baseband waveforms $A_{\mathrm{I},k}(t)$, $A_{\mathrm{Q},k}(t)$, IF frequency $\omega_{\mathrm{IF},k}$ and phase ϕ_k for each qubit k and the clocks for all IF oscillators are synchronized. All the XY lines share an identical LO but the delay introduced by lines add unknown phases to the microwave signals when they arrive to the qubits. In this case, without loss of generality, the drive voltages on the qubits can be written as

$$V_{\mathrm{d},k}(t) = a_k(t)\cos\left[(\omega_\mathrm{LO} + \omega_{\mathrm{IF},k})t + \varphi_k(t)\right], \tag{17}$$

where the phase $\varphi_k(t)$ can always be written as $\varphi_k(t) = \phi_k + \theta_k(t) + \varphi_{0,k}$. Here, $\varphi_{0,k} \equiv \phi_\mathrm{LO} + \phi_{d,k}$ is the sum of initial phase of LO and a constant added phase for qubit drive line k.

For the phase coherence of all the XY pulses, we would like the phase of microwave signals for all qubits to be constant at the beginning of each pulse sequence (realization of the experiment). For example, suppose that the m-th experiment E_m starts at an absolute time $t = t_m$, then the time relative to the start of this experiment is given by $\tau_m = t - t_m$. We want all the microwave signals $V_{\mathrm{d},k}(t)$ to start with identical phase when $\tau_m = 0$ (or $t = t_m$) which requires the argument inside the cosine function in Eq. (17) to be constant at the beginning of all experimental realizations E_m. i.e.,

$$\omega_\mathrm{LO} t_m + \omega_{\mathrm{IF},k} t_m + \varphi_k(t_m) = \mathrm{const}(k) \quad \text{for all } m,\, k.$$

While it is easy to set $\varphi_k(t_m)$ to be constant for all m by starting from identical modulated phase $\theta_k(t_m)$ at each sequence, finding times $t = t_m$ where $\omega_\mathrm{LO} t_m + \omega_{\mathrm{IF},k} t_m = \mathrm{const}(k)$ is challenging since the drive frequencies are in general incommensurate. Therefore it is necessary to reset the phase of all IF oscillators and the LO at the beginning $t = t_m$ of each sequence in order to start exactly at the identical phase between runs of experiments.

However, in case of using a common LO for upconversion, it is sufficient to only reset the phases of all IF oscillators at the beginning of the sequence without need for resetting the phase of LO. This is because if we reset phases of all IF oscillators at the beginning $t = t_m$ of each realization E_m of an experiment and perform amplitude a_k and phase modulation θ_k according to the relative time $\tau_m = t - t_m$, the drive voltage applied to qubits takes the identical form as in Eq. (17)

$$V_{\mathrm{d},k}(\tau_m) = a_k(\tau_m)\times \cos\left[(\omega_\mathrm{LO} + \omega_{\mathrm{IF},k})\tau_m + \varphi_k(\tau_m) + \phi_{\mathrm{LO},m}\right], \tag{18}$$

with additional running phase $\phi_{\mathrm{LO},m} = \omega_\mathrm{LO} t_m$ of LO independent of qubit index k. Since this factor doesn't affect the relative phase of drive between qubits, it can be gauged out in the representation of the Hamiltonian, without loss of generality.

[1] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, A quantum engineer's guide to superconducting qubits, Appl. Phys. Rev. **6**, 021318 (2019).

[2] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Charge-insensitive qubit design derived from the cooper pair box, Phys. Rev. A **76**, 042319 (2007).

[3] D. A. Steck, *Quantum and Atom Optics*, available online at `http://steck.us/teaching` (revision 0.13.10, 22 September 2021).

[4] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).