#1. ① Quantum Coin state $|s\rangle \in H_c = \{a_0|0\rangle + a_1|1\rangle : a_0, a_1 \in \mathbb{C}\}$

② Fair Coin Operator

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}\langle 0| + \frac{|0\rangle - |1\rangle}{\sqrt{2}}\langle 1|$$

③ Four sided Quantum Coin state $|s'\rangle \in H_c' = \{a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle : a_0, a_1, a_2, a_3 \in \mathbb{C}\}$

④ Four sided Fair Coin Operator

$$H' = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} = \frac{|0\rangle + |1\rangle + |2\rangle + |3\rangle}{\sqrt{4}}\langle 0| + \frac{|0\rangle + i|1\rangle - |2\rangle - i|3\rangle}{\sqrt{4}}\langle 1| + \frac{|0\rangle - |1\rangle + |2\rangle - |3\rangle}{\sqrt{4}}\langle 2| + \frac{|0\rangle - i|1\rangle - |2\rangle + i|3\rangle}{\sqrt{4}}\langle 3|$$

(Fourier Coin)

⑤ Fair check

$|H_{ij}| = |\langle i|H|j\rangle| = \frac{1}{\sqrt{2}}$, all matrix elements of H has same absolute value $\frac{1}{\sqrt{N}}$,

$|H'_{ij}| = |\langle i|H'|j\rangle| = \frac{1}{\sqrt{4}}$ thus unbiased coin.

#2. location on the board

$$|\psi\rangle \in H_p = \{\underbrace{\sum_{k=0}^{15} a_k|k\rangle}_{16\text{개 state}} : a_0, \cdots, a_{15} \in \mathbb{C}\}$$

encoding by $|j_3 j_2 j_1 j_0\rangle$

$|0000\rangle := |0\rangle$

$|0001\rangle := |1\rangle$

$\vdots$

$|1111\rangle := |15\rangle$

#3 quantum coin with shift operator

$$R = \begin{bmatrix} 0 & 0 & \cdots & & 1 \\ 1 & 0 & 0 & & 0 \\ \vdots & 1 & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad L = \begin{bmatrix} 0 & 1 & \cdots & & 0 & 0 \\ 0 & 0 & 1 & & & \vdots \\ \vdots & & 0 & \ddots & & \vdots \\ \vdots & & & & \ddots & \\ 0 & & & & & 1 \\ 1 & & \cdots & & & 0 \end{bmatrix}$$

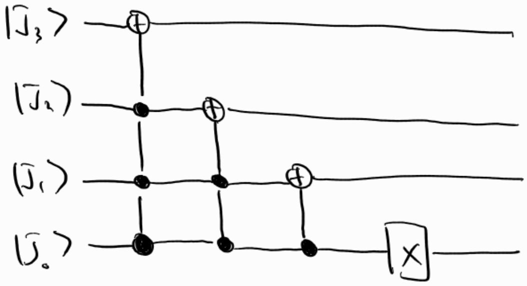
$$L = |15\rangle\langle 0| + \sum_{k=0}^{14} |k\rangle\langle k+1|$$

$$= |1111\rangle\langle 0000| + |0000\rangle\langle 0001| + |0001\rangle\langle 0010| + \cdots + |1110\rangle\langle 1111|$$

$$R = |0\rangle\langle 15| + \sum_{k=0}^{14} |k+1\rangle\langle k|$$

$$= |0000\rangle\langle 1111| + |0001\rangle\langle 0000| + |0010\rangle\langle 0001| + \cdots + |1111\rangle\langle 1110|$$

이 중 R을 사용



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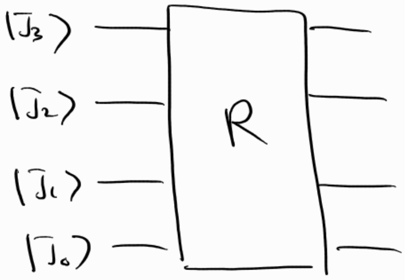


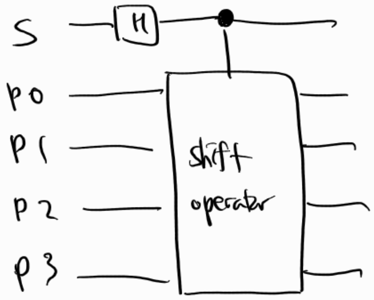
$$R = \left[I \otimes I \otimes I \otimes X\right]\left[I \otimes I \otimes \left(I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|\right)\right]$$

$$\left[I \otimes \left(I \otimes |00\rangle\langle 00| + I \otimes |01\rangle\langle 01| + I \otimes |10\rangle\langle 10| + X \otimes |11\rangle\langle 11|\right)\right]$$

$$\left[I \otimes |000\rangle\langle 000| + I \otimes |001\rangle\langle 001| + I \otimes |010\rangle\langle 010| + \cdots + X \otimes |111\rangle\langle 111|\right]$$

$$= \begin{bmatrix} 0 & 0 & \cdots & & 1 \\ 1 & 0 & 0 & & \vdots \\ & 1 & 0 & 0 & \vdots \\ & & 1 & \ddots & \\ \vdots & & & \ddots & 0 \; 0 \\ 0 & & \cdots & & 1 \; 0 \end{bmatrix}$$

when $\underbrace{|0000\rangle}_{|0\rangle} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $\underbrace{|0001\rangle}_{|1\rangle} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$, $\cdots$, $\underbrace{|1111\rangle}_{|15\rangle} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$

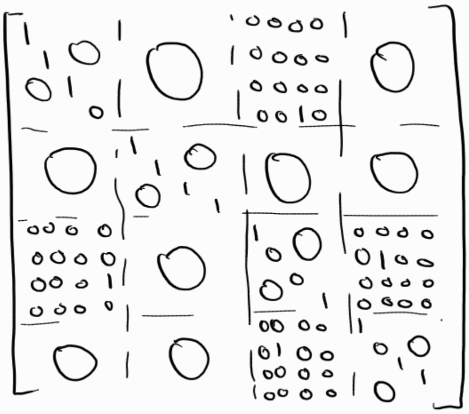


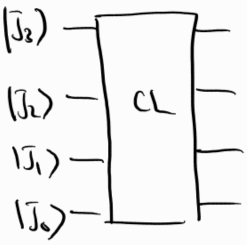
N repetition.

#4. Chutes & Ladder Operator : CL

$$CL = \sum_{k=0,\, k\neq 3,9,10,13}^{15} |k\rangle\langle k| + |10\rangle\langle 3| + |3\rangle\langle 10| + |13\rangle\langle 9| + |9\rangle\langle 13|$$

$3 \leftrightarrow 10$, $9 \leftrightarrow 13$





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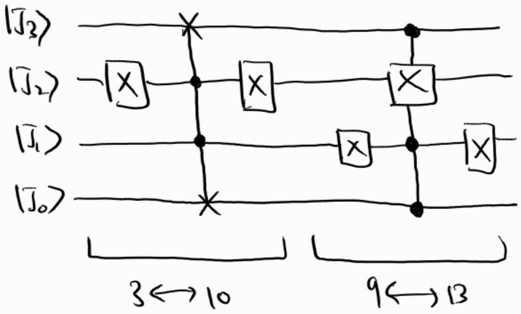

$|0011\rangle\langle 1010| + |1010\rangle\langle 0011|$

$|1001\rangle\langle 1101| + |1101\rangle\langle 1001|$

1 step