#1. $\vec{V} = \sum V_i \vec{e_i}$ (8 dimensional vector) $= [1,5,2,6,3,7,4,8]$

$$R_x = e^{-i\frac{\theta}{2}X} \quad X = |+X+| - |-X-| \rightarrow R_x = e^{-i\frac{\theta}{2}}|+X+| + e^{i\frac{\theta}{2}}|-X-|$$

$$R_x = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \qquad P_i = |\langle 1 | R_x^{(i)}(f(v_i)) | 0 \rangle|^2$$

vector length $= 8$

$$[1,5,2,6,3,7,4,8] \rightarrow \left[\frac{1}{24}, \frac{5}{2}, \frac{2}{2}, \frac{6}{2}, \frac{3}{2}, \frac{7}{2}, \frac{4}{2}, 1\right] \quad Z = \sqrt{204}, \text{ norm}$$

divide each entry by Length (8) to make each entry between 0 and 1

$$\langle 1 | R_x(\theta) | 0 \rangle = [0 \quad 1] \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -i\sin\frac{\theta}{2}, \quad P_i = \sin^2\frac{\theta}{2}, \quad i = 0,1,\cdots,7$$

encoding method : $|\psi\rangle = \prod_{i=0}^{7} R_x^i(f(v_i))|0\rangle$, when $P_i = \sin^2\left(\frac{f(v_i)}{2}\right) = \frac{V_i}{\text{norm}(V)}$

$$\Rightarrow f(v_i) = 2\arcsin\left(\sqrt{\frac{V_i}{\text{norm}(V)}}\right)$$

$$f(v_0) = 2\arcsin\left(\frac{1}{2\sqrt{2}}\right) \simeq 0.723$$

$$f(v_1) = 2\arcsin\left(\frac{\sqrt{5}}{2\sqrt{2}}\right) \simeq 1.82$$

$\vdots$

$$f(v_7) = 2\arcsin(1) \simeq \pi$$

#2 $Z = |V| = \sqrt{204}$

$$|\psi\rangle = \frac{1}{Z}\sum_{i=0}^{7} V_i |i\rangle = \frac{1}{\sqrt{204}}\left(1|000\rangle + 5|001\rangle + 2|010\rangle + 6|011\rangle + 3|100\rangle + 7|101\rangle + 4|110\rangle + 8|111\rangle\right)$$

$$\frac{1}{\sqrt{204}}\begin{bmatrix} 1 \\ 5 \\ 2 \\ 6 \\ 3 \\ 7 \\ 4 \\ 8 \end{bmatrix} = U|0\rangle = \frac{1}{\sqrt{204}}\begin{bmatrix} 1 & & \\ 5 & & \\ 2 & & \\ 6 & ? & \\ 3 & & \\ 7 & & \\ 4 & & \\ 8 & & \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f = \{[1,0,0,\cdots,0], [0,1,0,\cdots,0], \cdots, [0,0,\cdots,0,1]\}$$

$$g = \left\{\frac{1}{\sqrt{204}}[1,5,2,6,3,7,4,8], \cdots \right\}$$

$$1\cdot x_0 + 5x_1 + 2x_2 + 6x_3 + 3x_4 + 7x_5 + 4x_6 + 8x_7 = 0. \quad x_0 = -5x_1 - 2x_2 - 6x_3 - 3x_4 - 7x_5 - 4x_6 - 8x_7$$

$$
\begin{bmatrix}
1 & -5 & 0 & 0 & 0 & -10 & 0 & 0 \\
5 & 1 & 0 & 0 & 0 & -50 & 0 & 0 \\
2 & 0 & -6 & 0 & 0 & 0 & -1 & 0 \\
6 & 0 & 2 & 0 & 0 & 0 & -3 & 0 \\
3 & 0 & 0 & -1 & 0 & 0 & 0 & -30 \\
7 & 0 & 0 & 3 & 0 & 0 & 0 & -70 \\
4 & 0 & 0 & 0 & -8 & 13 & 1 & 29 \\
8 & 0 & 0 & 0 & 4 & 26 & 2 & 58
\end{bmatrix}
$$

