IonQ-SKKU Challenge
Your name here
Fall 2023

Problem Set 1 – Solution

1. *(Spin chains 101.)* Show that the spin interaction terms acting on a common pair of sites along different axes commute. Concretely, let

$$[A, B] = AB - BA$$

denote the *commutator* of A and B, let $S_j^\alpha = \sigma_j^\alpha \sigma_{j+1}^\alpha$ denote the spin-spin interaction term at the $(j, j+1)$-st sites along the α direction, and show that

$$[S_j^\alpha, S_j^\beta] = 0.$$

Now show that, conversely, spin interaction terms at adjacent pairs of sites do **not** commute; that is, prove that

$$[S_j^\alpha, S_{j+1}^\beta] \neq 0.$$

Solution

Fill this in!

2. *(Schrodinger dynamics.)* Suppose A is a complex $m \times m$ matrix. In addition, suppose that for each $t \geq 0$, $z(t)$ is a complex m-vector. Use the defining series for the matrix exponential to show that $z(t) = e^{tA} z(0)$ is a solution to

$$z'(t) = Az(t).$$

Solution

Fill this in!

3. *(Foundations: $N = 2$ case.)*

(a) Use the binomial theorem to show that if A and B are commuting $m \times m$ matrices then

$$e^A e^B = e^{A+B}.$$

In addition, show that this generally fails if A and B do **not** commute; that is, find two matrices A, B such that $[A, B] \neq 0$ and

$$e^A e^B \neq e^{A+B}.$$

Solution

Fill this in!

(b) Write $\mathcal{H}_S = \mathcal{H}_{\text{even}} + \mathcal{H}_{\text{odd}}$ as a sum of even- and odd-indexed spin interaction terms.

Briefly explain why the summands in H_{even} (H_{odd}) pairwise commute; in other words, justify the following equalities:

$$e^{-it\mathcal{H}_{\text{even}}} = \prod_j B_{2j}(\theta) \quad \text{and similarly} \quad e^{-itH_{\text{odd}}} = \prod_j B_{2j-1}(\theta),$$

with B_j acting as the block operator on the $(j, j+1)$-st sites.

Solution

Fill this in!

(c) What is the depth of the circuit illustrated above? Compare this to the depth of the circuit that directly Trotterizes

$$U(t) \approx \left(\prod_j B_j(\theta/n) \right)^n.$$

Solution

Fill this in!

4. *(Staggered magnetization.)* Suppose $\mathcal{O}$ is a Hermitian observable whose matrix with respect to the computational basis is diagonal. Let λ_x denote the eigenvalue of $\mathcal{O}$ corresponding to the computational basis state $|x\rangle$. Let $|\psi\rangle = \sum_x \alpha_x |x\rangle$ denote any quantum state, written as a superposition over the computational basis, and let $p_x = |\alpha_x|^2$ denote the probability of observing $|\psi\rangle$ in the state $|x\rangle$.

Show that the expectation value

$$\langle\psi| \mathcal{O} |\psi\rangle = \sum_x p_x \lambda_x$$

is simply a weighted average of the eigenvalues of $\mathcal{O}$.

Solution

Fill this in!

5. *(YBE-powered compression.)*

   (a) Comment on your results and include your plots showing four magnetization curves on the same set of axes for each N.

   Solution

   Fill this in!

   (b) Use Matsumoto's Monoid Lemma or an explicit basis of the (finite-dimensional) 0-Hecke algebra to prove our compression scheme existence theorem.

   Solution

   Fill this in!