$x_1 = 5x_0$

$x_3 = 3x_2$

$x_5 = \frac{7}{3}x_4$

$x_7 = 2x_6$

$3 + \frac{49}{3} \quad \frac{58}{3}$

$x_0 + 5x_1 + 2x_2 + 6x_3 + 3x_4 + 7x_5 + 4x_6 + 8x_7 = 0$

$26x_0 + 20x_2 + \frac{58}{3}x_4 + 20x_6 = 0$

$x_6 = -\frac{13}{10}x_0 - x_2 - \frac{29}{30}x_4$

Unitary Matrix

$$
\left[\frac{1}{Z_1}\begin{bmatrix} 1 \\ 5 \\ 2 \\ 6 \\ 3 \\ 7 \\ 4 \\ 8 \end{bmatrix}, \frac{1}{Z_2}\begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{Z_3}\begin{bmatrix} 0 \\ 0 \\ -6 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{Z_4}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{Z_5}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -8 \\ 4 \end{bmatrix}, \frac{1}{Z_6}\begin{bmatrix} -1 \\ -5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 13 \\ 26 \end{bmatrix}, \frac{1}{Z_7}\begin{bmatrix} \frac{10}{53} \\ \frac{50}{53} \\ -1 \\ -3 \\ 0 \\ 0 \\ \frac{40}{53} \\ \frac{80}{53} \end{bmatrix}, \frac{1}{Z_8}\begin{bmatrix} \frac{290}{73} \\ \frac{1450}{73} \\ \frac{580}{73} \\ \frac{1740}{73} \\ -30 \\ -70 \\ \frac{1160}{73} \\ \frac{2320}{73} \end{bmatrix} \right]
$$

#3. a) Task 1. ┌ pros : only 8 single rotating gates. Low cost in gate number

└ cons : one-hot encoding ⇒ 8 qubits used. High cost in qubit number

Task 2 ┌ pros : only 3 qubits used to encoding. Low cost in qubit number
binary encoding

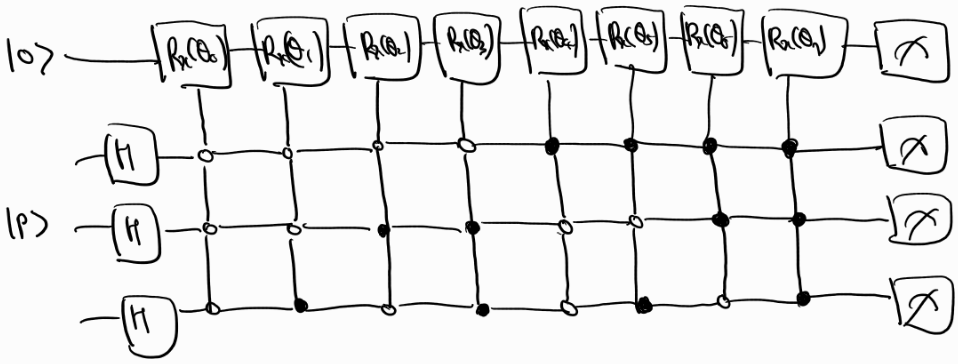
└ cons : Many many gates to construct unitary matrix

b) $|\psi\rangle = \sum_{i=0}^{n-1} \left(R_x^i(f(v_i))|0\rangle\right)|i\rangle$

$= \sum_{i=0}^{n-1} \left[\cos\left(\frac{f(v_i)}{2}\right)|0\rangle - i\sin\left(\frac{f(v_i)}{2}\right)|1\rangle\right]|i\rangle$, let $|i\rangle$ be position state $|p\rangle$

for $V = [1,5,2,6,3,7,4,8]$,

$|\psi\rangle = \frac{1}{2\sqrt{2}}\left(\left[\cos\left(\frac{f(v_0)}{2}\right)|0\rangle - i\sin\left(\frac{f(v_0)}{2}\right)|1\rangle\right]|000\rangle + \cdots + \left[\cos\left(\frac{f(v_7)}{2}\right)|0\rangle - i\sin\left(\frac{f(v_7)}{2}\right)|1\rangle\right]|111\rangle\right)$



#4

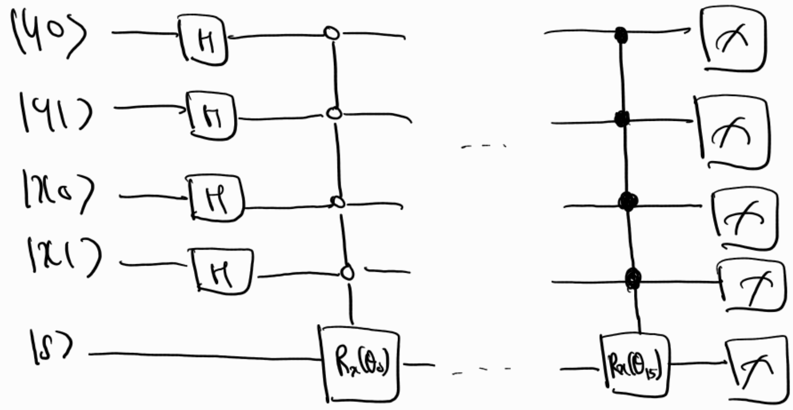
	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
$\|00\rangle$	0	0.5	0	0
$\|01\rangle$	0.8	1	0.8	0
$\|10\rangle$	0	0.5	0	0
$\|11\rangle$	0	0	0	0

$$|\psi\rangle = \frac{1}{\sqrt{2^4}} \sum_{0}^{3} \sum_{0}^{3} |f(y,x)\rangle \otimes |y\rangle|x\rangle$$

$R_x(f(y,x))|0\rangle$

2 qubits

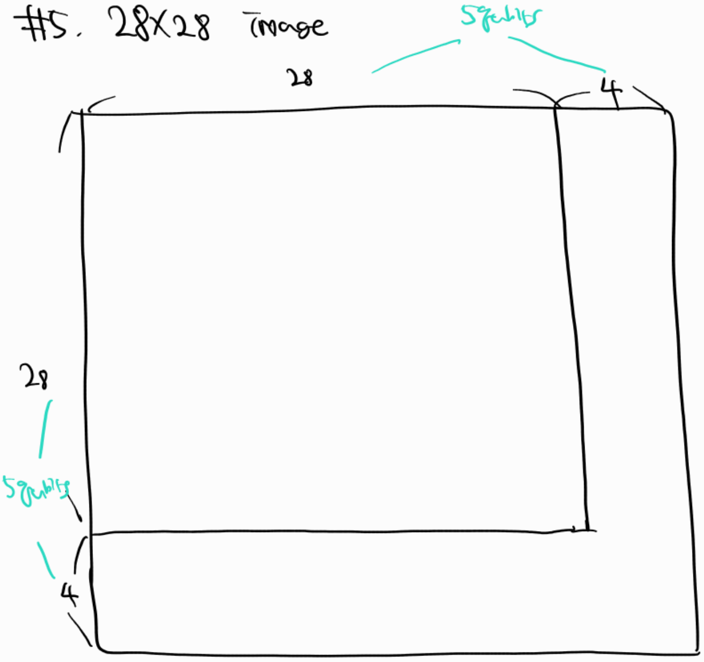
2 qubits



$\theta_i = f(v_i) = 2\arcsin(\sqrt{v_i})$

0, 0.5, 0.8, ⋯ etc

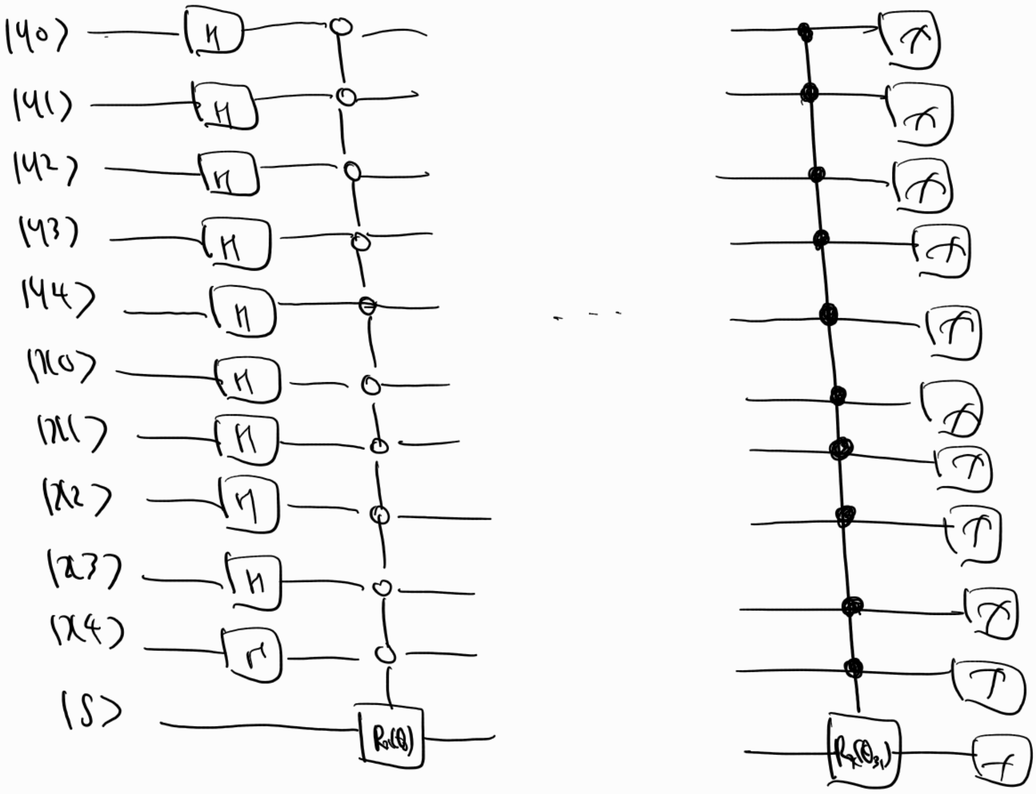
#5. 28X28 Image



1 qubit 5 qubits 5 qubits

$$|\psi\rangle = \frac{1}{\sqrt{2^5}} \sum_{y=0}^{2^5-1} \sum_{x=0}^{2^5-1} |f(y,x)\rangle \otimes |y\rangle|x\rangle$$

$R_x(f(x,y))|0\rangle$


|y0⟩
|y1⟩
|y2⟩
|y3⟩
|y4⟩
|x0⟩
|x1⟩
|x2⟩
|x3⟩
|x4⟩
|s⟩
H
Ry(θ)
Ry(θ31)